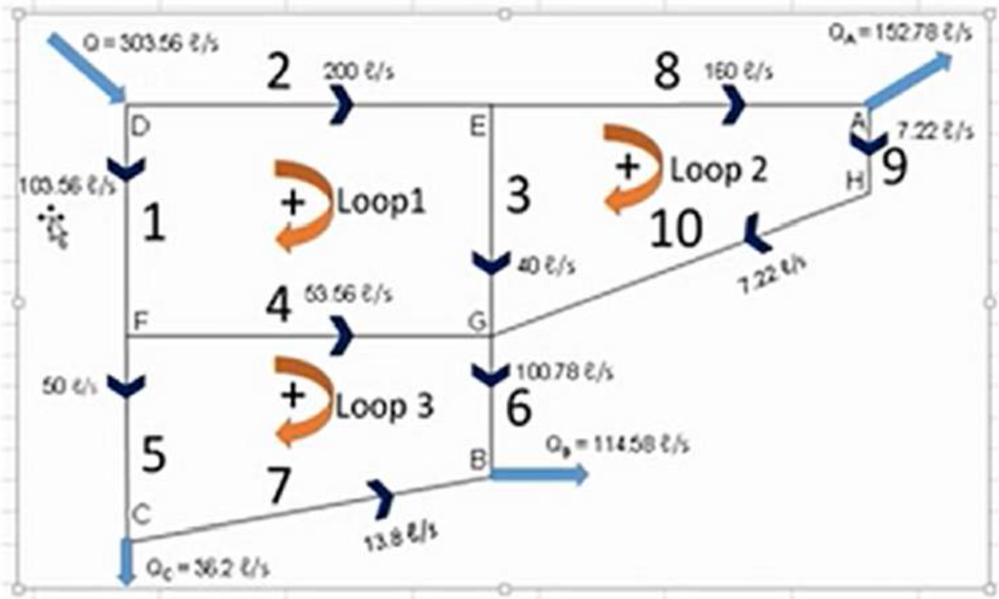
Example: Network Analysis by Hardy-Cross Method - With 3 LOOPS

appVersion(4) = "0.99.7739.40423"

$t_0 := \text{time}(0)$



Index of common pipes in LOOPS

Index of EG in LOOP 1

$n1 := 3$

Index of GE in LOOP 2

$n2 := 1$

Index of GF in LOOP 1

$n3 := 4$

Index of FG in LOOP 3

$n4 := 2$

Flows at Nodes

$$QJ := \begin{bmatrix} 303.56 \\ -152.78 \\ -114.58 \\ -36.2 \end{bmatrix} \frac{L}{s}$$

Continuity of the whole network. Sum of Nodal flows should be zero

$$\sum QJ = 0$$

Friction Factor

$f := 0.02$

Hazen William 'n'

$n := 1.85$

Flows in LOOPS should be balanced first to start with. Otherwise, will yield wrong answers

Assumed Flows for Continuity in LOOP 1

$$Q1 := \begin{bmatrix} -103.56 \\ 200 \\ 40 \\ -53.56 \end{bmatrix} \frac{L}{s}$$

1
2
3
4

Assumed Flows for Continuity in LOOP 2

$$Q2 := \begin{bmatrix} -40 \\ 160 \\ 7.22 \\ 7.22 \end{bmatrix} \frac{L}{s}$$

3
8
9
10

Assumed Flows for Continuity in LOOP 3

$$Q3 := \begin{bmatrix} -50 \\ 53.56 \\ 100.78 \\ -13.8 \end{bmatrix} \frac{L}{s}$$

5
4
6
7

Friction Factor

$f := 0.02$

Flows in LOOPS Sign Convention
1. Clockwise: +ve
2. Anticlockwise: -ve

Pipe no.	L(m)	D(m)	ks(m)
Bulk main	5000	0.65	0.00010
1	1000	0.40	0.00005
2	2000	0.45	0.00005
3	1000	0.30	0.00005
4	2000	0.30	0.00005
5	1000	0.40	0.00003
6	750	0.30	0.00003
7	2200	0.30	0.00003
8	2000	0.30	0.00003
9	500	0.25	0.00003
10	2200	0.25	0.00003

v= 1.13E-06 m^2/s

dq = -Σhf/(2Σ(hf/q))

