Example 13.1 : Page 400

Reinforced Concrete - Design Theory and Examples

Second Edition

T.J. Macginley & B.S. Choo

appVersion(−4) = "0.99.7822.147"

appVersion(4) = "0.99.7822.147"

$t_0 := \text{time}(0)$

https://www.academia.edu/34894173/Reinforced_Concrete_Design_Theory_and_Examples

Example 13.1 Simplified analysis of concrete framed building—vertical load

The application of the various simplified methods of analysis given in section 3.2 of the code is shown in the following example.

(a) Specification

The cross-section of a reinforced concrete building is shown in Fig. 13.5(a). The frames are at 4.5 m centres, the length of the building is 36 m and the column bases are fixed. Preliminary sections for the beams and columns are shown in Fig. 13.5(b). The floor and roof slabs are **designed to span one way** between the frames. Longitudinal beams are provided between external columns of the roof and floor levels only.:

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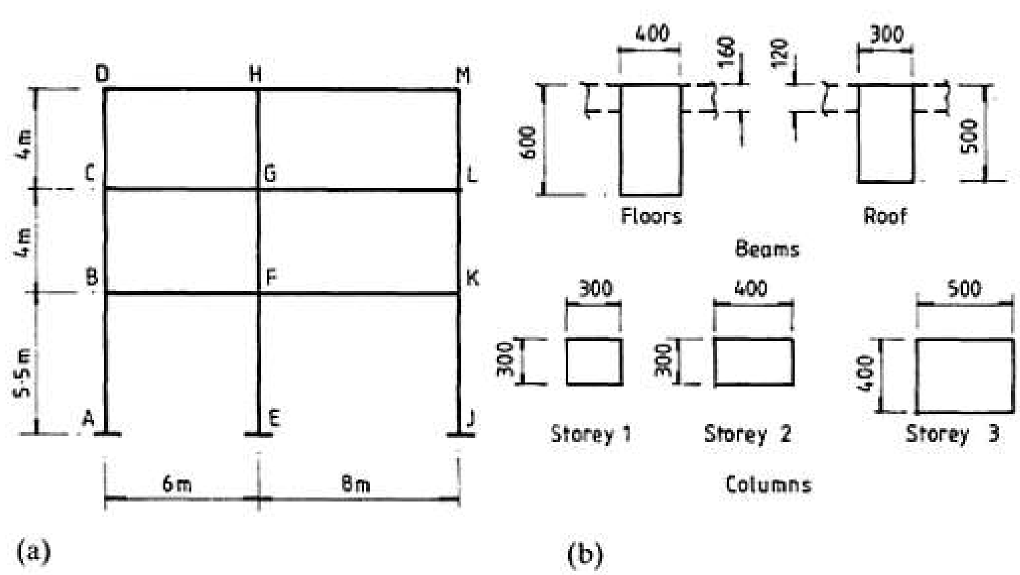


Fig. 13.5 (a) Cross-section; (b) assumed member sections.

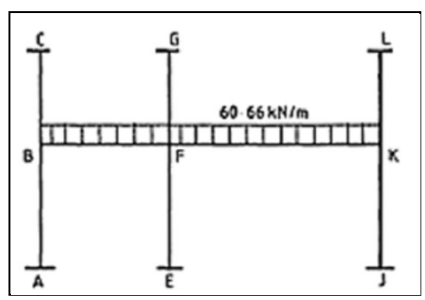
Frames are spaced at 4.5 m

$S_{frame} := 4.5\ \text{m}$

$L_{building} := 36\ \text{m}$

$\gamma_{cu} := 24\ \frac{\text{kN}}{\text{m}^3}$

This worked example shows only the analysis of the sub-frame shown. Hence, only Dead & Imposed loads on Beams BF & FK are considered.



More Iterations -----> more accuracy

For column **GF**

$L_{BF} := 6\ \text{m}$

$L_{FK} := 8\ \text{m}$

Number of iterations for Moment Iterations

$N_{Iter1} := 3$

$col_{FE\_b} := 400\ \text{mm}$ $col_{FE\_h} := 500\ \text{mm}$

$col_{GF\_b} := 300\ \text{mm}$ $col_{GF\_h} := 400\ \text{mm}$

$beam_{FK\_b} := 400\ \text{mm}$ $beam_{FK\_h} := 600\ \text{mm}$

Loadings

For the floors

Floors D/L

$g_{k\_floor} := 6.2 \ \frac{kN}{m^2}$

Floors I/L

$q_{k\_floor} := 3.0 \ \frac{kN}{m^2}$

Following load cases are required for the beam BFK for the braced frame.
But, only Case 1 is shown in this example.

