Recursions in SMath

SMath seems to have a limit of 66 for call recursive functions

```
f(n):= | n:=n+1
       | try
       |   f(n)
       | on error
       |   n
```

$f(0)=66$

Also, it could be very slow

$$F(n):=\begin{cases}0 & \text{if } n=0\\ 1 & \text{if } n=1\\ \text{eval}(F(n-1)+F(n-2)) & \text{otherwise}\end{cases}$$

$to:=\text{time}(0)$

$F(20)=6765$

$\text{time}(0)-to=2.905\ s$

This small piece of code can call a function for make a list of the recursion values

```
rec(f(2), a, N):= | v:=stack(a)
                  | for k ∈ [(cols(v)+1)..(N+1)]
                  |   v:=eval(augment(v, f(v, k)))
                  | v
```

rec Examples

Examples

- **Fibonacci numbers** $r(a,n):=a_{n-1}+a_{n-2}$

$rec(r(a,n),[0\ 1],15)=[0\ 1\ 1\ 2\ 3\ 5\ 8\ 13\ 21\ 34\ 55\ 89\ 144\ 233\ 377\ 610]$

$\phi:=\frac{1+\sqrt{5}}{2}$ $\quad F(n):=\overrightarrow{\frac{\phi^{n}-\cos(n\cdot\pi)\,\phi^{-n}}{\sqrt{5}}}$

$F([0..15])^{T}=[0\ 1\ 1\ 2\ 3\ 5\ 8\ 13\ 21\ 34\ 55\ 89\ 144\ 233\ 377\ 610]$

- **Lichtenberg sequence** $r(a,n):=2\cdot a_{n-2}+a_{n-1}+1$

$rec(r(a,n),[0\ 1],15)=[0\ 1\ 2\ 5\ 10\ 21\ 42\ 85\ 170\ 341\ 682\ 1365\ 2730\ 5461\ 10922\ 21845]$

$L(n):=\overrightarrow{\left\lfloor\frac{2^{n+1}-2}{3}\right\rfloor}$

$L([0..15])^{T}=[0\ 1\ 2\ 5\ 10\ 21\ 42\ 85\ 170\ 341\ 682\ 1365\ 2730\ 5461\ 10922\ 21845]$

- **Triangular numbers** $r(a,n):=n^{2}-a_{n-1}$

$rec(r(a,n),1,15)=[1\ 3\ 6\ 10\ 15\ 21\ 28\ 36\ 45\ 55\ 66\ 78\ 91\ 105\ 120\ 136]$

Also: $r(a,n):=a_{n-2}+2\cdot n-1$

$$rec\left(r(a,n),[1\ 3],15\right)=[1\ 3\ 6\ 10\ 15\ 21\ 28\ 36\ 45\ 55\ 66\ 78\ 91\ 105\ 120\ 136]$$

- **Somos-5 sequence**

$$r(a,n):=\frac{a_{n-1}\cdot a_{n-4}+a_{n-2}\cdot a_{n-3}}{a_{n-5}}$$

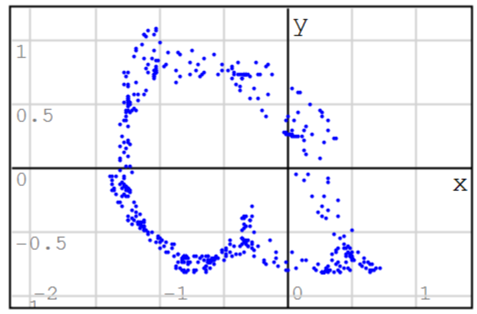
$$rec\left(r(a,n),[1\ 1\ 1\ 1\ 1],15\right)=[1\ 1\ 1\ 1\ 1\ 2\ 3\ 5\ 11\ 37\ 83\ 274\ 1217\ 6161\ 22833\ 165713]$$

- **Nonlinear map of the plane**

$$r(xy,n):=\text{eval}\left(\begin{bmatrix}0.7\cdot xy_{1\,n-1}+xy_{2\,n-1}\\ -0.7989995+\left(xy_{1\,n-1}\right)^2\end{bmatrix}\right)$$

where $\begin{cases}x_n=xy_{1\,n}\\ y_n=xy_{2\,n}\end{cases}$

$$P:=rec\left(r(a,n),\begin{bmatrix}0.142857\\0.33\end{bmatrix},500\right)$$



P^T

Also r could be

$$r(a,n):=\left\|\begin{array}{l}[x\ y]:=[\text{row}(a,1)\ \text{row}(a,2)]\\ \begin{bmatrix}0.7\cdot x_{n-1}+y_{n-1}\\ -0.7989995+\left(x_{n-1}\right)^2\end{bmatrix}\end{array}\right.$$