Lengths LOOP 1

$$L1 := \begin{bmatrix} 1000 \\ 2000 \\ 1000 \\ 2000 \end{bmatrix} m$$

Dia. LOOP 1

$$D1 := \begin{bmatrix} 0.4 \\ 0.45 \\ 0.30 \\ 0.30 \end{bmatrix} m$$

Lengths LOOP 2

$$L2 := \begin{bmatrix} 1000 \\ 2000 \\ 500 \\ 2200 \end{bmatrix} m$$

Dia. LOOP 2

$$D2 := \begin{bmatrix} 0.3 \\ 0.3 \\ 0.25 \\ 0.25 \end{bmatrix} m$$

Lengths LOOP 3

$$L3 := \begin{bmatrix} 1000 \\ 2000 \\ 750 \\ 2200 \end{bmatrix} m$$

Dia. LOOP 3

$$D3 := \begin{bmatrix} 0.4 \\ 0.3 \\ 0.3 \\ 0.3 \end{bmatrix} m$$

METHOD 1: Hardy Cross Method using Successive Steps of Corrections

PROGRAM 1 starts with assumed flow for both LOOPS

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Program 1: Using Vectorize Function.
Find 'Corrected Flow' for the 3 LOOPS
```

$$Calc\_\Delta Q\#(Q\#, L\#, D\#) :=$$

$$k\# := \overrightarrow{\frac{8 \cdot f \cdot L\#}{g_e \cdot (D\#)^5 \cdot \pi^2}}$$ <---------- K values for all pipes

$$HL\# := \overrightarrow{k\# \cdot Q\# \cdot (|Q\#|)^{n-1}}$$ <---------- Head Loss for all pipes

$$\delta HL\# := \left|\overrightarrow{\frac{HL\#}{Q\#}}\right|$$ <---------- Net Head Loss

$$\Delta Q\# := -\frac{\sum HL\#}{n \cdot \left(\sum \delta HL\#\right)}$$ <---------- Corrections for 3 LOOPS .

To be minimized with successive steps

1st correction (Starts with assumed flows in both LOOPS)

$$\Delta QQ := \overrightarrow{Calc\_\Delta Q\#\left(\begin{bmatrix} Q1 \\ Q2 \\ Q3 \end{bmatrix}, \begin{bmatrix} L1 \\ L2 \\ L3 \end{bmatrix}, \begin{bmatrix} D1 \\ D2 \\ D3 \end{bmatrix}\right)} = \begin{bmatrix} -5.73 \\ -60.28 \\ -27.75 \end{bmatrix} \frac{L}{s}$$

$$\Delta QQ_1 = -5.73 \frac{L}{s} \quad \Delta QQ_2 = -60.28 \frac{L}{s} \quad \Delta QQ_3 = -27.75 \frac{L}{s}$$

```
1st Corrected flows in LOOP 1
```

$$QQ1_{cor1} := Q1 + \Delta QQ_1 = \begin{bmatrix} -109.29 \\ 194.27 \\ 34.27 \\ -59.29 \end{bmatrix} \frac{L}{s}$$

```
1st Corrected flows in LOOP 2
```

$$QQ2_{cor1} := Q2 + \Delta QQ_2 = \begin{bmatrix} -100.28 \\ 99.72 \\ -53.06 \\ -53.06 \end{bmatrix} \frac{L}{s}$$

```
1st Corrected flows in LOOP 3
```

$$QQ3_{cor1} := Q3 + \Delta QQ_3 = \begin{bmatrix} -77.75 \\ 25.81 \\ 73.03 \\ -41.55 \end{bmatrix} \frac{L}{s}$$

Now Apply 1st Correction for Pipe GE common to loops 1 & 2

```
Adjusted flow for Pipe EG in LOOP 1
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$$\Delta HL1 := QQ1_{cor1_3} - \Delta QQ_2 = 94.55 \frac{L}{s}$$

```
Adjusted flow for Pipe GE in LOOP 2
```

$$QQ2_{cor1_1} := \Delta HL1$$

$$QQ1_{cor1_3} := \Delta HL1 \qquad QQ1_{cor1_3} = 34.265 \frac{L}{s}$$

$$QQ1_{cor1} = \begin{bmatrix} -109.29 \\ 194.27 \\ 94.55 \\ -59.29 \end{bmatrix} \frac{L}{s} \qquad QQ2_{cor1} = \begin{bmatrix} 94.55 \\ 99.72 \\ -53.06 \\ -53.06 \end{bmatrix} \frac{L}{s}$$

