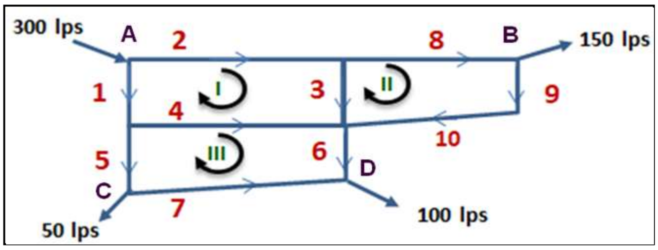
appVersion(4) = "1.0.8151.26392"

CheGuide

https://cheguide.com/pipe_network.html

Example

A water supply distribution system is shown in the figure below. All pipes are cast iron with lengths and diameters as provided in table below. Perform pipe network analysis and calculate water flow in all branches.



Pipe	Length m	Diameter m	e/D
1	1000	0.40	0.00087
2	2000	0.45	0.00104
3	1000	0.30	0.00130
4	2000	0.30	0.00130
5	1000	0.40	0.00130
6	750	0.30	0.00130
7	2200	0.30	0.00130
8	2000	0.30	0.00173
9	500	0.25	0.00130
10	2200	0.25	0.00173

$t_0 := \text{time}(0)$

Darcy-Weisbach 'n'

$n := 2$

Kinematic Viscosity of water

$\nu := 1 \cdot 10^{-6} \; \frac{m^2}{s}$

$cumec := \frac{m^3}{s}$

Length | Diameter | Relative roughness | Nodal flows at A,B,C,D

$$LL := \begin{bmatrix} 1000 \\ 2000 \\ 1000 \\ 2000 \\ 1000 \\ 750 \\ 2200 \\ 2000 \\ 500 \\ 2200 \end{bmatrix} m \qquad D := \begin{bmatrix} 0.4 \\ 0.45 \\ 0.30 \\ 0.30 \\ 0.40 \\ 0.30 \\ 0.30 \\ 0.30 \\ 0.25 \\ 0.25 \end{bmatrix} m \qquad Ks := \begin{bmatrix} 0.00087 \\ 0.00104 \\ 0.00130 \\ 0.00130 \\ 0.00130 \\ 0.00130 \\ 0.00130 \\ 0.00173 \\ 0.00130 \\ 0.00173 \end{bmatrix}$$

$$\begin{bmatrix} Q_A \\ Q_B \\ Q_C \\ Q_D \end{bmatrix} := \begin{bmatrix} 300 \\ -150 \\ -50 \\ -100 \end{bmatrix} \frac{L}{s}$$

Sanity check

$$\sum \begin{bmatrix} Q_A \\ Q_B \\ Q_C \\ Q_D \end{bmatrix} = 0$$

Define

$nr := [1..\text{rows}(D)]$ $\qquad n\_loops := 3$

PROGRAM 1: Find 'head loss' in a pipe

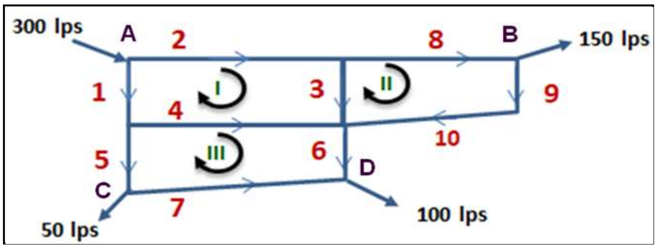
$$Calc\_HL(q, k, d, l) := \begin{cases} v := \dfrac{|q|}{\frac{\pi}{4} \cdot d^2} \\[2ex] Re := \left| \dfrac{\frac{v \cdot d}{\nu}}{\frac{m^2}{s}} \right| \\[2ex] f := \begin{cases} \dfrac{64}{Re} & \text{if } Re \le 2100 \\ 0.25 \cdot \left(\log10\left(\dfrac{k}{3.7} + \dfrac{5.74}{Re^{0.9}} \right) \right)^{-2} & \text{otherwise} \end{cases} \\[2ex] K\# := \dfrac{8 \cdot f \cdot l}{\left(\dfrac{g_e}{\frac{m}{s^2}} \right) \cdot d^5 \cdot \pi^2} \\[2ex] hL := K\# \cdot q \cdot (|q|)^{n-1} \end{cases}$$