- **Solving difference or differential equations.**

$$ic:=\begin{bmatrix}0\\1\end{bmatrix}\qquad vf:=\begin{bmatrix}y\\ x\cdot\left(1-x^2\right)\end{bmatrix}$$

for using RK methods:

$$D(t,u):=vf\Big|\begin{cases}x=u_1\\ y=u_2\end{cases}$$

Discretizing for Euler Method

$$\text{Clear}(\text{ho})=1$$

$$vfn:=\begin{bmatrix}x_{n-1}\\ y_{n-1}\end{bmatrix}+\text{ho}\cdot vf\Big|\begin{cases}x=x_{n-1}\\ y=y_{n-1}\end{cases}=\begin{bmatrix}x_{-1+n}+\text{ho}\cdot y_{-1+n}\\ y_{-1+n}+x_{-1+n}\cdot\left(1-x_{-1+n}^{\ 2}\right)\text{ho}\end{bmatrix}$$

Discretizing for Symplectic Euler Method

$$vfn':=vf\Big|\begin{cases}x=x_{n-1}\\ y=y_{n-1}\end{cases}\quad vfn'_1:=x_{n-1}+\text{ho}\cdot vfn'_1\quad vfn'_2:=y_{n-1}+\text{ho}\cdot vf_2\Big|\begin{cases}x=vfn'_1\\ y=y_{n-1}\end{cases}$$

$$vfn'=\begin{bmatrix}\text{ho}\cdot y_{-1+n}+x_{-1+n}\\ \text{ho}\cdot\left(\text{ho}\cdot y_{-1+n}+x_{-1+n}\right)\cdot\left(1-\left(\text{ho}\cdot y_{-1+n}+x_{-1+n}\right)^2\right)+y_{-1+n}\end{bmatrix}$$

Rewriting for $x_n=xy_{1\,n}$ and $y_n=xy_{2\,n}$

$$vfn(xy,n):=\begin{bmatrix}\text{ho}\cdot xy_{2\,n-1}+xy_{1\,n-1}\\ xy_{2\,n-1}+xy_{1\,n-1}\cdot\left(1-\left(xy_{1\,n-1}\right)^2\right)\text{ho}\end{bmatrix}$$

$$vfn'(xy,n) := \begin{bmatrix} ho \cdot xy_{2\,n-1} + xy_{1\,n-1} \\ ho \cdot \left(ho \cdot xy_{2\,n-1} + xy_{1\,n-1}\right) \cdot \left(1 - \left(ho \cdot xy_{2\,n-1} + xy_{1\,n-1}\right)^2\right) + xy_{2\,n-1} \end{bmatrix}$$

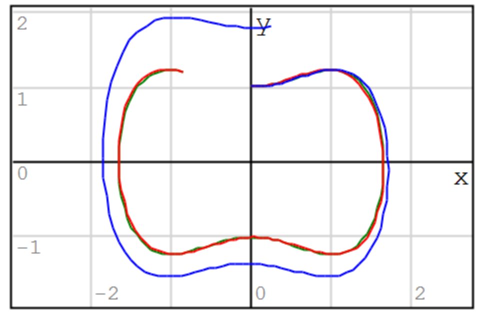
$ho := 0.06$

$N := 100$

$TXY := \text{rkfixed}(ic, 0, N\,ho, N, D(t,u))$

$$Plot := \begin{cases} rec(vfn(xy,n), ic, N)^T \\ rec(vfn'(xy,n), ic, N)^T \\ \text{augment}(\text{col}(TXY,2), \text{col}(TXY,3)) \end{cases}$$

blue,
red,
green.