Now Apply 1st Correction for Pipe GE common to loops 1 & 3

```
Adjusted flow for Pipe GF in LOOP 1
```

$$\Delta HL2 := QQ3_{cor1_4} - \Delta QQ_3 = -13.8 \frac{L}{s} \qquad QQ3_{cor1_2} := -\Delta HL2$$

$$QQ1_{cor1_4} := \Delta HL2 \qquad QQ3_{cor1_2} = 13.8 \frac{L}{s}$$

$$QQ1_{cor1} = \begin{bmatrix} -109.29 \\ 194.27 \\ 94.55 \\ -13.8 \end{bmatrix} \frac{L}{s} \qquad QQ2_{cor1} = \begin{bmatrix} 94.55 \\ 99.72 \\ -53.06 \\ -53.06 \end{bmatrix} \frac{L}{s} \qquad QQ3_{cor1} = \begin{bmatrix} -77.75 \\ 13.8 \\ 73.03 \\ -41.55 \end{bmatrix} \frac{L}{s}$$

2nd correction (Starts with flows in correction 1)

$$\Delta QQ2 := \overrightarrow{Calc\_\Delta Q\#\left(\begin{bmatrix} QQ1_{cor1} \\ QQ2_{cor1} \\ QQ3_{cor1} \end{bmatrix}, \begin{bmatrix} L1 \\ L2 \\ L3 \end{bmatrix}, \begin{bmatrix} D1 \\ D2 \\ D3 \end{bmatrix}\right)} = \begin{bmatrix} -38.88 \\ -6.38 \\ 2.77 \end{bmatrix} \frac{L}{s}$$

2nd Corrected flows in LOOP 1

$$QQ1_{cor2} := QQ1_{cor1} + \Delta QQ2_1 = \begin{bmatrix} -148.17 \\ 155.39 \\ 55.67 \\ -52.68 \end{bmatrix} \frac{L}{s}$$

2nd Corrected flows in LOOP 2

$$QQ2_{cor2} := QQ2_{cor1} + \Delta QQ2_2 = \begin{bmatrix} 88.16 \\ 93.34 \\ -59.44 \\ -59.44 \end{bmatrix} \frac{L}{s}$$

2nd Corrected flows in LOOP 3

$$QQ3_{cor2} := QQ3_{cor1} + \Delta QQ2_3 = \begin{bmatrix} -74.98 \\ 16.57 \\ 75.8 \\ -38.78 \end{bmatrix} \frac{L}{s}$$

Now Apply 2nd Correction for Pipe GE common to loops 1 & 2

Adjusted flow for Pipe EG in LOOP 1

$$\Delta HL3 := QQ1_{cor2\,3} - \Delta QQ2_2 = 62.05 \frac{L}{s}$$

$$QQ1_{cor2\,3} := \Delta HL3 \qquad QQ1_{cor2\,3} = 55.67 \frac{L}{s}$$

Adjusted flow for Pipe GE in LOOP 2

$$QQ2_{cor2\,1} := -\Delta HL3$$

$$QQ1_{cor2} = \begin{bmatrix} -148.17 \\ 155.39 \\ 62.05 \\ -52.68 \end{bmatrix} \frac{L}{s} \qquad QQ2_{cor2} = \begin{bmatrix} -62.05 \\ 93.34 \\ -59.44 \\ -59.44 \end{bmatrix} \frac{L}{s} \qquad QQ3_{cor2} = \begin{bmatrix} -74.98 \\ 16.57 \\ 75.8 \\ -38.78 \end{bmatrix} \frac{L}{s}$$

Now Apply 2nd Correction for Pipe GE common to loops 1 & 3

Adjusted flow for Pipe GF in LOOP 1

$$\Delta HL4 := QQ3_{cor2\,4} - \Delta QQ2_3 = -41.55 \frac{L}{s}$$

$$QQ3_{cor2\,2} := -\Delta HL4$$

$$QQ1_{cor2\,4} := \Delta HL4$$

$$QQ3_{cor2\,2} = 41.5465 \frac{L}{s}$$

$$QQ1_{cor2} = \begin{bmatrix} -148.17 \\ 155.39 \\ 62.05 \\ -41.55 \end{bmatrix} \frac{L}{s} \qquad QQ2_{cor2} = \begin{bmatrix} -62.05 \\ 93.34 \\ -59.44 \\ -59.44 \end{bmatrix} \frac{L}{s} \qquad QQ3_{cor2} = \begin{bmatrix} -74.98 \\ 41.55 \\ 75.8 \\ -38.78 \end{bmatrix} \frac{L}{s}$$

