⊞—Util

⊞—Numerical Inverse Laplace Transform

⊞—equrep

⊞—isol

⊞—laplace

⊟—Laplace ODE examples

Solving ODE's by Laplace Transform

Shorthands

$$plot := \begin{cases} x_{lt}(x) \\ \text{augment}\left(\text{col}\left(x_{rk}, 1\right), \text{col}\left(x_{rk}, 2\right), ".", 10, \text{"green"}\right) \end{cases}$$

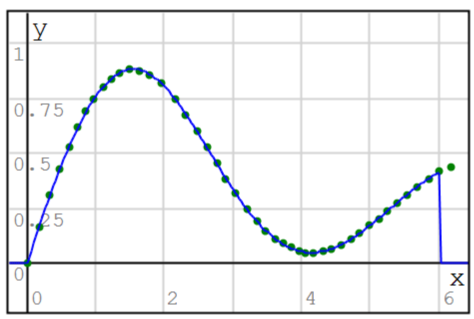
$$N := 25$$

$$ode := a \cdot \frac{d^2}{dt^2} x(t) + b \cdot \frac{d}{dt} x(t) + c \cdot x(t) = \varphi(t)$$

$$RK\left(t_{end}\right) := \left| \begin{array}{l} D(t, x) := \frac{1}{a} \cdot \begin{bmatrix} a \cdot x_2 \\ \varphi(t) - c \cdot x_1 - b \cdot x_2 \end{bmatrix} \\ RK23\left(D(t, x), \begin{bmatrix} 0 & t_{end} \end{bmatrix}, \begin{bmatrix} x_o & x'_o \end{bmatrix}, N, 10^{-3}\right) \end{array} \right.$$

$$\varphi(t) := 1 - \frac{\exp(-t)}{2} \cdot \cos(3 \cdot t) \qquad \begin{bmatrix} a & b & c \end{bmatrix} := \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \qquad \begin{bmatrix} x_o & x'_o \end{bmatrix} := \begin{bmatrix} 0 & 1 \end{bmatrix} \qquad t_{end} := 6$$

$$\begin{cases} X_{lt}(s) := isol\left(lt(ode), X(s)\right) \\ x_{lt} := ILT\left(X_{lt}(s), 0, t_{end}, N\right) \\ x_{lt}(t) := cinterp\left(x_{lt}, t\right) \\ x_{rk} := RK\left(t_{end}\right) \end{cases}$$



Where

$$ode = 2 \cdot \frac{d}{dt} \frac{d}{dt} x(t) + 3 \cdot x(t) + \frac{d}{dt} x(t) = \frac{2 - \exp(-t) \cdot \cos(3 \cdot t)}{2}$$

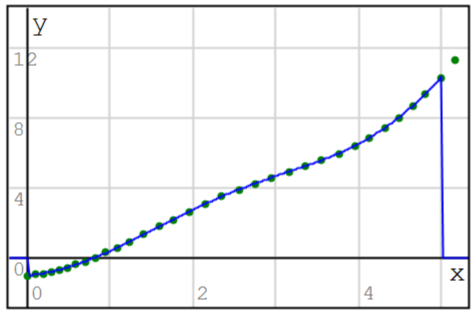
$$lt(ode) = X(s) \cdot (3 + s) + 2 \cdot \left(-1 + s^2 \cdot X(s)\right) = \frac{2 \cdot \left(9 + (1 + s)^2\right) - (1 + s) \cdot s}{2 \cdot \left(9 + (1 + s)^2\right) \cdot s}$$

$$isolate\left(lt(ode), X(s)\right) = X(s) = \frac{2 \cdot \left(9 + (1 + s)^2\right) \cdot (1 + 2 \cdot s) - (1 + s) \cdot s}{2 \cdot \left(9 + (1 + s)^2\right) \cdot s \cdot (3 + s \cdot (1 + 2 \cdot s))}$$

$\varphi(t) := t^2 + \sin(t)$

$[a\ b\ c] := [1\ 0\ 2]$ $[x_o\ x'_o] := [-1\ 0.5]$ $t_{end} := 5$

$$\begin{cases} X_{lt}(s) := isol(lt(ode), X(s)) \\ x_{lt} := ILT(X_{lt}(s), 0, t_{end}, N) \\ x_{lt}(t) := cinterp(x_{lt}, t) \\ x_{rk} := RK(t_{end}) \end{cases}$$