Case 1 $1.4 \cdot G_k + 1.6 \cdot Q_k$ on the whole beam

Case 2 $1.4 \cdot G_k + 1.6 \cdot Q_k$ on BF and $1.6 \cdot G_k$ on FK

Case 3 $1.0 \cdot G_k$ on BF and $1.4 \cdot G_k + 1.6 \cdot Q_k$ on FK

Loads on Beams

Dead loads on Beams

$G_{k\_floor} := g_{k\_floor} \cdot S_{frame} = 27.9 \ \frac{kN}{m}$

Imposed loads on Beams

$Q_{k\_floor} := q_{k\_floor} \cdot S_{frame} = 13.5 \ \frac{kN}{m}$

Calculation of Moments of Inertia

$I_{Col\_FE} := \frac{1}{12} \cdot col_{FE\_b} \cdot \left(col_{FE\_h}\right)^3 = 4.1667 \cdot 10^9 \ mm^4$

$I_{Col\_FG} := \frac{1}{12} \cdot col_{GF\_b} \cdot \left(col_{GF\_h}\right)^3 = 1.6 \cdot 10^9 \ mm^4$

$I_{beam\_FK} := \frac{1}{12} \cdot beam_{FK\_b} \cdot \left(beam_{FK\_h}\right)^3 = 7.2 \cdot 10^9 \ mm^4$

$I_{beam\_FB} := I_{beam\_FK} = 7.2 \cdot 10^9 \ mm^4$

Beam BF and FK : same cross section

Moment of Inertia

$$I := \begin{bmatrix} I_{Col\_FE} \\ I_{Col\_FG} \\ I_{beam\_FK} \\ I_{beam\_FB} \end{bmatrix} = \begin{bmatrix} 4.1667 \cdot 10^9 \\ 1.6 \cdot 10^9 \\ 7.2 \cdot 10^9 \\ 7.2 \cdot 10^9 \end{bmatrix} mm^4$$

Length Cols & Beams

$$L := \begin{bmatrix} 5500 \\ 4000 \\ 8000 \\ 6000 \end{bmatrix} mm$$

Stiffness

$$\begin{bmatrix} K_{FE} \\ K_{FG} \\ K_{FK} \\ K_{FB} \end{bmatrix} := \overrightarrow{\frac{I}{L}} = \begin{bmatrix} 7.5758 \\ 4 \\ 9 \\ 12 \end{bmatrix} 10^5 \ mm^3$$

(d) : Subframe analysis for braced frames

The subframe consists of the beams at first floor level and the columns above and below that level with ends fixed.
The frame is analysed for the dead and imposed load cases given in above.
The distribution factors at the joints of the subframe are, for joint **B, F, K are as follows**

Distribution Factors at the joints of the subframe for

Joint B

$$K_B := \begin{bmatrix} K_{FG} \\ K_{FB} \\ K_{FE} \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 7.5758 \end{bmatrix} mm^3 \cdot 10^5$$

Joint F

$$K_F := \begin{bmatrix} K_{FE} \\ K_{FG} \\ K_{FK} \\ K_{FB} \end{bmatrix} = \begin{bmatrix} 7.5758 \\ 4 \\ 9 \\ 12 \end{bmatrix} mm^3 \cdot 10^5$$

Joint K

$$K_K := \begin{bmatrix} K_{FG} \\ K_{FK} \\ K_{FE} \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 7.5758 \end{bmatrix} mm^3 \cdot 10^5$$

$K_{B\_tot} := \sum K_B = 2.36 \cdot 10^6\ mm^3$ $K_{F\_tot} := \sum K_F = 32.58\ 10^5\ mm^3$ $K_{KF\_tot} := \sum K_K = 20.58\ 10^5\ mm^3$