3rd correction (Starts with flows in correction 1)

$$\Delta QQ3 := \overrightarrow{Calc\_\Delta Q\#\left(\begin{bmatrix} QQ1_{cor2} \\ QQ2_{cor2} \\ QQ3_{cor2} \end{bmatrix}, \begin{bmatrix} L1 \\ L2 \\ L3 \end{bmatrix}, \begin{bmatrix} D1 \\ D2 \\ D3 \end{bmatrix}\right)} = \begin{bmatrix} -2.87 \\ 9.59 \\ -6.43 \end{bmatrix} \frac{L}{s}$$

Similarlr, continue as above

Hardy Cross Method (1936, Procedure)

- 1. In this method a trial distribution of discharges is made arbitrary but in such a way that continuity equation is satisfied at each Junction.
- 2. With the assumed the value of Q the head loss in each pipe is calculated according to Darcy Weisbach equation.
- 3. Now consider any loop the algebraic sum of head losses round each loop must be zero. This means that in each loop the loss of head due to flow in in clockwise direction must be equal to the loss of head due to flow in anticlockwise direction.
- 4. If the net head loss due to assumed value of Q round the loop is zero, then the assumed value of Q in that loop is correct. But if the net head loss due to assumed values of Q is not zero then the assumed the values of Q are corrected by introducing a correction factor.

Procedure Cont...

- 5. Apply the correction factor to each pipe if the value of correction factor is not zero.
- 6. After the corrections have been applied to each pipe in a loop and to all loops, a second trial calculation is made for all loops. The procedure is repeated till the correction factor becomes negligible.

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PROGRAM 2: Calculates 'Corrected Flows' & 'Corrections'
CALLS PROGRAM 1
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$$Calc\_All\_\Delta(q1, q2, q3) :=$$

$$\Delta q := \overrightarrow{Calc\_\Delta Q\#\left(\begin{bmatrix} q1 \\ q2 \\ q3 \end{bmatrix}, \begin{bmatrix} L1 \\ L2 \\ L3 \end{bmatrix}, \begin{bmatrix} D1 \\ D2 \\ D3 \end{bmatrix}\right)}$$

$$q1_{cor} := q1 + \Delta q_1$$

$$q2_{cor} := q2 + \Delta q_2$$

$$q3_{cor} := q3 + \Delta q_3$$

$$\Delta cd1 := {q1_{cor}}_{n1} - \Delta q_2$$

$${q1_{cor}}_{n1} := \Delta cd1$$ $n1 = 3$

$${q2_{cor}}_{n2} := -\Delta cd1$$ $n2 = 1$

"--------------------"

$$\Delta cd2 := {q3_{cor}}_{n3} - \Delta q_3$$

$${q1_{cor}}_{n3} := \Delta cd2$$ $n3 = 4$

$${q3_{cor}}_{n4} := -\Delta cd2$$ $n4 = 2$

$$\Delta q_{cor} := \overrightarrow{Calc\_\Delta Q\#\left(\begin{bmatrix} q1_{cor} \\ q2_{cor} \\ q3_{cor} \end{bmatrix}, \begin{bmatrix} L1 \\ L2 \\ L3 \end{bmatrix}, \begin{bmatrix} D1 \\ D2 \\ D3 \end{bmatrix}\right)}$$

$$\begin{bmatrix} \begin{bmatrix} q1_{cor} \\ q2_{cor} \\ q3_{cor} \end{bmatrix} & \Delta q_{cor} \end{bmatrix}$$

Corrections at start

$$\overrightarrow{Calc\_\Delta Q\#\left(\begin{bmatrix} Q1 \\ Q2 \\ Q3 \end{bmatrix}, \begin{bmatrix} L1 \\ L2 \\ L3 \end{bmatrix}, \begin{bmatrix} D1 \\ D2 \\ D3 \end{bmatrix}\right)} = \begin{bmatrix} -5.735 \\ -60.2804 \\ -27.7465 \end{bmatrix} \frac{L}{s}$$