Where

$$ode = 2\cdot x(t) + \frac{d}{dt}\frac{d}{dt}x(t) = t^2 + \sin(t)$$

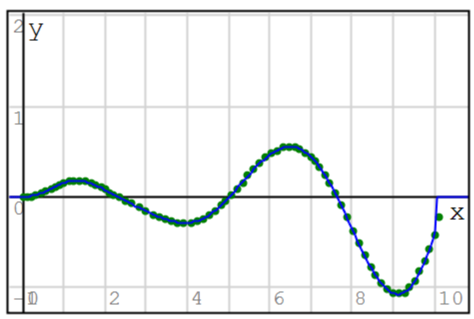
$$lt(ode) = \frac{-1 + 2\cdot(s\cdot(1 + s\cdot X(s)) + 2\cdot X(s))}{2} = \frac{s^3 + 2\cdot(1+s^2)}{s^3\cdot(1+s^2)}$$

$$isolate(lt(ode), X(s)) = X(s) = \frac{(1+s^2)\cdot s^3\cdot(1-2\cdot s) + 2\cdot(s^3 + 2\cdot(1+s^2))}{2\cdot(1+s^2)\cdot s^3\cdot(2+s^2)}$$

$\varphi(t) := \exp(-0.5\cdot t)\cdot\sin(3\cdot t - 2)$

$[a\ b\ c] := [-2\ 1\ -3]$ $[x_o\ x'_o] := [0\ 0]$ $t_{end} := 10$

$$\begin{cases} X_{lt}(s) := isol(lt(ode), X(s)) \\ x_{lt} := ILT(X_{lt}(s), 0, t_{end}, N) \\ x_{lt}(t) := cinterp(x_{lt}, t) \\ x_{rk} := RK(t_{end}) \end{cases}$$



Where

$$ode = -\left(2\cdot\frac{d}{dt}\frac{d}{dt}x(t) + 3\cdot x(t) - \frac{d}{dt}x(t)\right) = \exp\left(-\frac{t}{2}\right)\cdot\sin(-2 + 3\cdot t)$$

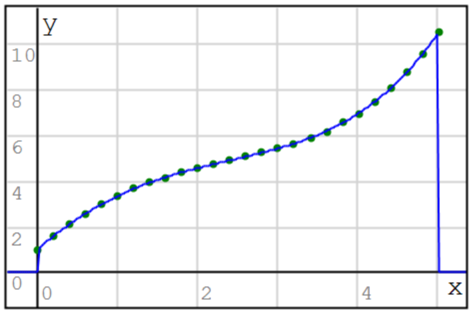
$$lt(ode) = -X(s)\cdot(3 + s\cdot(-1 + 2\cdot s)) = \frac{2\cdot(-(1+2\cdot s)\cdot\sin(2) + 6\cdot\cos(2))}{36 + (1 + 2\cdot s)^2}$$

$$isolate(lt(ode), X(s)) = X(s) = -\frac{2\cdot(-(1+2\cdot s)\cdot\sin(2) + 6\cdot\cos(2))}{(36 + (1+2\cdot s)^2)\cdot(3 + s\cdot(-1 + 2\cdot s))}$$

$\varphi(t) := t^2 \cdot \cosh\left(\frac{t}{5}\right)$

$[a \; b \; c] := [5 \; 2 \; 1]$ $\quad [x_o \; x'_o] := [1 \; 3]$ $\quad t_{end} := 5$

$$\begin{cases} X_{lt}(s) := isol\left(lt(ode), X(s)\right) \\ x_{lt} := ILT\left(X_{lt}(s), 0, t_{end}, N\right) \\ x_{lt}(t) := cinterp\left(x_{lt}, t\right) \\ x_{rk} := RK\left(t_{end}\right) \end{cases}$$



Where

$$ode = 5 \cdot \frac{d}{d\,t}\frac{d}{d\,t} x(t) + 2 \cdot \frac{d}{d\,t} x(t) + x(t) = t^2 \cdot \cosh\left(\frac{t}{5}\right)$$

$$lt(ode) = 2 \cdot (-1 + s \cdot X(s)) + 5 \cdot (-3 + s \cdot (-1 + s \cdot X(s))) + X(s) = -\frac{250 \cdot ((5 \cdot s \cdot \cosh(0) + \sinh(0)) \cdot (-1 + 2 \ldots}{\ldots}$$

$$isolate(lt(ode), X(s)) = X(s) = \frac{2 \cdot \left(\left(-1 + 25 \cdot s^2\right)^3 - 125 \cdot \left((5 \cdot s \cdot \cosh(0) + \sinh(0)) \cdot \left(-1 + 25 \cdot s^2\right) + 10 \cdot (\ldots\right.\right.}{\left(-1 + 25 \ldots\right.}$$

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