The procedure for constructing a circle by three points in 3D

The calculation algorithm
1. The three points are vertices of the triangle. Write three linear equations:
two equations of planes passing through the midpoints of the two sides of the triangle
perpendicular to these sides: f1 = 0, f3 = 0
and the equation of the plane which contains the triangle: f2 = 0.
2. Solve the system of equations and find the center of a circle with coordinates Cen xCen, ..
3. Find the circle radius R of the equation of a sphere with radius R and center at Cen
4. Construct a circle as the line of intersection of the sphere with the plane of the triangle

⊞—Поворот вокруг осей координат

Координаты заданных точек

$p1 := \begin{bmatrix} 2 & -20 & 0.1 \end{bmatrix}^T \qquad p2 := \begin{bmatrix} 10 & -3 & 1 \end{bmatrix}^T \qquad p3 := \begin{bmatrix} -3 & 1.2 & -2 \end{bmatrix}^T$

Алгоритм расчета

**1.Три заданые точки являются вершинами треугольника.Составляем три линйных уравнения:
два уравнения плоскостей, проходящих через середины двух сторон треугольника
перпендикулярно этим сторонам: f1 = 0, f3 = 0
и уравнение плоскости, в которой лежит треугольник: f2 = 0 .**

$$\left(f_1 := \left(p2_1 - p1_1\right)\cdot\left(\frac{p1_1 + p2_1}{2} - x_1\right) + \left(p2_2 - p1_2\right)\cdot\left(\frac{p1_2 + p2_2}{2} - x_2\right) + \left(p2_3 - p1_3\right)\cdot\left(\frac{p1_3 + p2_3}{2} - x_3\right)\right) \equiv 0$$

$$f_3 := \left(p3_1 - p1_1\right)\cdot\left(\frac{p1_1 + p3_1}{2} - x_1\right) + \left(p3_2 - p1_2\right)\cdot\left(\frac{p1_2 + p3_2}{2} - x_2\right) + \left(p3_3 - p1_3\right)\cdot\left(\frac{p1_3 + p3_3}{2} - x_3\right)$$

$$f_2 := \begin{vmatrix} x_1 - p1_1 & x_2 - p1_2 & x_3 - p1_3 \\ p2_1 - p1_1 & p2_2 - p1_2 & p2_3 - p1_3 \\ p3_1 - p1_1 & p3_2 - p1_2 & p3_3 - p1_3 \end{vmatrix}$$

2.Решаем эту систему уравнений и находим центр окружности "center"

$$center := \operatorname{roots}\left(\begin{bmatrix} f_1 \equiv 0 \\ f_2 \equiv 0 \\ f_3 \equiv 0 \end{bmatrix}, \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) \qquad center = \begin{bmatrix} 0.9082 \\ -9.0396 \\ -0.6644 \end{bmatrix}$$

3.Находим радиус окружности "rad" из уравнения сферы с радусом rad и центром в точке "center"

$$rad := \sqrt{\left(p1_1 - center_1\right)^2 + \left(p1_2 - center_2\right)^2 + \left(p1_3 - center_3\right)^2}$$

$$\left(x_1 - center_1\right)^2 + \left(x_2 - center_2\right)^2 + \left(x_3 - center_3\right)^2 \equiv rad^2$$

4.Строим окружность как линию пересечения сферы с плоскостью треугольника(Методом Драгилева)

The procedure for constructing a circle by three points in 3D,p1, p2, p3 - vectors of points

$$\text{Circle3d}(p1, p2, p3) :=$$

$$f_1 := (p2_1 - p1_1)\cdot\left(\frac{p1_1 + p2_1}{2} - x_1\right) + (p2_2 - p1_2)\cdot\left(\frac{p1_2 + p2_2}{2} - x_2\right) + (p2_3 - p1_3)\cdot\left(\frac{p1_3 + p2_3}{2} - x_3\right)$$

$$f_3 := (p3_1 - p1_1)\cdot\left(\frac{p1_1 + p3_1}{2} - x_1\right) + (p3_2 - p1_2)\cdot\left(\frac{p1_2 + p3_2}{2} - x_2\right) + (p3_3 - p1_3)\cdot\left(\frac{p1_3 + p3_3}{2} - x_3\right)$$

$$f_2 := \begin{vmatrix} x_1 - p1_1 & x_2 - p1_2 & x_3 - p1_3 \\ p2_1 - p1_1 & p2_2 - p1_2 & p2_3 - p1_3 \\ p3_1 - p1_1 & p3_2 - p1_2 & p3_3 - p1_3 \end{vmatrix}$$

$$\text{center} := \text{roots}\left(\begin{bmatrix} f_1 = 0 \\ f_2 = 0 \\ f_3 = 0 \end{bmatrix}, \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right)$$

$$\text{rad} := \text{eval}\left(\sqrt{(p1_1 - \text{center}_1)^2 + (p1_2 - \text{center}_2)^2 + (p1_3 - \text{center}_3)^2}\right)$$

$$f_1 := (x_1 - \text{center}_1)^2 + (x_2 - \text{center}_2)^2 + (x_3 - \text{center}_3)^2$$

