Pseudo random number between 0 and 1

$rnd := \left| 10^{-9} \cdot \text{random}\left(10^{9}\right) \right.$

Pseudo random number between a and b

$rnd(a, b) := \left| a + rnd \cdot (b - a) \right.$

Vector of n Pseudo random number between a and b

$rnd(n, a, b) := \left| \begin{bmatrix} ans := 0 & k := [1..n] \end{bmatrix} \right.$
$\left| ans_{k} := rnd(a, b) \right.$

Find the ones postitions in the column vector V

$Find_1(V) := \left| \text{col}\left(\text{findrows}\left(\text{eval}\left(\text{augment}\left(V, [1..\text{length}(V)]\right)\right), 1, 1\right), 2\right) \right.$

Shorthand for 2D plots

$Plot := \begin{cases} f(x\ \text{m}, y\ \text{m}) \\ x \cdot y \\ \text{augment}(X, Y, ".", 1, "green") \\ \text{augment}\left(\begin{bmatrix} x_G & y_G \end{bmatrix}, "x", 10\right) \end{cases}$

max utility

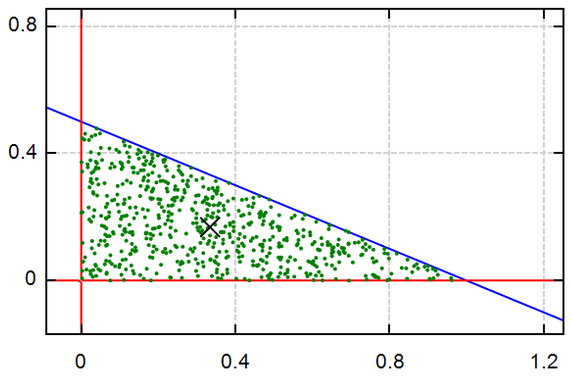
$max(a, b) := \begin{cases} a & \text{if } a > b \\ b & \text{otherwise} \end{cases}$

$max(a, b, c) := max(max(a, b), c)$

$max(a, b, c, d) := max(max(a, b, c), d)$

$\left| \begin{array}{l} f(x, y) := y + 0.5 \cdot x - 0.5\ \text{m} \\ n := 2500 \\ \begin{bmatrix} X & Y \end{bmatrix} := \text{eval}\left(\begin{bmatrix} rnd(n, 0, 1) & rnd(n, 0, 1) \end{bmatrix} \text{m}\right) \\ K := Find_1\left(\overrightarrow{f(X, Y) < 0}\right) \\ \begin{bmatrix} X & Y \end{bmatrix} := \begin{bmatrix} X_K & Y_K \end{bmatrix} \\ \begin{bmatrix} x_G & y_G \end{bmatrix} := \begin{bmatrix} \text{Mean}(X) & \text{Mean}(Y) \end{bmatrix} \end{array} \right.$

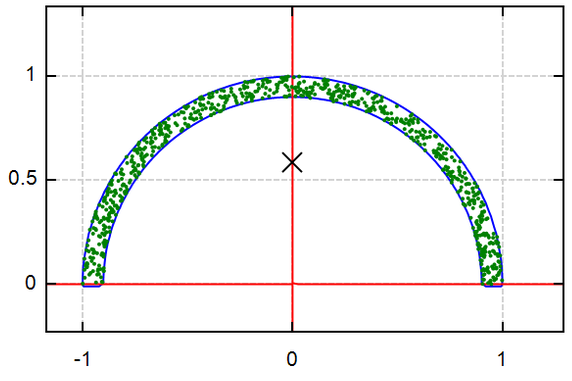
$\begin{bmatrix} x_G & y_G \end{bmatrix} = \begin{bmatrix} 33.43 & 16.83 \end{bmatrix} \text{cm}$



Plot

$\left| \begin{array}{l} \begin{bmatrix} R & r \end{bmatrix} := \begin{bmatrix} 1 & 0.9 \end{bmatrix} \text{m} \\ f(x, y) := \left(x^2 + y^2 - R^2\right) \cdot \left(x^2 + y^2 - r^2\right) \cdot (y > 0) \\ n := 10000 \\ \begin{bmatrix} X & Y \end{bmatrix} := \text{eval}\left(\begin{bmatrix} rnd(n, -1, 1) & rnd(n, -1, 1) \end{bmatrix} \text{m}\right) \\ K := Find_1\left(\overrightarrow{f(X, Y) < 0}\right) \\ \begin{bmatrix} X & Y \end{bmatrix} := \begin{bmatrix} X_K & Y_K \end{bmatrix} \\ \begin{bmatrix} x_G & y_G \end{bmatrix} := \begin{bmatrix} \text{Mean}(X) & \text{Mean}(Y) \end{bmatrix} \end{array} \right.$

