Гравитационный поезд из Москвы в Питер (Gravity Train)

Cauchy problem

$$eq := \begin{cases} m \cdot x''(t) = -m \cdot g \cdot \dfrac{x(t)}{R} \\ x(0) = -\dfrac{L}{2} \\ D(x)(0) = 0 \end{cases}$$

$$x'(t) := \frac{d}{dt} x(t) \qquad x''(t) := \frac{d^2}{dt^2} x(t)$$

$$x(t) := \text{maple}\left[rhs\left[dsolve\left(\begin{pmatrix} eq_1 \\ eq_2 \\ eq_3 \end{pmatrix} \right) \right] \right] = -\frac{L \cdot \cos\left(\frac{\sqrt{g \cdot R} \cdot t}{R} \right)}{2}$$

Check

$$eq_1 = \frac{m \cdot L \cdot g \cdot \cos\left(\frac{\sqrt{g \cdot R} \cdot t}{R} \right)}{2 \cdot R} = \frac{m \cdot g \cdot L \cdot \cos\left(\frac{\sqrt{g \cdot R} \cdot t}{R} \right)}{2 \cdot R}$$

$x(0) = -\frac{L}{2}$

$x'(0) = 0$

Travel time

$$\tau := \text{maple}\left(\text{solve}\left(x(t) = -x(0),\ t \right) \right) = \frac{\pi \cdot R}{\sqrt{g \cdot R}}$$

$x(\tau) = \frac{L}{2}$

$x'(\tau) = 0$

Values

$R := 6371\ \text{km}$ $g := g_e$ $\boxed{\tau = 42.2029\ \text{min}}$

Earth

$$Earth(\theta, \varphi) := \begin{bmatrix} R \cdot \sin(\theta) \cdot \cos(\varphi) \\ R \cdot \sin(\theta) \cdot \sin(\varphi) \\ R \cdot \cos(\theta) \end{bmatrix}$$

Cities

Moscov $\lambda_M := 55.76°$ $\phi_M := 37.62°$ $M := Earth(90° - \lambda_M, \phi_M)$

Leningrad $\lambda_P := 59.94°$ $\phi_P := 30.33°$ $P := Earth(90° - \lambda_P, \phi_P)$

Distances

$\boxed{L := \text{norme}(M - P) = 633.1535\ \text{km}}$ Rect line, gravity train lenght

$\Lambda := 2 \cdot R \cdot \text{asin}\left(\frac{L}{2 \cdot R} \right) = 633.4143\ \text{km}$ Geodesic distance

Max Speed and acc

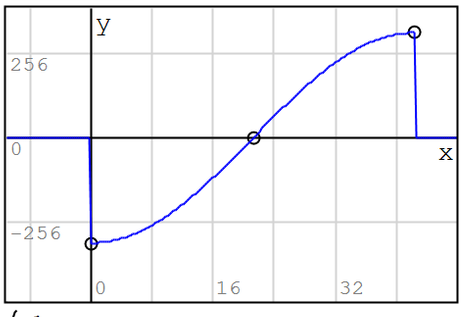
$x'(0.5 \cdot \tau) = 1413.9629\ \frac{\text{km}}{\text{hr}}$

$x''(0) = 0.0497\ g_e$

$T := \tau \cdot \text{stack}(0, 0.5, 1)$

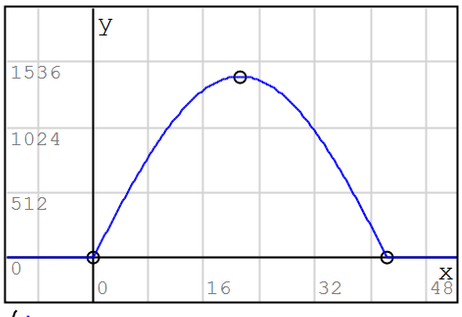
$k := [1..3]$

Position



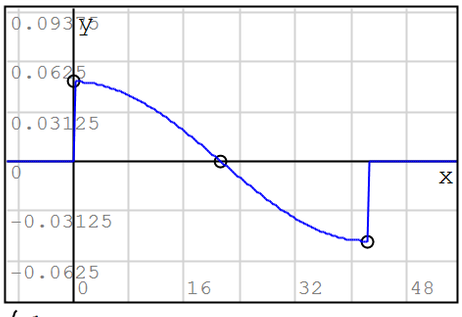
$$\begin{cases} \frac{1}{\text{km}} \cdot x(x\ \text{min}) \cdot 0 \le x\ \text{min} \le \tau \\ \text{augment}\left(\frac{T}{\text{min}}, A_k := x(T_k) \cdot \frac{1}{\text{km}}, \text{"o"} \right) \end{cases}$$

Speed



$$\begin{cases} \frac{\text{hr}}{\text{km}} \cdot x'(x\ \text{min}) \cdot 0 \le x\ \text{min} \le \tau \\ \text{augment}\left(\frac{T}{\text{min}}, A_k := x'(T_k) \frac{\text{hr}}{\text{km}}, \text{"o"} \right) \end{cases}$$

Acceleration

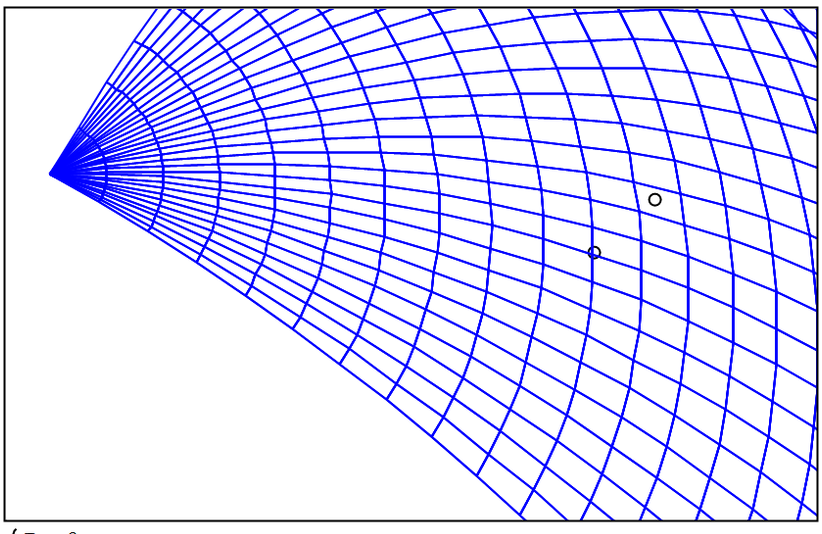


$$\begin{cases} \frac{1}{g_e} \cdot x''(x\ \text{min}) \cdot 0 \le x\ \text{min} \le \tau \\ \text{augment}\left(\frac{T}{\text{min}}, A_k := x''(T_k) \cdot \frac{1}{g_e}, \text{"o"} \right) \end{cases}$$

Cities

$E := \text{CreateMesh}(Earth, 0, 0.5 \cdot \pi, 0, 0.5 \cdot \pi, 30, 30)$

$$\gamma 2 := \begin{bmatrix} 0.866 & -0.433 \\ 0.5 & 0.75 \\ 0 & 0.5 \end{bmatrix}$$



$$\begin{cases} E \cdot \gamma 2 \\ \text{augment}\left(M^{T} \cdot \gamma 2, \text{"}\circ\text{"}\right) \\ \text{augment}\left(P^{T} \cdot \gamma 2, \text{"}\circ\text{"}\right) \end{cases}$$

Alvaro

$\text{appVersion}(4) = \text{"1.0.8348.30405"}$