Note:
Formula for f as given in the spreadsheet example.
(See link above)

Head Loss (HL)

$guess_{[1..10]} := 0.1$

For testing intermediate claculationls only



$$guess = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix} \qquad Q_{init} := \begin{bmatrix} 100 \\ 200 \\ 30 \\ 30 \\ 70 \\ 80 \\ 20 \\ 170 \\ 20 \\ 20 \end{bmatrix} \frac{L}{s}$$

Head Loss in all pipes

$$HL(Q) := \overrightarrow{Calc\_HL\left(Q, Ks, \frac{D}{m}, \frac{LL}{m}\right)}$$

Testing only

$$HL\left(\frac{Q_{init}}{cumec}\right) = \begin{bmatrix} 1.6249 \\ 7.3303 \\ 0.7014 \\ 1.4028 \\ 0.8774 \\ 3.5615 \\ 0.7083 \\ 45.0635 \\ 0.3944 \\ 1.8302 \end{bmatrix}$$

Flows at Nodes

$$eq_{nodes}(Q) := \begin{bmatrix} \frac{Q_A}{cumec} - Q_1 - Q_2 \\ \frac{Q_B}{cumec} + Q_8 - Q_9 \\ \frac{Q_C}{cumec} + Q_5 - Q_7 \\ \frac{Q_D}{cumec} + Q_6 + Q_7 \\ Q_1 - Q_4 - Q_5 \\ Q_2 - Q_3 - Q_8 \\ Q_9 - Q_{10} \\ Q_4 + Q_3 + Q_{10} - Q_6 \end{bmatrix}$$

Clockwise flow + ve
Anticlockwise flow - ve

Head loss in the 3 loops

$$eq_{Loops}(Q) := \begin{bmatrix} -HL(Q)_1 + HL(Q)_2 + HL(Q)_3 - HL(Q)_4 \\ -HL(Q)_3 + HL(Q)_8 + HL(Q)_9 + HL(Q)_{10} \\ HL(Q)_4 - HL(Q)_5 + HL(Q)_6 - HL(Q_7)_7 \end{bmatrix}$$

Testing only

$$eq_{Loops}\left(\frac{Q_{init}}{cumec}\right) = \begin{bmatrix} 5.004 \\ 46.5867 \\ 3.3786 \end{bmatrix}$$

Define the system of 11 equations (8 Nodes + 3 Loops) ; 10 Variables (pipe flows) Hence, Overdetermimed Nonlinear System

$$f(Q) := \text{stack}\left(eq_{nodes}(Q), eq_{Loops}(Q)\right)$$

$$result := \text{al\_nleqsolve}\left(guess, 10^{-4}, f(Q)\right)$$

$Q_{final} := result$

Excel Result

93	Iteration		
94	5	Pipe	Flow
95			m3/s
96		1	0.1442
97		2	0.1558
98		3	0.0560
99		4	0.0482
100		5	0.0960
101		6	0.0540
102		7	0.0460
103		8	0.0999
104		9	-0.0501
105		10	-0.0501

$$Q_{final} = \begin{bmatrix} 0.1442 \\ 0.1558 \\ 0.056 \\ 0.0482 \\ 0.096 \\ 0.054 \\ 0.046 \\ 0.0999 \\ -0.0501 \\ -0.0501 \end{bmatrix}$$

Final head loss in each pipe

$$HL\left(Q_{final}\right) = \begin{bmatrix} 3.326 \\ 4.4834 \\ 2.3583 \\ 3.5157 \\ 1.6287 \\ 1.6483 \\ 3.5354 \\ 15.6995 \\ -2.3468 \\ -10.9945 \end{bmatrix}$$

In Method 1, "f" for each pipe is directly calculated using the formula shown in Program 1. Each pipe has a diffrent "f" value.