$D_{BC} := \frac{K_{FG}}{K_{B\_tot}} = 0.17$

$D_{BF} := \frac{K_{FB}}{K_{B\_tot}} = 0.51$

$D_{BA} := \frac{K_{FE}}{K_{B\_tot}} = 0.32$

$D_{FE} := \frac{K_{FE}}{K_{F\_tot}} = 0.23$

$D_{FG} := \frac{K_{FG}}{K_{F\_tot}} = 0.12$

$D_{FK} := \frac{K_{FK}}{K_{F\_tot}} = 0.28$

$D_{FB} := \frac{K_{FB}}{K_{F\_tot}} = 0.37$

$D_{KL} := \frac{K_{FG}}{K_{KF\_tot}} = 0.19$

$D_{KF} := \frac{K_{FK}}{K_{KF\_tot}} = 0.44$

$D_{KJ} := \frac{K_{FE}}{K_{KF\_tot}} = 0.37$

Sanity check

$D_{BC} + D_{BF} + D_{BA} = 1$

Sanity check

$D_{FE} + D_{FG} + D_{FK} + D_{FB} = 1$

Sanity check

$D_{KL} + D_{KF} + D_{KJ} = 1$

Fixed End Moments

Case I

The FEMs are for case1 (F1) fon both beams

$F1 := 1.4 \cdot G_{k\_floor} + 1.6 \cdot Q_{k\_floor} = 60.66\ \frac{kN}{m}$

Case II (Not considered in this example)

$F2 := 1.0 \cdot G_{k\_floor} = 27.9\ \frac{kN}{m}$

FEM for span BF: Case I

$FEM_{BF} := -F1 \cdot \frac{L_{BF}^2}{12} = -181.98\ kN\ m$

FEM for FB: Case I

$FEM_{FB} := -FEM_{BF} = 181.98\ kN\ m$

FEM for span FK: Case I

$FEM_{FK} := -F1 \cdot \frac{L_{FK}^2}{12} = -323.52\ kN\ m$

FEM for span KF: Case I

$FEM_{KF} := -FEM_{FK} = 323.52\ kN\ m$

These programs in Yellow Color) are applicable to any Moment Distribution Method used below

To find column totals

$$tot\_BMM(M\#) := \begin{array}{|l} j := [1..cols(M\#)] \\ A\#_j := \sum col(M\#, j) \end{array}$$

To stack nested matrix

$$Stack\_M(M) := \begin{array}{|l} A := M_1 \\ \text{for } j \in [2..rows(M)] \\ \quad A := stack(A, M_j) \\ A \end{array}$$

$N_{Iter1} := 3$

$N_{Iter2} := 6$

Arrange result matrix for Table

$$Arrange(d, fem\#, df) := \begin{array}{|l} d := \dfrac{Stack\_M(d)}{kN\ m} \\ A1 := d_{[1..(rows(d)-1)][1..cols(d)]} \\ A2 := stack\left(\dfrac{fem\#}{kN\ m}, A1\right) \\ A3 := tot\_BMM(A2)^T \\ A4 := stack(df^T, A2, A3) \end{array}$$

To replace zeros with blank space.
(To improve readability in Table).

$$format(M\#) := \begin{array}{|l} \text{for } j \in [1..rows(M\#)] \\ \quad \text{for } k \in [1..cols(M\#)] \\ \quad\quad \text{if } (M\#_{j\,k} = 0) \\ \quad\quad\quad BB_{j\,k} := "" \\ \quad\quad \text{else} \\ \quad\quad\quad BB_{j\,k} := M\#_{j\,k} \\ BB \end{array}$$

$P1 := \begin{bmatrix} "Bal" \\ "CO" \end{bmatrix}$ $P2 := \begin{bmatrix} "DF" \\ "FEM" \end{bmatrix}$

$P3 := [\ "Final\ BM"\ ]$

Create Left Stub for Table

$$Create\_LS(n) := \begin{array}{|l} B := P1 \\ \text{for } j \in [1..(n-2)] \\ \quad B := stack(B, P1) \\ B := B_{[1..(rows(B)-1)][1..cols(B)]} \\ stack(P2, B, P3) \end{array}$$

