⊞—GaussLegendre Coefficients

$$err := \left| I_{GL} - I \right|$$

⊟—Integral examples

$$Int_{GL}(f(1), a, b, GL) := \left| \begin{array}{l} \left[c := \frac{a+b}{2} \quad h := \frac{b-a}{2} \quad X := eval(c + h \cdot UnitsOf(c) \cdot col(GL, 1)) \right] \\ h \cdot \left(\sum_{n = Clear(n)}^{rows(GL)} GL_{n\,2} \cdot f(X_n) \right) \end{array} \right.$$

$$f(x) := x \cdot \cos\left(x^2\right) \qquad I := \frac{\sin(81) - \sin(4)}{2} = 0.063457251$$

$$I_{GL} := Int_{GL}\left(f(x), 2, 9, GL_{20}\right) = -15.77416142 \qquad err = 15.84$$

$$I_{GL} := Int_{GL}\left(f(x), 2, 9, GL_{50}\right) = 0.063457251 \qquad err = 1.24 \cdot 10^{-13}$$

$$f(x) := \frac{4}{1 + x^2} \qquad I := \pi = 3.141592654$$

$$I_{GL} := Int_{GL}\left(f(x), 0, 1, GL_{20}\right) = 3.141592654 \qquad err = 1.04 \cdot 10^{-14}$$

$$I_{GL} := Int_{GL}\left(f(x), 0, 1, GL_{50}\right) = 3.141592654 \qquad err = 3.22 \cdot 10^{-15}$$

⊟—Double Integral Examples

$$DInt_{GL}(f(2), a, b, c, d, GL) := \left| \begin{array}{l} \left[Cx := \frac{a+b}{2} \quad hx := \frac{b-a}{2} \quad X := eval(Cx + hx \cdot UnitsOf(Cx) \cdot col(GL, 1)) \right] \\ \left[Cy := \frac{c+d}{2} \quad hy := \frac{d-c}{2} \quad Y := eval(Cy + hy \cdot UnitsOf(Cy) \cdot col(GL, 1)) \right] \\ \left[N := rows(GL) \quad Clear(m, n) \right] \\ hy \cdot hx \cdot \sum_{m=1}^{N} \left(GL_{m\,2} \cdot \sum_{n=1}^{N} GL_{n\,2} \cdot f(X_n, Y_m) \right) \end{array} \right.$$

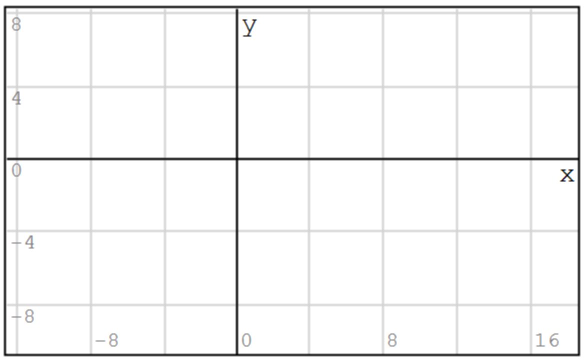
Double Integrals

SMath can Integrate $f(x, y) := x^2 + y^2$ over the region defined between $\begin{cases} a(x) := e^x \\ b(x) := 2 \cdot x + 2 \end{cases}$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} := sort(solve(b(x) - a(x), x)) = \begin{bmatrix} -0.768 \\ 1.6783 \end{bmatrix}$$

$$t_0 := time(0)$$

$$I_0 := eval\left(\int_{x_1}^{x_2} \int_{a(x)}^{b(x)} f(x, y)\, d y\, d x \right) = 18.57686864$$



$\mathrm{time}(0) - t_0 = 1.885\ s$

Define some utilities

$$\mathrm{Err\&T}(t_0) := \left[\Delta t := \mathrm{time}(0) - t_0 \quad t_0 := \mathrm{time}(0) \quad \begin{bmatrix} |I - I_0| \\ \Delta t \end{bmatrix} \right]$$

Now, we try to extend the above method for solve this problem parametrizing the ode solver from the SMath plugins ...

$$\mathrm{DInt}_{GL50}(f(2), x_1, x_2, y_1, y_2) := \mathrm{DInt}_{GL}(f(xx, yy), x_1, x_2, y_1, y_2, GL_{50})$$

$$\mathrm{DInt}_{GL20}(f(2), x_1, x_2, y_1, y_2) := \mathrm{DInt}_{GL}(f(xx, yy), x_1, x_2, y_1, y_2, GL_{20})$$

$$g(x, y) := f(x, y) \cdot (y \le b(x)) \cdot (y \ge a(x)) \qquad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} := \begin{bmatrix} 0 \\ 6 \end{bmatrix} \qquad t_0 := \mathrm{time}(0)$$

$$I := \int_{x_1}^{x_2} \int_{y_1}^{y_2} g(x, y)\, d y\, d x = 18.5832591 \qquad \mathrm{Err\&T}(t_0) = \begin{bmatrix} 0.0064 \\ 5.338\ s \end{bmatrix}$$

$$I := \mathrm{DInt}_{GL20}(g(x, y), x_1, x_2, y_1, y_2) = 20.12561745 \qquad \mathrm{Err\&T}(t_0) = \begin{bmatrix} 1.5487 \\ 0.073\ s \end{bmatrix}$$

$$I := \mathrm{DInt}_{GL50}(g(x, y), x_1, x_2, y_1, y_2) = 18.716591 \qquad \mathrm{Err\&T}(t_0) = \begin{bmatrix} 0.1397 \\ 0.639\ s \end{bmatrix}$$

As better approach, using a change of variables $y = y_1 + u \cdot (y_2 - y_1)$ $\quad dy = (y_2 - y_1) \cdot du$

we can integrate from 0 to 1 for y, and isn't necessary to guess the limits of integration, as in the first method.

$$h(x, u) := f(x, a(x) + u \cdot (b(x) - a(x))) \cdot (b(x) - a(x))$$

$$I := \int_{x_1}^{x_2} \int_{0}^{1} h(x, y)\, d y\, d x = 18.56839179 \qquad \mathrm{Err\&T}(t_0) = \begin{bmatrix} 0.0085 \\ 9.891\ s \end{bmatrix}$$

$$I := \mathrm{DInt}_{GL20}(h(x, y), x_1, x_2, 0, 1) = 18.57686882 \qquad \mathrm{Err\&T}(t_0) = \begin{bmatrix} 1.8437 \cdot 10^{-7} \\ 0.12\ s \end{bmatrix}$$

$$I := \mathrm{DInt}_{GL50}(h(x, y), x_1, x_2, 0, 1) = 18.57686882 \qquad \mathrm{Err\&T}(t_0) = \begin{bmatrix} 1.8437 \cdot 10^{-7} \\ 0.798\ s \end{bmatrix}$$

Alvaro