⊞—d

⊞—equrep

⊞—isol

⊞—Differentials samples

⊟—Lagrangians

Lagrangian Dynamics

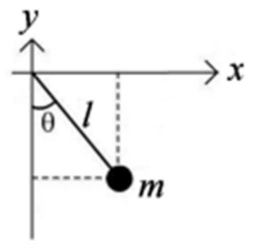
dSymbol := "∂"

$\mathrm{Clear}(a, g, \theta, \theta', \theta'') = 1$

Simple pendulum

Constants $\partial a := 0$ $\partial m := 0$ $\partial g := 0$

Gen Coords $\partial\theta := \theta' \cdot \partial t$ $\partial\theta' := \theta'' \cdot \partial t$



Position $x := a \cdot \sin(\theta)$ $y := -a \cdot \cos(\theta)$

$\dot{x} := \frac{d(x)}{\partial t}$ $\dot{y} := \frac{d(y)}{\partial t}$

Lagrangian $L := \frac{1}{2} \cdot m \cdot \left(\dot{x}^2 + \dot{y}^2\right) - m \cdot g \cdot y$

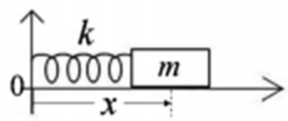
Equations of motion $eq := \left\{ isol\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial(L, \theta')}{\partial\theta'} \right) - \frac{\partial(L, \theta)}{\partial\theta}, \theta'' \right) \right. = \left\{ -\frac{g \cdot \sin(\theta)}{a} \right.$

$\mathrm{Clear}(k, m, x, \dot{x}, \ddot{x}) = 1$

Harmonic Oscillator

Constants $\partial k := 0$ $\partial m := 0$

Gen Coords $\partial x := \dot{x} \cdot \partial t$ $\partial\dot{x} := \ddot{x} \cdot \partial t$



Position x

Lagrangian $L := \frac{1}{2} \cdot m \cdot \dot{x}^2 - \frac{1}{2} \cdot k \cdot x^2$

Equations of motion $eq := \left\{ isol\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial(L, \dot{x})}{\partial\dot{x}} \right) - \frac{\partial(L, x)}{\partial x}, \ddot{x} \right) \right. = \{ \partial t \cdot \dot{x} \cdot (\ddot{x} \cdot m + x \cdot k) + m \cdot \dot{x} \cdot \partial\ddot{x}$

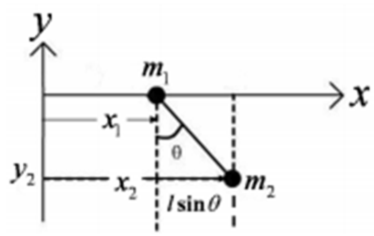
$\mathrm{Clear}(a, m_1, m_2, g, \theta, \theta', \theta'', x_1, \dot{x}_1, \ddot{x}_1) = 1$

Pendulum with Horizontal support

Constants $\partial a := 0$ $\partial m_1 := 0$ $\partial m_2 := 0$ $\partial g := 0$

Gen Coords $\partial\theta := \theta' \cdot \partial t$ $\partial\theta' := \theta'' \cdot \partial t$

$\partial x_1 := \dot{x}_1 \cdot \partial t$ $\partial\dot{x}_1 := \ddot{x}_1 \cdot \partial t$



Position $x_2 := x_1 + a \cdot \sin(\theta)$ $y_2 := -a \cdot \cos(\theta)$

$\dot{x}_2 := \frac{d(x_2)}{\partial t}$ $\dot{y}_2 := \frac{d(y_2)}{\partial t}$

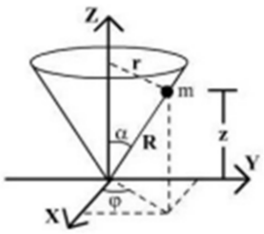
Lagrangian $L := \frac{1}{2} \cdot m_1 \cdot \dot{x}_1^2 + \frac{1}{2} \cdot m_2 \cdot \left(\dot{x}_2^2 + \dot{y}_2^2\right) - m_2 \cdot g \cdot y_2$