1. METHOD 1 : Solution : With Far End Column Moments

Number of members

$N_{members1} := 16$

PROGRAM : Moment Balancing

$$Bal\_M1(M) := \begin{array}{|l} \begin{bmatrix} dM1 := 0 \\ dM5 := 0 \\ dM6 := 0 \\ dM11 := 0 \\ dM12 := 0 \\ dM16 := 0 \end{bmatrix} \\ dM2 := -M_1 \cdot [\,D_{BA}\ D_{BF}\ D_{BC}\,] \\ dM7 := -(M_2 + M_3) \cdot [\,D_{FE}\ D_{FB}\ D_{FG}\ D_{FK}\,] \\ dM13 := -(M_4) \cdot [\,D_{KJ}\ D_{KF}\ D_{KL}\,] \\ A := augment(dM1, dM2, dM5, dM6, dM7) \\ A := augment(A, dM11, dM12, dM13, dM16) \end{array}$$

Moment Carry Over

$$CO\_M1(M, BL) := \begin{array}{|l} M_1 := 0.5 \cdot BL_2 \\ M_3 := 0.5 \cdot BL_8 \\ M_5 := 0.5 \cdot BL_4 \\ M_6 := 0.5 \cdot BL_7 \\ M_8 := 0.5 \cdot BL_3 \\ M_{10} := 0.5 \cdot BL_{14} \\ M_{11} := 0.5 \cdot BL_9 \\ M_{12} := 0.5 \cdot BL_{13} \\ M_{14} := 0.5 \cdot BL_{10} \\ M_{16} := 0.5 \cdot BL_{15} \end{array}$$

A	B			C	E	F				G	J	K			L
AB	BA	BF	BC	CB	EF	FE	FB	FG	FK	GF	JK	KJ	KF	KL	LK

Moment Distribution for METHOD 1
iter = Iterations , mem = Number of Elements

```
Find_BMM1(iter, mem) := | "Initialize all row vector"
                        | [ fem_{1 mem} := 0
                        |   ba_{1 mem} := 0
                        |   co_{1 mem} := 0 ]
                        | fem_3 := FEM_BF
                        | fem_8 := − fem_3
                        | fem_10 := FEM_FK
                        | fem_14 := − fem_10
                        | for j ∈ [1..iter]
                        |   | if j = 1
                        |   |   | ba_j := Bal_M1([[fem_3, fem_8, fem_10, fem_14]])
                        |   |   | co1_j := CO_M1(co, ba_j)
                        |   | else
                        |   |   | ba_j := Bal_M1([[co_3, co_8, co_10, co_14]])
                        |   |   | co1_j := CO_M1(co, ba_j)
                        |   | D_j := stack(ba_j, co1_j)
                        | Arrange(D, fem, DF1)
```

DF for METHOD 1

$$DF1 := \begin{bmatrix} 0 \\ D_{BA} \\ D_{BF} \\ D_{BC} \\ 0 \\ 0 \\ D_{FE} \\ D_{FB} \\ D_{FG} \\ D_{FK} \\ 0 \\ 0 \\ D_{KJ} \\ D_{KF} \\ D_{KL} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.32 \\ 0.51 \\ 0.17 \\ 0 \\ 0 \\ 0.23 \\ 0.37 \\ 0.12 \\ 0.28 \\ 0 \\ 0 \\ 0.37 \\ 0.44 \\ 0.19 \\ 0 \end{bmatrix}$$

$LH1 := Create\_LS\left(N_{Iter1} + 1\right)$

$$LH1 = \begin{bmatrix} \text{"DF"} \\ \text{"FEM"} \\ \text{"Bal"} \\ \text{"CO"} \\ \text{"Bal"} \\ \text{"CO"} \\ \text{"Bal"} \\ \text{"Final BM"} \end{bmatrix}$$

$N_{Iter1} = 3$ $N_{members1} = 16$ $\text{rows}(LH1) = 8$

$MD1 := Find\_BMM1\left(N_{Iter1}, N_{members1}\right)$ **16 members in Methd 1** $\text{cols}(MD1) = 16$