$\text{time}(0) - t_0 = 7.3\ s$

Method 2: Using Colebrook-White equation

Define $\varepsilon_f := 10^{-10}$ $t1 := \mathrm{time}(0)$

Numerical solution of Colebrook-White equation using SMath Studio 'NewtonRaphson' function

$$\frac{1}{\sqrt{f}} = -2\cdot\log10\left(\frac{k}{3.7\cdot D} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$$

Substitute $x = \frac{1}{\sqrt{f}}$ → $x + 2\cdot\log10\left(\frac{k}{3.7\cdot d} + \frac{2.51}{Re}\cdot x\right) = 0$

```
PROGRAM 2: 'NewtonRaphson' function to find 'f' of a pipe
```

$$NR\_SOLVE(k, d, Re) := \left| \begin{array}{l} F(x\#) := x\# + 2\cdot\log10\left(\frac{k}{3.7\cdot d} + \frac{2.51}{Re}\cdot x\#\right) \\ x_{init} := \frac{1}{\sqrt{0.01}} \\ x_{nr} := \mathrm{NewtonRaphson}\left(F(x\#), x_{init}, \varepsilon_f\right) \\ f_{nr} := \frac{1}{x_{nr}^{2}} \end{array} \right.$$

Colebrook_White Eqn

Start with **min 'f'** value in Moody chart

Use "Newton Raphson" method to solve for 'x

Note: "LevenbergMarquadt" method is slow.

Friction factor of pipe

```
PROGRAM 3: Find 'head loss' in a pipe using PROGRAM 2
```

$$Calc\_HL\_NR(q, k, d, l) := \left| \begin{array}{l} v := \frac{|q|}{\frac{\pi}{4}\cdot d^{2}} \\ Re := \frac{v\cdot d}{\frac{\nu}{\frac{m^{2}}{s}}} \\ f := \begin{cases} \frac{64}{Re} & \text{if } Re \le 2100 \\ NR\_SOLVE(k, d, Re) & \text{otherwise} \end{cases} \\ K\# := \frac{8\cdot f\cdot l}{\frac{g_e}{\frac{m}{s^{2}}}\cdot (d)^{5}\cdot \pi^{2}} \\ hL := K\#\cdot q\cdot (|q|)^{n-1} \end{array} \right.$$

Velocity in pipe

Kinamatic viscosity

Reynold number $\nu = 1\cdot 10^{-6}\ \frac{m^{2}}{s}$

friction factor (if laminar flow)

friction factor (call Program 2 if Re>2100)

$n = 2$ Darcy-Weisbach

Head loss

```
Head loss in all pipes
```

$$HL\_NR(Q) := \left| \overrightarrow{Calc\_HL\_NR\left(Q, Ks, \frac{D}{m}, \frac{LL}{m}\right)} \right.$$

Testing only

$$HL\_NR\left(\frac{Q_{init}}{cumec}\right) = \begin{bmatrix} 1.9768 \\ 8.8302 \\ 0.9157 \\ 1.8313 \\ 1.0814 \\ 4.8014 \\ 0.9069 \\ 62.6084 \\ 0.5354 \\ 2.5575 \end{bmatrix}$$

Clockwise flow + ve
Anticlockwise flow - ve

```
Head loss in the 3 loops
```

$$eq\_NR_{Loops}(Q) := \left| \begin{bmatrix} -HL\_NR(Q)_1 + HL\_NR(Q)_2 + HL\_NR(Q)_3 - HL\_NR(Q)_4 \\ -HL\_NR(Q)_3 + HL\_NR(Q)_8 + HL\_NR(Q)_9 + HL\_NR(Q)_{10} \\ HL\_NR(Q)_4 - HL\_NR(Q)_5 + HL\_NR(Q)_6 - HL\_NR(Q_7)_7 \end{bmatrix} \right.$$