Equations of motion

$$eq := \begin{cases} isol\left(\frac{1}{\partial t}\cdot d\left(\frac{\partial(L, \dot{x}_1)}{\partial \dot{x}_1}\right) - \frac{\partial(L, x_1)}{\partial x_1}, \ddot{x}_1\right) \\ isol\left(\frac{1}{\partial t}\cdot d\left(\frac{\partial(L, \theta')}{\partial \theta'}\right) - \frac{\partial(L, \theta)}{\partial \theta}, \theta''\right)\end{cases} = \begin{cases} \frac{a\cdot m_2\cdot\left(-\sin(\theta)\cdot\theta'^2 + \cos(\theta)\cdot\theta''\right)}{m_1+m_2} \\ -\frac{\sin(\theta)\cdot g + \cos(\theta)\cdot\ddot{x}_1}{a}\end{cases}$$

$$\text{Clear}(\alpha, m, g, \varphi, \varphi', \varphi'', r, \dot{r}, \ddot{r}) = 1$$

Particle in a Cone



Constants $\partial\alpha := 0 \quad \partial m := 0 \quad \partial g := 0$

Gen Coords $\partial\varphi := \varphi'\cdot\partial t \quad \partial\varphi' := \varphi''\cdot\partial t$

$\partial r := \dot{r}\cdot\partial t \quad \partial\dot{r} := r''\cdot\partial t$

Position $x := r\cdot\cos(\varphi) \quad y := r\cdot\sin(\varphi) \quad z := r\cdot\cot(\alpha)$

$$\dot{x} := \frac{d(x)}{\partial t} \quad \dot{y} := \frac{d(y)}{\partial t} \quad \dot{z} := \frac{d(z)}{\partial t}$$

Lagrangian $L := \frac{1}{2}\cdot m\cdot\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - m\cdot g\cdot z$

Equations of motion

$$eq := \begin{cases} isol\left(\frac{1}{\partial t}\cdot d\left(\frac{\partial(L, \varphi')}{\partial \varphi'}\right) - \frac{\partial(L, \varphi)}{\partial \varphi}, \varphi''\right) \\ isol\left(\frac{1}{\partial t}\cdot d\left(\frac{\partial(L, \dot{r})}{\partial \dot{r}}\right) - \frac{\partial(L, r)}{\partial r}, r''\right)\end{cases}$$

$\alpha := 45\ \text{deg}$

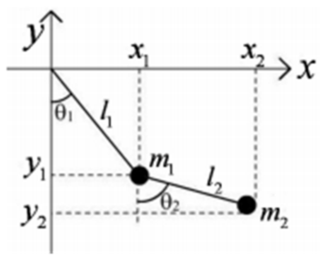
$$eq = \begin{cases} -\frac{\left(-\sin(\varphi)\cdot(\cos(\varphi)\cdot\dot{r} - r\cdot\sin(\varphi)\cdot\varphi') + \cos(\varphi)\cdot(\sin(\varphi)\cdot\dot{r} + r\cdot\cos(\varphi)\cdot\varphi') + r\cdot\varphi'\right)\cdot\dot{r}}{r^2} \\ \frac{-g\cdot\cot(\alpha) + \varphi'\cdot\left(-\sin(\varphi)\cdot(\cos(\varphi)\cdot\dot{r} - r\cdot\sin(\varphi)\cdot\varphi') + \cos(\varphi)\cdot(\sin(\varphi)\cdot\dot{r} + r\cdot\cos(\varphi)\cdot\varphi')\right)}{\cos(\varphi)^2 + \sin(\varphi)^2 + \cot(\alpha)^2}\end{cases}$$

For small angles

$$EquRep\left(eq, \begin{bmatrix}\sin(a) & 0 \\ \cos(a) & 1\end{bmatrix}\right) = \begin{cases} -\frac{2\cdot\varphi'\cdot\dot{r}}{r} \\ \frac{-g\cdot\cot(\alpha) + \varphi'^2\cdot r}{1+\cot(\alpha)^2}\end{cases}$$

$$\text{Clear}(a_1, a_2, m_1, m_2, g, \theta_1, \theta'_1, \theta''_1, \theta_2, \theta''_2) = 1$$

Double Pendulum



Constants $\partial m_1 := 0 \quad \partial m_2 := 0 \quad \partial g := 0$

$\partial a_1 := 0 \quad \partial a_2 := 0$

Gen Coords $\partial\theta_1 := \theta'_1\cdot\partial t \quad \partial\theta'_1 := \theta''_1\cdot\partial t$