Plot

- **Linear ODE with constant coefficients**

$u'' + 2 \cdot u' + 3 \cdot u = 0$

$vf := \begin{bmatrix} y \\ -2 \cdot y - 3 \cdot x \end{bmatrix}$ $D(t,u) := vf \left| \begin{cases} x = u_1 \\ y = u_2 \end{cases} \right.$

$ic := \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ Clear$(ho) = 1$

Euler

$$vfn(xy,n) := \begin{bmatrix} ho \cdot xy_{2\,n-1} + xy_{1\,n-1} \\ xy_{2\,n-1} - ho \cdot \left(xy_{2\,n-1} + 3 \cdot xy_{1\,n-1}\right) \end{bmatrix}$$

Same process than the previous example

Symplectic Euler

$$vfn'(xy,n) := \begin{bmatrix} ho \cdot xy_{2\,n-1} + xy_{1\,n-1} \\ -ho \cdot \left(2 \cdot xy_{2\,n-1} + 3 \cdot \left(ho \cdot xy_{2\,n-1} + xy_{1\,n-1}\right)\right) + xy_{2\,n-1} \end{bmatrix}$$

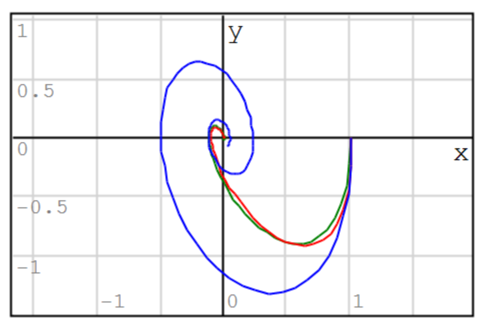
$ho := 0.08$

$N := 100$

$TXY := \text{rkfixed}(ic, 0, N\,ho, N, D(t,u))$

$$Plot := \begin{cases} rec(vfn(xy,n), ic, N)^T \\ rec(vfn'(xy,n), ic, N)^T \\ \text{augment}(\text{col}(TXY,2), \text{col}(TXY,3)) \end{cases}$$

blue,
red,
green.



Plot

- **Iterate a matrix recurrence**

$$r(M,n) := \left\| \left[M_{n-1} \cdot \begin{bmatrix} 0.1 & 1 \\ 0 & 0.1 \end{bmatrix} \cdot M_{n-1} \right] \right.$$

$$rec\left(r(M,n), \left[\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\right], 3\right) = \left[\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -0.1 & 0 \\ 1 & -0.1 \end{bmatrix} \begin{bmatrix} -0.099 & 0.01 \\ 0.98 & -0.099 \end{bmatrix} \begin{bmatrix} -0.0951 & 0.0096 \\ 0.941 & -0.0951 \end{bmatrix} \right]$$

- **Cliff random number generator**

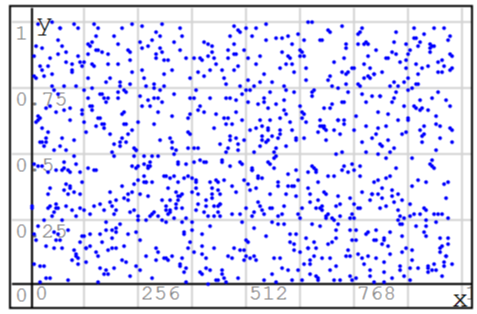
$Cliff(x,n) := \text{eval}\left(\left|\text{mod}\left(100 \cdot \text{log10}\left(x_{n-1}\right), 1\right)\right|\right)$ $seed := 0.3$

$rec(Cliff(x,n), seed, 6) = [\,0.3\ 0.2879\ 0.0797\ 0.8672\ 0.1885\ 0.4593\ 0.7912\,]$

$\left[N := 1000 \;\; r := [1..(N+1)] \right]$

$\rho := rec\left(Cliff(x, n), seed, N\right)^{T}$

$Plot := \text{augment}(r, \rho)$



Plot

Compare with
SMath's random

$\rho'_{r} := \text{eval}\left(10^{-9} \cdot \text{random}\left(10^{9}\right)\right)$

$\text{Mean}(\rho) = 0.4856$	$\text{Mean}(\rho') = 0.4862$
$\text{Kurtosis}(\rho) = 1.7938$	$\text{Kurtosis}(\rho') = 1.7933$
$\text{Skewness}(\rho) = 0.0654$	$\text{Skewness}(\rho') = 0.0697$

Alvaro

ToDO: rec for 2 index vars.