Finding Distance of a Point from Curve with al_nleqsolve

Plot, λ

$$
\begin{aligned}
&Dist\left(x_o, h, \varepsilon\right) := \\
&\quad \left[a := x_o \;\; r := 0 \;\; c := [1..2] \;\; s := 0\right] \\
&\quad \left[NA := \text{eval}\left(\text{norme}(A)\right) \; \text{Clear}(x)\right] \\
&\quad \phi(x) := \begin{bmatrix} \left(x_1 - A_1\right)^2 + \left(x_2 - A_2\right)^2 - \left(NA - h \cdot k\right)^2 \\ f\left(x_1, x_2\right) \end{bmatrix} \\
&\quad G(x) := \begin{bmatrix} \frac{d}{d\,x_1} f\left(x_1, x_2\right) & \frac{d}{d\,x_2} f\left(x_1, x_2\right) \end{bmatrix} \\
&\quad \text{for } \; k \in \left[2..\frac{NA}{h}\right] \\
&\qquad \left[a := \text{al\_nleqsolve}\left(a^T, \phi(x\#)\right)^T\right] \\
&\qquad \text{if } \left|f\left(a_1, a_2\right)\right| \le \varepsilon \\
&\qquad\quad \left[r := r + 1 \;\; s_{r\,c} := a_c\right] \\
&\qquad \text{else} \\
&\qquad\quad 0 \\
&\quad \left[a := s_{r\,c} \;\; a\lambda := \text{eval}\left(\text{augment}\left(a, -\text{norme}(A - a)\right)\right)\right] \\
&\quad L(x) := \left(x_1 - A_1\right)^2 + \left(x_2 - A_2\right)^2 + x_3 \cdot f\left(x_1, x_2\right) \\
&\quad \Phi(x) := \begin{bmatrix} \frac{d}{d\,x_1} L(x) & \frac{d}{d\,x_2} L(x) & \frac{d}{d\,x_3} L(x) \end{bmatrix}^T \\
&\quad LSol := \text{al\_nleqsolve}\left(a\lambda^T, \Phi(x)\right) \\
&\quad \left[a := \begin{bmatrix} LSol_1 & LSol_2 \end{bmatrix} \;\; G := G(a) \;\; \delta := \text{norme}(A - a)\right] \\
&\quad \text{eval}\left(\left[a \;\; \delta \;\; \text{stack}\left(a, a + \frac{\delta}{2} \cdot \frac{G}{\text{norme}(G)}\right) \;\; s \;\; LSol_3\right]\right)
\end{aligned}
$$

Lagrangian Multipliers:
Minimize the distance
subject to f(x,y)=0

$$L = \delta^2 + \lambda \cdot f(x, y)$$

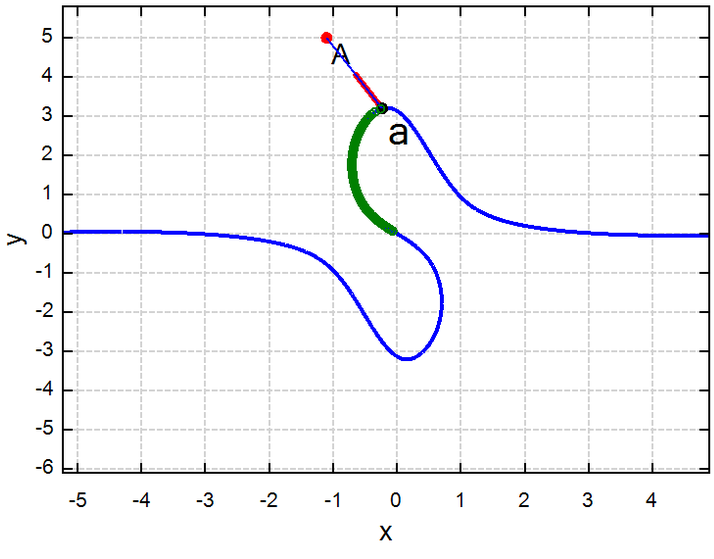
$$\Phi = \begin{bmatrix} \frac{d}{d\,x} L & \frac{d}{d\,y} L & \frac{d}{d\,\lambda} L \end{bmatrix}^T$$