$MD1 =$

0	0.32	0.51	0.17	0	0	0.23	0.37	0.12	0.28	0	0	0.37	0.44
0	0	−181.98	0	0	0	0	181.98	0	−323.52	0	0	0	323.52
0	58.48	92.63	30.88	0	0	32.92	52.14	17.38	39.1	0	0	−119.12	−141.51
29.24	0	26.07	0	15.44	16.46	0	46.31	0	−70.76	8.69	−59.56	0	19.55
0	−8.38	−13.27	−4.42	0	0	5.68	9	3	6.75	0	0	−7.2	−8.55
−4.19	0	4.5	0	−2.21	2.84	0	−6.63	0	−4.28	1.5	−3.6	0	3.38
0	−1.45	−2.29	−0.76	0	0	2.54	4.02	1.34	3.01	0	0	−1.24	−1.48
25.05	48.65	−74.34	25.69	13.23	19.3	41.14	286.82	21.72	−349.68	10.19	−63.16	−127.56	194.91

...

```
Format results.
(replace zeros with blanks
```

$C1 := format(MD1)$

```
Heading for Results Table
```

$Hd1 := [$ "AB" "BA" "BF" "BC" "CB" "EF" "FE" "FB" "FG" "FK" "GF" "JK" "KJ" "KF" "KL" "LK" $]$

METHOD 1 : With Far End Column Moments - 3 Iterations as in Text Book
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	AB	BA	BF	BC	CB	EF	FE	FB	FG	FK	GF	JK	KJ	KF	KL	LK
DF		0.32	0.51	0.17			0.23	0.37	0.12	0.28			0.37	0.44	0.19	
FEM			-181.98					181.98		-323.52				323.52		
Bal		58.48	92.63	30.88			32.92	52.14	17.38	39.1			-119.12	-141.51	-62.89	
CO	29.24		26.07		15.44	16.46		46.31		-70.76	8.69	-59.56		19.55		-31.45
Bal		-8.38	-13.27	-4.42			5.68	9	3	6.75			-7.2	-8.55	-3.8	
CO	-4.19		4.5		-2.21	2.84		-6.63		-4.28	1.5	-3.6		3.38		-1.9
Bal		-1.45	-2.29	-0.76			2.54	4.02	1.34	3.01			-1.24	-1.48	-0.66	
Final BM	25.05	48.65	-74.34	25.69	13.23	19.3	41.14	286.82	21.72	-349.68	10.19	-63.16	-127.56	194.91	-67.35	-33.35

C1

$time(0) - t_0 = 0.12\ s$

A	B			C	E	F				G	J	K			L
AB	BA	BF	BC	CB	EF	FE	FB	FG	FK	GF	JK	KJ	KF	KL	LK
	0·32	0·51	0·17			0·23	0·37	0·12	0·28			0·37	0·44	0·19	
	58·2	-181·9 92·8	30·9			32·5	181·9 52·3	16·9	-323·2 39·6			-119·6	323·2 -142·2	-61·4	
29·1	-8·3	26·2 -13·4	-4·5	15·5	16·3	5·7	46·4 9·1	3·0	-71·1 6·9	8·5	-59·8	-7·3	19·8 -8·7	-3·8	-30·7
-4·2	-1·5	4·6 -2·3	-0·8	-2·3	2·9	2·6	-6·7 4·1	1·3	-4·4 3·1	1·5	-3·7	-1·3	3·5 -1·5	-0·7	-1·9
24·9	48·4	-74·0	25·6	13·3	19·2	40·8	267·1	21·2	-349·1	10·0	-63·5	-128·2	194·1	-65·9	-32·6

Text book results : This gives moments of far end of columns also.

Fig. 13.6

2. METHOD 1 : With More Iterations

$N_{Iter2} = 6$ $N_{members1} = 16$ $LH2 := Create\_LS\left(N_{Iter2} + 1\right)$