"Dragilev method"

$$n := 3$$

for $k \in 1 .. n$

$$u_k := x_k$$

$$b := (-1)\cdot \text{submatrix}(\text{Jacobian}(f, u), 1, n-1, n, n)$$

$$A := \text{submatrix}(\text{Jacobian}(f, u), 1, n-1, 1, n-1)$$

$$\det_1 := -\left|\text{augment}(b, \text{submatrix}(A, 1, n-1, 2, n-1))\right|$$

$$\det_{n-1} := -\left|\text{augment}(\text{submatrix}(A, 1, n-1, 1, n-2), b)\right|$$

$$\det_n := -|A|$$

for $i \in 1 .. n$

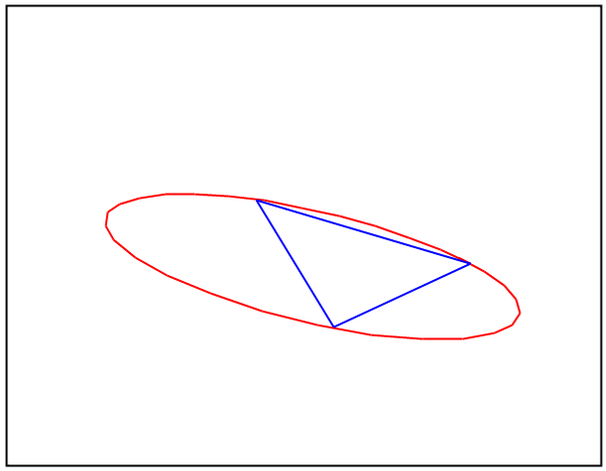
$$D2_i := \frac{\det_i}{\sqrt{(\det_1)^2 + (\det_2)^2 + (\det_3)^2}}$$

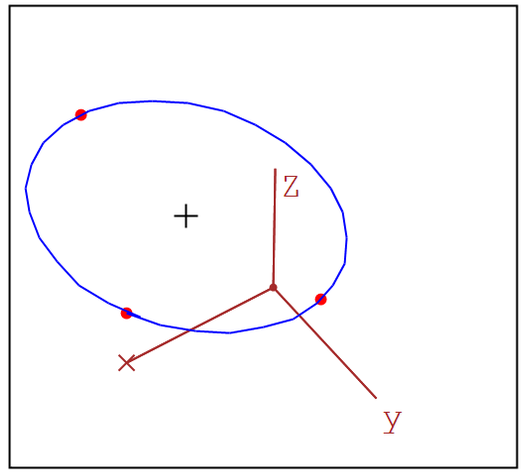
$$D1(t, x) := D2$$

$$\text{plot} := \text{submatrix}(\text{rkfixed}(p2, 0, 2.1\cdot\pi\cdot\text{rad}, 30, D1(t, x)), 1, 30, 2, 4)$$

$$\begin{bmatrix} \text{"plotCircle"} & \text{"center"} & \text{"radius"} \\ \text{plot} & \text{center} & \text{rad} \end{bmatrix}$$

$$\text{plot} := \text{col}(\text{Circle3d}(p1, p2, p3), 1)_2 \qquad \text{center} := \text{col}(\text{Circle3d}(p1, p2, p3), 2)_2$$





$$\text{Circle3d}(p1\ ,\ p2\ ,\ p3)=\begin{bmatrix} \text{"plotCircle"} & \text{"center"} & \text{"radius"} \\ \begin{bmatrix} 10 & -3 & 1 \\ 8.5116 & -1.1324 & 0.5895 \\ 6.657 & 0.3543 & 0.1187 \\ 4.5255 & 1.3886 & -0.3899 \\ 2.2197 & 1.9206 & -0.9117 \\ -0.1492 & 1.9247 & -1.4216 \\ -2.4672 & 1.4007 & -1.8951 \\ -4.6226 & 0.3738 & -2.3092 \\ -6.5116 & -1.1065 & -2.6441 \\ -8.0432 & -2.9689 & -2.8837 \\ -9.1436 & -5.1237 & -3.0164 \\ -9.7599 & -7.4671 & -3.0358 \\ -9.8624 & -9.8863 & -2.9409 \\ -9.4461 & -12.2646 & -2.7364 \\ -8.531 & -14.4876 & -2.4322 \\ -7.1613 & -16.4483 & -2.0427 \\ -5.4029 & -18.052 & -1.5869 \\ -3.3405 & -19.2217 & -1.0867 \\ -1.0735 & -19.9009 & -0.5661 \\ 1.289 & -20.057 & -0.0502 \\ 3.6331 & -19.6824 & 0.436 \\ 5.846 & -18.7951 & 0.8693 \\ 7.821 & -17.438 & 1.2287 \\ 9.4631 & -15.6764 & 1.4969 \\ 10.6931 & -13.595 & 1.661 \\ 11.4518 & -11.2943 & 1.7131 \\ 11.7027 & -8.885 & 1.6507 \\ 11.4336 & -6.4831 & 1.4767 \\ 10.6576 & -4.2043 & 1.1997 \\ 9.4119 & -2.1585 & 0.8328 \end{bmatrix} & \begin{bmatrix} 0.9082 \\ -9.0396 \\ -0.6644 \end{bmatrix} & 11.0412 \end{bmatrix}$$