$\begin{bmatrix} x_G & y_G \end{bmatrix} = \begin{bmatrix} -0.17 & 58.61 \end{bmatrix} \text{cm}$



Plot

Triangle $f_T(x, y) := -\max\left(-(y + 0.7 \cdot x - 90\ \text{cm}),\ y + 0.1 \cdot x - 90\ \text{cm},\ x + 0.3 \cdot y - 90\ \text{cm}\right)$

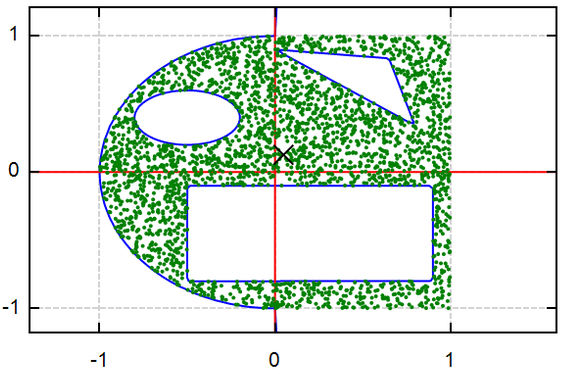
Rectangle $f_R(x, y) := -\max\left(-(x + 50\ \text{cm}),\ x - 90\ \text{cm},\ -(80\ \text{cm} + y),\ y + 10\ \text{cm}\right)$

Ellipse $f_E(x, y) := \left(\frac{x + 50\ \text{cm}}{30\ \text{cm}}\right)^2 + \left(\frac{y - 40\ \text{cm}}{20\ \text{cm}}\right)^2 - 1$

Left $f_L(x, y) := \begin{cases} x^2 + y^2 - (1\ \text{m})^2 & \text{if } x < 0 \\ -1 & \text{otherwise} \end{cases}$

$f(x, y) := f_T(x, y) \cdot f_E(x, y) \cdot f_R(x, y) \cdot f_L(x, y)$
$n := 5000$
$[X\ Y] := \text{eval}\left(\left[rnd(n, -1, 1)\ rnd(n, -1, 1)\right]\text{m}\right)$
$K := Find_1\left(\overrightarrow{f(X, Y) < 0}\right)$
$[X\ Y] := \left[X_K\ Y_K\right]$
$\left[x_G\ y_G\right] := \left[\text{Mean}(X)\ \text{Mean}(Y)\right]$

$\left[x_G\ y_G\right] = [4.83\ 12.69]\ \text{cm}$



Plot

$R := 1$
$f(x, y, z) := \left(x^2 + y^2 + z^2 - R^2\right) \cdot (z > 0)$
$n := 10000$
$[X\ Y\ Z] := \text{eval}\left(\left[rnd(n, -1, 1)\ rnd(n, -1, 1)\ rnd(n, -1, 1)\right]\right)$
$K := Find_1\left(\overrightarrow{f(X, Y, Z) < 0}\right)$
$[X\ Y\ Z] := \left[X_K\ Y_K\ Z_K\right]$
$\left[x_G\ y_G\ z_G\right] := \left[\text{Mean}(X)\ \text{Mean}(Y)\ \text{Mean}(Z)\right]$

$\begin{bmatrix} x_G \\ y_G \\ z_G \end{bmatrix} = \begin{bmatrix} 0.002 \\ 0.0219 \\ 0.3737 \end{bmatrix}$

$\frac{3}{8} \cdot R = 0.375$

$[R := 1\ \ r := 0.5]$
$f_1(x, y, z, w) := \left(\left(\left((x > -0.9) \wedge (y > -0.8)\right) \wedge (z > -0.7)\right) \wedge (w > -0.6)\right) \wedge \left(x^2 + y^2 + z^2 + w^2 < R^2\right)$
$f_2(x, y, z, w) := (x - 0.3)^2 + (y + 0.2)^2 + (z - 0.1)^2 + (w + 0.2)^2 > r^2$
$f(x, y, z, w) := \left(f_1(x, y, z, w) \wedge f_2(x, y, z, w)\right)$
$n := 10000$
$[X\ Y\ Z\ W] := \text{eval}\left(\left[rnd(n, -1, 1)\ rnd(n, -1, 1)\ rnd(n, -1, 1)\ rnd(n, -1, 1)\right]\right)$
$K := Find_1\left(\overrightarrow{f(X, Y, Z, W)}\right)$
$[X\ Y\ Z\ W] := \left[X_K\ Y_K\ Z_K\ W_K\right]$
$\left[x_G\ y_G\ z_G\ w_G\right] := \left[\text{Mean}(X)\ \text{Mean}(Y)\ \text{Mean}(Z)\ \text{Mean}(W)\right]$

$\left[x_G\ y_G\ z_G\ w_G\right] = [-0.0186\ 0.0291\ 0.0336\ 0.0896]$ Using just booleans

Alvaro