$MD2 := Find\_BMM1\left(N_{Iter2}, N_{members1}\right)$

$C2 := format\left(MD2\right)$

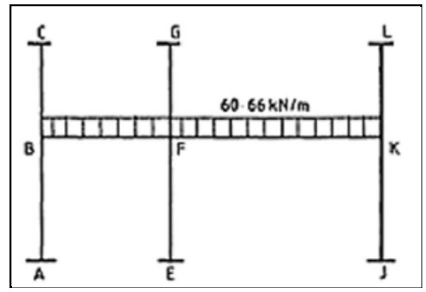
Header same as in Method 1

$Hd1$

Dist Factors same as in Method1

$DF1$

$$LH2 = \begin{bmatrix} \text{"DF"} \\ \text{"FEM"} \\ \text{"Bal"} \\ \text{"CO"} \\ \text{"Bal"} \\ \text{"CO"} \\ \text{"Bal"} \\ \text{"CO"} \\ \text{"Bal"} \\ \text{"CO"} \\ \text{"Bal"} \\ \text{"CO"} \\ \text{"Bal"} \\ \text{"Final BM"} \end{bmatrix}$$



$$MD2 = \begin{bmatrix} 0 & 0.32 & 0.51 & 0.17 & 0 & 0 & 0.23 & 0.37 & 0.12 & 0.28 & 0 & 0 & 0.37 & 0.44 \\ 0 & 0 & -181.98 & 0 & 0 & 0 & 0 & 181.98 & 0 & -323.52 & 0 & 0 & 0 & 323.52 \\ 0 & 58.48 & 92.63 & 30.88 & 0 & 0 & 32.92 & 52.14 & 17.38 & 39.1 & 0 & 0 & -119.12 & -141.51 \\ 29.24 & 0 & 26.07 & 0 & 15.44 & 16.46 & 0 & 46.31 & 0 & -70.76 & 8.69 & -59.56 & 0 & 19.55 \\ 0 & -8.38 & -13.27 & -4.42 & 0 & 0 & 5.68 & 9 & 3 & 6.75 & 0 & 0 & -7.2 & -8.55 \\ -4.19 & 0 & 4.5 & 0 & -2.21 & 2.84 & 0 & -6.63 & 0 & -4.28 & 1.5 & -3.6 & 0 & 3.38 \\ 0 & -1.45 & -2.29 & -0.76 & 0 & 0 & 2.54 & 4.02 & 1.34 & 3.01 & 0 & 0 & -1.24 & -1.48 \\ -0.72 & 0 & 2.01 & 0 & -0.38 & 1.27 & 0 & -1.15 & 0 & -0.74 & 0.67 & -0.62 & 0 & 1.51 \\ 0 & -0.65 & -1.02 & -0.34 & 0 & 0 & 0.44 & 0.69 & 0.23 & 0.52 & 0 & 0 & -0.55 & -0.66 \\ -0.32 & 0 & 0.35 & 0 & -0.17 & 0.22 & 0 & -0.51 & 0 & -0.33 & 0.12 & -0.28 & 0 & 0.26 \\ 0 & -0.11 & -0.18 & -0.06 & 0 & 0 & 0.2 & 0.31 & 0.1 & 0.23 & 0 & 0 & -0.1 & -0.11 \\ -0.06 & 0 & 0.15 & 0 & -0.03 & 0.1 & 0 & -0.09 & 0 & -0.06 & 0.05 & -0.05 & 0 & 0.12 \\ 0 & -0.05 & -0.08 & -0.03 & 0 & 0 & 0.03 & 0.05 & 0.02 & 0.04 & 0 & 0 & -0.04 & -0.05 \\ 23.95 & 47.85 & -73.11 & 25.26 & 12.64 & 20.89 & 41.81 & 286.13 & 22.07 & -350.01 & 11.03 & -64.1 & -128.25 & 195.97 \end{bmatrix} \cdots$$