After 1st correction

$$X1 := Calc\_All\_\Delta(Q1, Q2, Q3) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -109.295 \\ 194.265 \\ 94.5454 \\ -13.8 \end{bmatrix} \\ \begin{bmatrix} -94.5454 \\ 99.7196 \\ -53.0604 \\ -53.0604 \end{bmatrix} \\ \begin{bmatrix} -77.7465 \\ 13.8 \\ 73.0335 \\ -41.5465 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} -38.8794 \\ 7.8059 \\ 2.7664 \end{bmatrix} \end{bmatrix} \frac{L}{s}$$

$$QQ1_{cor1} = \begin{bmatrix} -109.295 \\ 194.265 \\ 94.5454 \\ -13.8 \end{bmatrix} \frac{L}{s}$$

$$QQ2_{cor1} = \begin{bmatrix} 94.5454 \\ 99.7196 \\ -53.0604 \\ -53.0604 \end{bmatrix} \frac{L}{s}$$

$$QQ3_{cor1} = \begin{bmatrix} -77.7465 \\ 13.8 \\ 73.0335 \\ -41.5465 \end{bmatrix} \frac{L}{s}$$

After 2nd correction

$$X2 := Calc\_All\_\Delta\left({X1_1}_1, {X1_1}_2, {X1_1}_3\right) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -148.1743 \\ 155.3857 \\ 47.8601 \\ -41.5465 \end{bmatrix} \\ \begin{bmatrix} -47.8601 \\ 107.5255 \\ -45.2545 \\ -45.2545 \end{bmatrix} \\ \begin{bmatrix} -74.9801 \\ 41.5465 \\ 75.7999 \\ -38.7801 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} 0.8354 \\ -4.2791 \\ -6.4306 \end{bmatrix} \end{bmatrix} \frac{L}{s}$$

$$QQ1_{cor2} = \begin{bmatrix} -148.1743 \\ 155.3857 \\ 62.0474 \\ -41.5465 \end{bmatrix} \frac{L}{s}$$

$$QQ2_{cor2} = \begin{bmatrix} -62.0474 \\ 93.3383 \\ -59.4417 \\ -59.4417 \end{bmatrix} \frac{L}{s}$$

$$QQ3_{cor2} = \begin{bmatrix} -74.9801 \\ 41.5465 \\ 75.7999 \\ -38.7801 \end{bmatrix} \frac{L}{s}$$

After 3rd correction

$$X3 := Calc\_All\_\Delta\left({X2_1}_1, {X2_1}_2, {X2_1}_3\right) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -147.3389 \\ 156.2211 \\ 52.9747 \\ -38.7801 \end{bmatrix} \\ \begin{bmatrix} -52.9747 \\ 103.2464 \\ -49.5336 \\ -49.5336 \end{bmatrix} \\ \begin{bmatrix} -81.4108 \\ 38.7801 \\ 69.3692 \\ -45.2108 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} -1.9015 \\ 0.1582 \\ -1.1582 \end{bmatrix} \end{bmatrix} \frac{L}{s}$$

METHOD 2: Hardy Cross Method: For Any Given Number of Iterations

PROGRAM 3: Repeated Iteration Method. CALLS PROGRAM 2

```
Q(q1, q2, q3, iter) := | for j ∈ [1..iter]
                       |   | if j = 1
                       |   |    P_j := Calc_All_Δ(q1, q2, q3)
                       |   | else
                       |   |    P_j := Calc_All_Δ((P_{j-1})_{1_1}, (P_{j-1})_{1_2}, (P_{j-1})_{1_3})
                       | P_iter
```

With 10 iterations

$$Q(Q1, Q2, Q3, 10) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -140.68 \\ 162.88 \\ 58.93 \\ -53.02 \end{bmatrix} \\ \begin{bmatrix} -58.93 \\ 103.95 \\ -48.83 \\ -48.83 \end{bmatrix} \\ \begin{bmatrix} -89.82 \\ 53.02 \\ 60.96 \\ -53.62 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} 0.84 \\ 0.1 \\ -0.5 \end{bmatrix} \end{bmatrix} \frac{L}{s}$$