Testing only

$$eq\_NR_{Loops}\left(\frac{Q_{init}}{cumec}\right) = \begin{bmatrix} 5.9378 \\ 64.7857 \\ 4.6444 \end{bmatrix}$$

Define the system of 11 equations (8 Nodes + 3 Loops) ; 10 Variables (pipe flows).
Hence, Overdetermimed Nonlinear System

$$f\_NR(Q) := \left| \mathrm{stack}\left(eq_{nodes}(Q), eq\_NR_{Loops}(Q)\right) \right.$$

$$result\_NR := \left| al\_nleqsolve\left(guess, 10^{-4}, f\_NR(Q)\right) \right.$$

$$Q\_NR_{final} := result\_NR$$

In **Method 2**, finding "f" for each pipe involves the numerical solution of **Colebrook-White** equation as per ***Program 2***. Hence, it takes a **longer time** than Method 1.

$\mathrm{time}(0) - t1 = 14.8\ s$

Summary

Method 1 Final head loss in each pipe

$$Q_{final} = \begin{bmatrix} 0.1442 \\ 0.1558 \\ 0.056 \\ 0.0482 \\ 0.096 \\ 0.054 \\ 0.046 \\ 0.0999 \\ -0.0501 \\ -0.0501 \end{bmatrix} \quad HL\left(Q_{final}\right) = \begin{bmatrix} 3.326 \\ 4.4834 \\ 2.3583 \\ 3.5157 \\ 1.6287 \\ 1.6483 \\ 3.5354 \\ 15.6995 \\ -2.3468 \\ -10.9945 \end{bmatrix}$$

$$\text{normi}\left(f\left(Q_{final}\right)\right) = 6.794 \cdot 10^{-6}$$

$$f\left(Q_{final}\right) = \begin{bmatrix} 2.9447 \cdot 10^{-7} \\ -4.8125 \cdot 10^{-8} \\ -1.9665 \cdot 10^{-7} \\ 2.2789 \cdot 10^{-7} \\ -3.8278 \cdot 10^{-8} \\ 1.8542 \cdot 10^{-8} \\ -2.6455 \cdot 10^{-7} \\ 6.6995 \cdot 10^{-9} \\ 7.0598 \cdot 10^{-7} \\ -6.794 \cdot 10^{-6} \\ 2.463 \cdot 10^{-7} \end{bmatrix}$$

Method 2 Final head loss in each pipe

$$Q\_NR_{final} = \begin{bmatrix} 0.1435 \\ 0.1565 \\ 0.0561 \\ 0.0475 \\ 0.0961 \\ 0.0539 \\ 0.0461 \\ 0.1004 \\ -0.0496 \\ -0.0496 \end{bmatrix} \quad HL\left(Q\_NR_{final}\right) = \begin{bmatrix} 3.2966 \\ 4.5202 \\ 2.3646 \\ 3.4147 \\ 1.6309 \\ 1.6444 \\ 3.5451 \\ 15.8781 \\ -2.2946 \\ -10.7492 \end{bmatrix}$$

$$\text{normi}\left(f\_NR\left(Q\_NR_{final}\right)\right) = 1.8377 \cdot 10^{-6}$$

$$\text{time}(0) - t_0 = 22.5 \text{ s}$$

$$f\_NR\left(Q\_NR_{final}\right) = \begin{bmatrix} -1.5181 \cdot 10^{-8} \\ -2.3648 \cdot 10^{-8} \\ -4.4888 \cdot 10^{-9} \\ 9.2153 \cdot 10^{-8} \\ 1.5986 \cdot 10^{-8} \\ -6.3011 \cdot 10^{-8} \\ -1.9469 \cdot 10^{-9} \\ 1.366 \cdot 10^{-10} \\ 4.2784 \cdot 10^{-7} \\ -1.8377 \cdot 10^{-6} \\ 1.3972 \cdot 10^{-7} \end{bmatrix}$$