METHOD 1 : With Far End Column Moments - With 6 Iterations
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	AB	BA	BF	BC	CB	EF	FE	FB	FG	FK	GF	JK	KJ	KF	KL	LK
DF		0.32	0.51	0.17			0.23	0.37	0.12	0.28			0.37	0.44	0.19	
FEM			-181.98					181.98		-323.52				323.52		
Bal		58.48	92.63	30.88			32.92	52.14	17.38	39.1			-119.12	-141.51	-62.89	
CO	29.24		26.07		15.44	16.46		46.31		-70.76	8.69	-59.56		19.55		-31.45
Bal		-8.38	-13.27	-4.42			5.68	9	3	6.75			-7.2	-8.55	-3.8	
CO	-4.19		4.5		-2.21	2.84		-6.63		-4.28	1.5	-3.6		3.38		-1.9
Bal		-1.45	-2.29	-0.76			2.54	4.02	1.34	3.01			-1.24	-1.48	-0.66	
CO	-0.72		2.01		-0.38	1.27		-1.15		-0.74	0.67	-0.62		1.51		-0.33
Bal		-0.65	-1.02	-0.34			0.44	0.69	0.23	0.52			-0.55	-0.66	-0.29	
CO	-0.32		0.35		-0.17	0.22		-0.51		-0.33	0.12	-0.28		0.26		-0.15
Bal		-0.11	-0.18	-0.06			0.2	0.31	0.1	0.23			-0.1	-0.11	-0.05	
CO	-0.06		0.15		-0.03	0.1		-0.09		-0.06	0.05	-0.05		0.12		-0.03
Bal		-0.05	-0.08	-0.03			0.03	0.05	0.02	0.04			-0.04	-0.05	-0.02	
Final BM	23.95	47.85	-73.11	25.26	12.64	20.89	41.81	286.13	22.07	-350.01	11.03	-64.1	-128.25	195.97	-67.72	-33.85

C2

3. METHOD 2 : Without Far End Column Moments

DF for METHOD 2

$$DF2 := \begin{bmatrix} D_{BA} \\ D_{BC} \\ D_{BF} \\ D_{FB} \\ D_{FE} \\ D_{FG} \\ D_{FK} \\ D_{KF} \\ D_{KJ} \\ D_{KL} \end{bmatrix} = \begin{bmatrix} 0.32 \\ 0.17 \\ 0.51 \\ 0.37 \\ 0.23 \\ 0.12 \\ 0.28 \\ 0.44 \\ 0.37 \\ 0.19 \end{bmatrix}$$

PROGRAM : Moment Balancing

$$Bal\_M2(M) := \left| \begin{array}{l} dM1 := -M_1 \cdot \begin{bmatrix} D_{BA} & D_{BC} & D_{BF} \end{bmatrix} \\ dM2 := \left(-\left(M_2 + M_3\right)\right) \cdot \begin{bmatrix} D_{FB} & D_{FE} & D_{FG} & D_{FK} \end{bmatrix} \\ dM3 := -M_4 \cdot \begin{bmatrix} D_{KF} & D_{KJ} & D_{KL} \end{bmatrix} \\ \text{augment}(dM1, dM2, dM3) \end{array} \right.$$

Carry Over Moment

$$CO\_M2(M, BL) := \left| \begin{array}{l} M_3 := 0.5 \cdot BL_4 \\ M_4 := 0.5 \cdot BL_3 \\ M_7 := 0.5 \cdot BL_8 \\ M_8 := 0.5 \cdot BL_7 \end{array} \right.$$

Moment Distribution for METHOD 2
iter = Iterations , mem = Number of Elements

```
Find_BMM2(iter, mem) := | "Initialize all row vector"
                        | [ fem_(1 mem) := 0
                        |   ba_(1 mem) := 0
                        |   co_(1 mem) := 0 ]
                        | fem_3 := FEM_BF
                        | fem_4 := -fem_3
                        | fem_7 := FEM_FK
                        | fem_8 := -fem_7
                        | for j ∈ [1..iter]
                        |   | if j = 1
                        |   |   | ba_j := Bal_M2([[fem_3, fem_4, fem_7, fem_8]])
                        |   |   | co1_j := CO_M2(co, ba_j)
                        |   | else
                        |   |   | ba_j := Bal_M2([[co_3, co_4, co_7, co_8]])
                        |   |   | co1_j := CO_M2(co, ba_j)
                        |   | D_j := stack(ba_j, co1_j)
                        | Arrange(D, fem, DF2)
```

$$LH3 := Create\_LS\left(N_{Iter2} + 1\right)$$

$$LH3 = \begin{bmatrix} \text{"DF"} \\ \text{"FEM"} \\ \text{"Bal"} \\ \text{"CO"} \\ \text{"Bal"} \\ \text{"CO"} \\ \text{"Bal"} \\ \text{"CO"} \\ \text{"Bal"} \\ \text{"CO"} \\ \text{"Bal"} \\ \text{"CO"} \\ \text{"Bal"} \\ \text{"Final BM"} \end{bmatrix} \qquad \text{rows}(LH3) = 14$$

