Piecewise continue functions

appVersion(4) = "0.99.7822.147"

Redefine sign as $sign(x\#) := \frac{|x\#|}{x\# + \varepsilon}$

with $\varepsilon := 0$ ... for now.

Shorthand $\Phi(x\#) := \text{Heaviside}(x\#)$

Notice that

$$\text{Heaviside}(x) = \frac{1 + \text{sign}(x)}{2}$$

Changes to

$$\text{Heaviside}(x) = \frac{x + \varepsilon + |x|}{2 \cdot (x + \varepsilon)}$$

and ...

$$\text{Heaviside}(x) = \frac{x + |x|}{2 \cdot x}$$

Example Let $y_1 := x^2 - 3 \cdot x + 1$ $y_2 := x + 1$ $xo := 2$

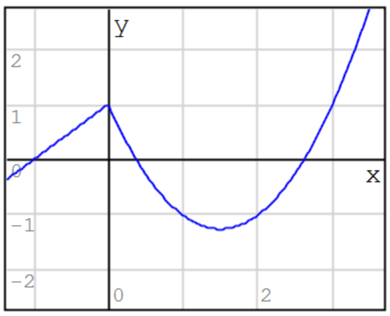
You can try to define without line too. (but f needs it fin integrals and "zero" case)

$f(x) := y_1 \cdot \Phi(x) + y_2 \cdot \Phi(-x)$

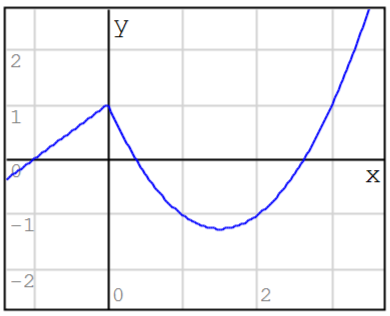
This is the correct way to define a piecewise function.

$g(x) :=$ if $x > 0$ y_1 else y_2

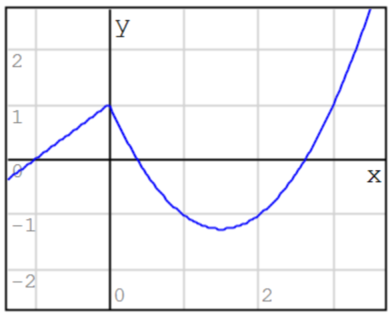
$h(x) := \begin{cases} y_1 & \text{if } x > 0 \\ y_2 & \text{otherwise} \end{cases}$



f(x)



g(x)



h(x)

We have only two reds for f:

$f(0) =$ ∎ $g(0) = 1$ $h(0) = 1$

Unknowns(f(x)) = [x] Unknowns(g(x)) = ∎ Unknowns(h(x)) = [x]

$\text{solve}(f(x), x) = \begin{bmatrix} -1 \\ 0.382 \\ 2.618 \end{bmatrix}$ solve(g(x), x) = ∎ $\text{solve}(h(x), x) = \begin{bmatrix} -1 \\ 0.382 \\ 2.618 \end{bmatrix}$

roots(f(x), x, xo) = 2.618 roots(g(x), x, xo) = ∎ roots(h(x), x, xo) = ∎

Broyden(f(x), xo) = 2.618 Broyden(g(x), xo) = ∎ Broyden(h(x), xo) = 2.618

$\frac{d}{d\,xo} f(xo) = 1$ $\frac{d}{d\,xo} g(xo) =$ ∎ $\frac{d}{d\,xo} h(xo) =$ ∎

$$\text{Jacob}(f(x), x) = \left[\frac{x - |-x| + (3 + 2 \cdot (-3 + x)) \cdot (x + |x|)}{2 \cdot x} \right]$$

Jacob(g(x), x) = ∎ Jacob(h(x), x) = ∎

$$\int_{-2}^{2} f(\xi)\,d\xi = \blacksquare \qquad \int_{-2}^{2} g(\xi)\,d\xi = \blacksquare \qquad \int_{-2}^{2} h(\xi)\,d\xi = -1.3333$$

We can correct the two errors for f with $\varepsilon := 0.00000000001$

$$f(0) = 1 \qquad \int_{-2}^{2} f(\xi)\,d\xi = -1.3333$$

Notice that all previous values remains unchanged.

Also, only for f: $\varepsilon := 0$

$$df(x) := \text{maple}\left(convert\left(\frac{d}{d\,x} f(x),\, Heaviside\right)\right)$$

As shown, Maple recognize the behavior of f, and this is why I say that it's the only "well defined" piecewise function.

$$df(xo) = 1 \qquad df(x) = \frac{x + |x| - 6 + 2\cdot x}{2}$$

$$adf(x) := \text{maple}\left(convert\left(\int f(x)\,d\,x,\, Heaviside\right)\right)$$

Same thing, more or less, with Maxima.

$$adf(2) - adf(-2) = -1.3333 \qquad adf(x) = \frac{x\cdot(3\cdot(x - 2\cdot(-1 + x + |x|)) + (x + |x|)\cdot x)}{6}$$

Conclussion: Notice that I only redefine **sign** for get correct answers for all the usual numeric and symbolics procedures. The errors related with cases could be hard to leave:

$$\frac{d}{d\,xo} h(xo) = \blacksquare \qquad$$ **lastError** = "xo - not defined."

$$\text{roots}(h(x), x, xo) = \blacksquare \qquad$$ **lastError** = "Inequalities are not supported."

$$\text{Jacob}(h(x), x) = \blacksquare \qquad$$ **lastError** = "Inequalities are not supported."

Also we can do some algebra with f, but not with h:

$$h(x) - h(-x) = \begin{cases} 1 + x\cdot(-3 + x) & \text{if } \quad x > 0 \\ 1 + x & \text{otherwise} \end{cases} - \begin{cases} 1 + x\cdot(3 + x) & \text{if } \quad -x > 0 \\ 1 - x & \text{otherwise} \end{cases}$$

$$f(x) - f(-x) = -\frac{-(1 + x\cdot(-3 + x))\cdot(x + |x|) + x\cdot(2 + x)\cdot(x - |-x|) + (1 - x)\cdot(x + |x|)}{2\cdot x}$$

and with SMath having a little more skills, we can se things like

$$\text{maple}(simplify(f(x) - f(-x))) = x\cdot(-2 + |x|)$$

Issues using Φ: First of all, time: with Φ the program evaluate both cases, with if only one. And maybe some errors when evaluate the body if it isn't the case, like in.

$$y_1 := \frac{1}{x + 1} \qquad f(-1) = \blacksquare \qquad g(-1) = 0 \qquad h(-1) = 0$$

and that's can't be solved with ε $\varepsilon := 0.00000000001$ $f(-1) = \blacksquare$ but isn't continue then. (see the title of this)

Finally, there are antoher "kind" of piecewise functions, where the test is foncion of the variable too, and ... well, there are a lot of more cases.

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