$\partial\theta_2 := \theta'_2\cdot\partial t \quad \partial\theta'_2 := \theta''_2\cdot\partial t$

Position $x_1 := a_1\cdot\sin(\theta_1) \quad y_1 := -a_1\cdot\cos(\theta_1)$

$x_2 := a_1\cdot\sin(\theta_1) + a_2\cdot\sin(\theta_2) \quad y_1 := -a_1\cdot\cos(\theta_1) - a_2\cdot\cos(\theta_2)$

$$\dot{x}_1 := \frac{d(x_1)}{\partial t} \quad \dot{y}_1 := \frac{d(y_1)}{\partial t} \quad \dot{x}_2 := \frac{d(x_2)}{\partial t} \quad \dot{y}_2 := \frac{d(y_2)}{\partial t}$$

Lagrangian

$$L := \frac{1}{2} \cdot m_1 \cdot \left(\dot{x}_1{}^2 + \dot{y}_1{}^2\right) + \frac{1}{2} \cdot m_2 \cdot \left(\dot{x}_2{}^2 + y'_2{}^2\right) - m_1 \cdot g \cdot y_1 - m_2 \cdot g \cdot y_2$$

Equations of motion

$$eq := \begin{cases} isol\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial\left(L, \theta'_1\right)}{\partial \theta'_1}\right) - \frac{\partial\left(L, \theta_1\right)}{\partial \theta_1}, \theta''_1\right) \\ isol\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial\left(L, \theta'_2\right)}{\partial \theta'_2}\right) - \frac{\partial\left(L, \theta_2\right)}{\partial \theta_2}, \theta''_2\right) \end{cases}$$

Too long for display. For m.1 = 0, it's independent for the other mass too

$$EquRep\left(eq, \left[\, m_1 \;\; 0 \,\right]\right) = \begin{cases} -\frac{a_2 \cdot \left(-\theta'_2{}^2 \cdot \sin\left(\theta_2\right) + \cos\left(\theta_2\right) \cdot \theta''_2\right) - \theta'_1{}^2 \cdot \sin\left(\theta_1\right) \cdot a_1}{\cos\left(\theta_1\right) \cdot a_1} \\ -\frac{-\theta'_2{}^2 \cdot \sin\left(\theta_2\right) \cdot a_2 + a_1 \cdot \left(-\theta'_1{}^2 \cdot \sin\left(\theta_1\right) + \cos\left(\theta_1\right) \cdot \theta''_1\right)}{\cos\left(\theta_2\right) \cdot a_2} \end{cases}$$

For small angles

$$EquRep\left(eq, \begin{bmatrix} \sin(a) & 0 \\ \cos(a) & 1 \end{bmatrix}\right) = \begin{cases} -\frac{a_2 \cdot m_2 \cdot \theta''_2}{a_1 \cdot \left(m_1 + m_2\right)} \\ -\frac{a_1 \cdot \theta''_1}{a_2} \end{cases}$$

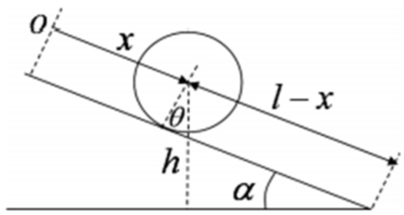
$$\mathrm{Clear}\left(m, a, R, \alpha, g, x, \dot{x}, \ddot{x}\right) = 1$$

Hoop rolling without sliding down an inclined plane

Constants $\quad \partial m := 0 \quad \partial a := 0 \quad \partial g := 0$

$\partial R := 0 \quad \partial \alpha := 0$

Gen Coords $\quad \partial x := \dot{x} \cdot \partial t \quad \partial \dot{x} := \ddot{x} \cdot \partial t$



Restrictions

$$\theta' := \frac{\dot{x}}{R}$$

Non sliding condition.