$N_{Iter2} = 6$ $N_{members1} := 10$

10 members in Method 2

$$MD3 := Find\_BMM2\left(N_{Iter2}, N_{members1}\right)$$

$$MD3 = \begin{bmatrix} 0.32 & 0.17 & 0.51 & 0.37 & 0.23 & 0.12 & 0.28 & 0.44 & 0.37 & 0.19 \\ 0 & 0 & -181.98 & 181.98 & 0 & 0 & -323.52 & 323.52 & 0 & 0 \\ 58.48 & 30.88 & 92.63 & 52.14 & 32.92 & 17.38 & 39.1 & -141.51 & -119.12 & -62.89 \\ 0 & 0 & 26.07 & 46.31 & 0 & 0 & -70.76 & 19.55 & 0 & 0 \\ -8.38 & -4.42 & -13.27 & 9 & 5.68 & 3 & 6.75 & -8.55 & -7.2 & -3.8 \\ 0 & 0 & 4.5 & -6.63 & 0 & 0 & -4.28 & 3.38 & 0 & 0 \\ -1.45 & -0.76 & -2.29 & 4.02 & 2.54 & 1.34 & 3.01 & -1.48 & -1.24 & -0.66 \\ 0 & 0 & 2.01 & -1.15 & 0 & 0 & -0.74 & 1.51 & 0 & 0 \\ -0.65 & -0.34 & -1.02 & 0.69 & 0.44 & 0.23 & 0.52 & -0.66 & -0.55 & -0.29 \\ 0 & 0 & 0.35 & -0.51 & 0 & 0 & -0.33 & 0.26 & 0 & 0 \\ -0.11 & -0.06 & -0.18 & 0.31 & 0.2 & 0.1 & 0.23 & -0.11 & -0.1 & -0.05 \\ 0 & 0 & 0.15 & -0.09 & 0 & 0 & -0.06 & 0.12 & 0 & 0 \\ -0.05 & -0.03 & -0.08 & 0.05 & 0.03 & 0.02 & 0.04 & -0.05 & -0.04 & -0.02 \\ 47.85 & 25.26 & -73.11 & 286.13 & 41.81 & 22.07 & -350.01 & 195.97 & -128.25 & -67.72 \end{bmatrix}$$

```
Heading for Results Table
```

Hd3 := augment("BA", "BC", "BF", "FB", "FE", "FG", "FK", "KF", "KJ", "KL")

```
Format results.
(replace zeros with blanks
```

C3 := format(MD3)

METHOD 2 : Without Far End Column Moments - 6 Iterations
Example 13.1 : Page 398
Reinforced Concrete - Design Theory and Examples
Second Edition - T.J. Macginley & B.S. Choo

	BA	BC	BF	FB	FE	FG	FK	KF	KJ	KL
DF	0.32	0.17	0.51	0.37	0.23	0.12	0.28	0.44	0.37	0.19
FEM			-181.98	181.98			-323.52	323.52		
Bal	58.48	30.88	92.63	52.14	32.92	17.38	39.1	-141.51	-119.12	-62.89
CO			26.07	46.31			-70.76	19.55		
Bal	-8.38	-4.42	-13.27	9	5.68	3	6.75	-8.55	-7.2	-3.8
CO			4.5	-6.63			-4.28	3.38		
Bal	-1.45	-0.76	-2.29	4.02	2.54	1.34	3.01	-1.48	-1.24	-0.66
CO			2.01	-1.15			-0.74	1.51		
Bal	-0.65	-0.34	-1.02	0.69	0.44	0.23	0.52	-0.66	-0.55	-0.29
CO			0.35	-0.51			-0.33	0.26		
Bal	-0.11	-0.06	-0.18	0.31	0.2	0.1	0.23	-0.11	-0.1	-0.05
CO			0.15	-0.09			-0.06	0.12		
Bal	-0.05	-0.03	-0.08	0.05	0.03	0.02	0.04	-0.05	-0.04	-0.02
Final BM	47.85	25.26	-73.11	286.13	41.81	22.07	-350.01	195.97	-128.25	-67.72

C3

$time(0) - t_0 = 0.38\ s$