Example 1

$$
\begin{cases}
A := \text{eval}\left(\begin{bmatrix} \left(0.45 \cdot \text{random}(15) - 2\right) \cdot \lambda \\ 5 \cdot \lambda 1 \end{bmatrix}^T\right) \\
f(x, y) := \left(x^2 \cdot y - \sin(x + y)\right) \cdot (1) \\
B := Dist\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}^T, 0.04, 10^{-3}\right) \\
\left[a \;\; \delta \;\; N \;\; s \;\; \lambda o\right] := B
\end{cases}
$$

$$a = \begin{bmatrix} -0.2322 & 3.2004 \end{bmatrix} \qquad \delta = 1.9979$$

$$\text{rows}(s) = 77$$

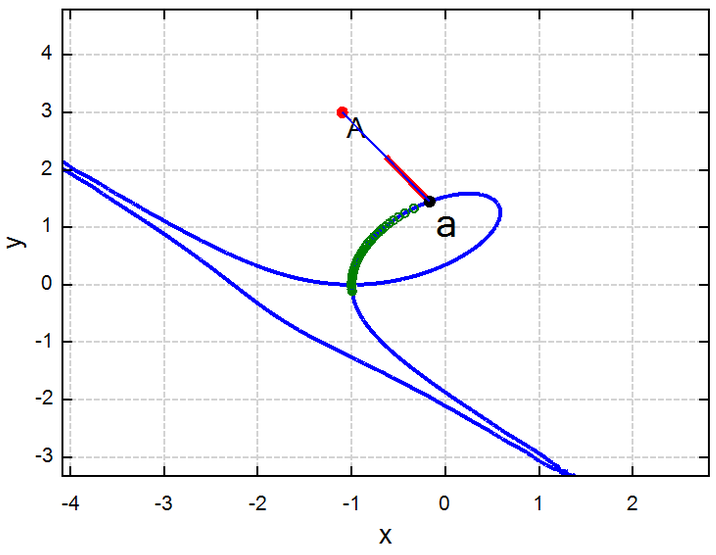


Example 2

$$A := \text{eval}\left(\left[\left(0.45\cdot\text{random}(10)-2\right)\cdot\lambda\ 3\cdot 1\right]\right)$$
$$f(x,y) := \ln\left(\left|1+(1+x)^3-3\cdot y\cdot(1+x)+y^3\right|\right)$$
$$B := Dist\left(\left[A_1\ 2\right], 0.04, 10^{-3}\right)$$
$$[a\ \delta\ N\ s\ \lambda o] := B$$

$a = [-0.1697\ 1.4476]$ $\delta = 1.8098$

$\text{rows}(s) = 33$

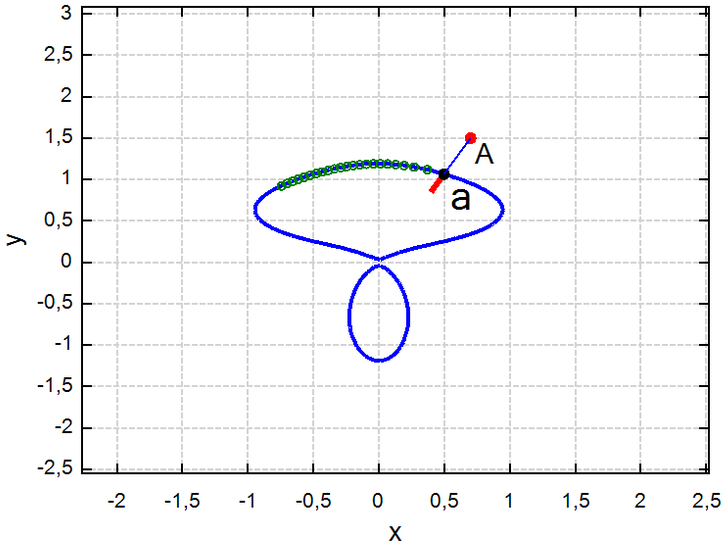


Example 3

$$A := \text{eval}\left(\left[\left(0.45\cdot\text{random}(10)-2\right)\cdot\lambda\ 1.5\cdot\lambda 1\right]\right)$$
$$\alpha := 0.842$$
$$f(x,y) := \frac{1}{\left(x^2+y^2-\alpha^2\right)^2+\left(2\cdot x\cdot(y-\alpha)\right)^2}-2$$
$$B := Dist\left(\left[0\ A_2\right], 0.05, 10^{-4}\right)$$
$$[a\ \delta\ N\ s\ \lambda o] := B$$

$a = [0.4976\ 1.0644]$ $\delta = 0.4803$

$\text{rows}(s) = 22$



Example 4

$$A := \text{eval}\left(\left[\left(0.45\cdot\text{random}(20)-0\right)\cdot\lambda\ 6\cdot\lambda 1\right]\right)$$
$$f(x,y) := e^{-\frac{1}{3}\cdot\sqrt{x^2+y^2}}\cdot\frac{x^2-2\cdot y^2}{81\cdot\sqrt{6\cdot\pi}}+0.005$$
$$B := Dist\left(\left[0\ A_2\right], 0.4, 10^{-4}\right)$$
$$[a\ \delta\ N\ s\ \lambda o] := B$$

$a = [6.1232\ 5.7672]$ $\delta = 0.359$

$\text{rows}(s) = 19$