Position $\quad x$

Lagrangian

$$L := \frac{1}{2} \cdot m \cdot \dot{x}^2 + \frac{1}{2} \cdot m \cdot R^2 \cdot \theta'^2 - m \cdot g \cdot (a - x) \cdot \sin(\alpha)$$

Equations of motion

$$eq := \left\{ isol\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial\left(L, \dot{x}\right)}{\partial \dot{x}}\right) - \frac{\partial\left(L, x\right)}{\partial x}, \ddot{x}\right) \right\} = \left\{ -\frac{g \cdot \sin(\alpha)}{2} \right.$$

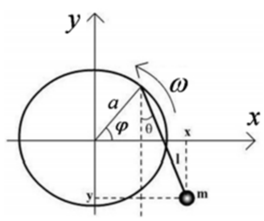
$$\mathrm{Clear}\left(\partial a, \partial \alpha, \partial m, \partial R, \partial g, \theta, \theta', \theta''\right) = 1$$

Pendulum whose support rotates with constant angular speed

Constants $\quad \partial m := 0 \quad \partial a := 0 \quad \partial g := 0$

$\partial R := 0 \quad \partial \omega := 0$

Gen Coords $\quad \partial \theta := \theta' \cdot \partial t \quad \partial \theta' := \theta'' \cdot \partial t$



Position

$$x := R \cdot \cos(\omega \cdot t) + a \cdot \sin(\theta)$$

$$y := R \cdot \sin(\omega \cdot t) - a \cdot \cos(\theta) \qquad \dot{x} := \frac{d(x)}{\partial t} \qquad \dot{y} := \frac{d(y)}{\partial t}$$

Lagrangian

$$L := \frac{1}{2} \cdot m \cdot \left(\dot{x}^2 + \dot{y}^2\right) - m \cdot g \cdot y$$

Equations of motion

$$eq := \left\{ isol\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial\left(L, \theta'\right)}{\partial \theta'}\right) - \frac{\partial\left(L, \theta\right)}{\partial \theta}, \theta''\right) \right.$$

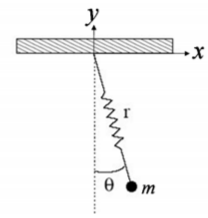
$$eq = \left\{ -\frac{-\omega^{2} \cdot R \cdot \left(\cos(\omega \cdot t) \cdot \cos(\theta) + \sin(\omega \cdot t) \cdot \sin(\theta)\right) + g \cdot \sin(\theta)}{a} \right.$$

For zero angular speed, we recover the simple pendulum

$$EquRep(eq, \omega, 0) = \left\{ -\frac{g \cdot \sin(\theta)}{a} \right.$$

$$\text{Clear}(m, a, k, g, \theta, \theta', \theta'') = 1$$

Pendulum whose support rotates with constant angular speed



Constants $\partial m := 0 \quad \partial a := 0 \quad \partial g := 0$

Gen Coords $\partial \theta := \theta' \cdot \partial t \quad \partial \theta' := \theta'' \cdot \partial t$

Position

$$x := r \cdot \cos(\theta) \qquad y := -r \cdot \sin(\theta)$$

$$\dot{x} := \frac{d(x)}{\partial t} \qquad \dot{y} := \frac{d(y)}{\partial t}$$

Lagrangian

$$L := \frac{1}{2} \cdot m \cdot \left(\dot{x}^{2} + \dot{y}^{2}\right) - m \cdot g \cdot y - \frac{1}{2} \cdot k \cdot (r - a)^{2}$$

Equations of motion

$$eq := \left\{ isol\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial(L, \theta')}{\partial \theta'} \right) - \frac{\partial(L, \theta)}{\partial \theta}, \theta'' \right) \right.$$

$$eq = \left\{ \frac{-\left(-\sin(\theta) \cdot \left(\cos(\theta) \cdot \dot{r} - r \cdot \sin(\theta) \cdot \theta'\right) + \cos(\theta) \cdot \left(\sin(\theta) \cdot \dot{r} + r \cdot \cos(\theta) \cdot \theta'\right) + \theta' \cdot r\right) \cdot \dot{r} + r \cdot g \cdot \cos(\theta)}{r^{2}} \right.$$

For small angles

$$EquRep\left(eq, \begin{bmatrix} \sin(a) & 0 \\ \cos(a) & 1 \end{bmatrix}\right) = \left\{ \frac{g - 2 \cdot \theta' \cdot \dot{r}}{r} \right.$$

Alvaro