With 20 iterations

$$Q(Q1, Q2, Q3, 20) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -136.6 \\ 166.96 \\ 62.51 \\ -55.92 \end{bmatrix} \\ \begin{bmatrix} -62.51 \\ 104.45 \\ -48.33 \\ -48.33 \end{bmatrix} \\ \begin{bmatrix} -92.2 \\ 55.92 \\ 58.58 \\ -56 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} 0.12 \\ 0.02 \\ -0.07 \end{bmatrix} \end{bmatrix} \frac{L}{s}$$

With 25 iterations

$$Q(Q1, Q2, Q3, 25) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -136.17 \\ 167.39 \\ 62.89 \\ -56.22 \end{bmatrix} \\ \begin{bmatrix} -62.89 \\ 104.5 \\ -48.28 \\ -48.28 \end{bmatrix} \\ \begin{bmatrix} -92.45 \\ 56.22 \\ 58.33 \\ -56.25 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} 0.05 \\ 0.01 \\ -0.03 \end{bmatrix} \end{bmatrix} \frac{L}{s}$$

With 30 iterations

$$Q(Q1, Q2, Q3, 30) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -136 \\ 167.56 \\ 63.03 \\ -56.34 \end{bmatrix} \\ \begin{bmatrix} -63.03 \\ 104.52 \\ -48.26 \\ -48.26 \end{bmatrix} \\ \begin{bmatrix} -92.55 \\ 56.34 \\ 58.23 \\ -56.35 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} 0.02 \\ 0 \\ -0.01 \end{bmatrix} \end{bmatrix} \frac{L}{s}$$

With 35 iterations

$$Q(Q1, Q2, Q3, 35) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -135.94 \\ 167.62 \\ 63.09 \\ -56.38 \end{bmatrix} \\ \begin{bmatrix} -63.09 \\ 104.53 \\ -48.25 \\ -48.25 \end{bmatrix} \\ \begin{bmatrix} -92.59 \\ 56.38 \\ 58.19 \\ -56.39 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} 0.01 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \frac{L}{s}$$

With 40 iterations

$$Q(Q1, Q2, Q3, 40) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -135.91 \\ 167.65 \\ 63.11 \\ -56.4 \end{bmatrix} \\ \begin{bmatrix} -63.11 \\ 104.54 \\ -48.24 \\ -48.24 \end{bmatrix} \\ \begin{bmatrix} -92.6 \\ 56.4 \\ 58.18 \\ -56.4 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \frac{L}{s}$$

METHOD 3: Hardy Cross Method: Using While Loop

PROGRAM 4: Using 'While' loop :
CALLS PROHRAM 2

```
Z(q1, q2, q3) := | A := Calc_All_Δ(q1, q2, q3)
                 | ΔQ := A_2
                 | qq1 := A_(1_1)
                 | qq2 := A_(1_2)
                 | qq3 := A_(1_3)
                 | Δ := 0.001 L/s
                 | while ((|ΔQ_1| ≥ Δ) ∧ (|ΔQ_2| ≥ Δ)) ∧ (|ΔQ_3| ≥ Δ)
                 |     | B := Calc_All_Δ(qq1, qq2, qq3)
                 |     | qq1 := B_(1_1)
                 |     | qq2 := B_(1_2)
                 |     | qq3 := B_(1_3)
                 |     | ΔQ := B_2
                 | B
```

Using "while" LOOP - PROGRAM 4

$$Z(Q1, Q2, Q3) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -135.94 \\ 167.62 \\ 63.09 \\ -56.38 \end{bmatrix} \\ \begin{bmatrix} -63.09 \\ 104.53 \\ -48.25 \\ -48.25 \end{bmatrix} \\ \begin{bmatrix} -92.59 \\ 56.38 \\ 58.19 \\ -56.39 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} 0.01 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \frac{L}{s}$$

Compare

Using 35 iterations - PROGRAM 3

$$Q(Q1, Q2, Q3, 35) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -135.94 \\ 167.62 \\ 63.09 \\ -56.38 \end{bmatrix} \\ \begin{bmatrix} -63.09 \\ 104.53 \\ -48.25 \\ -48.25 \end{bmatrix} \\ \begin{bmatrix} -92.59 \\ 56.38 \\ 58.19 \\ -56.39 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} 0.01 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \frac{L}{s}$$

$time(0) - t_0 = 7.1\ s$