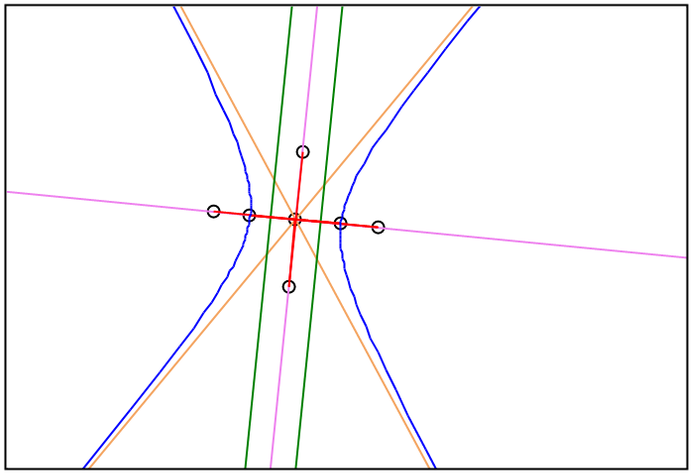
Conics

Gallery

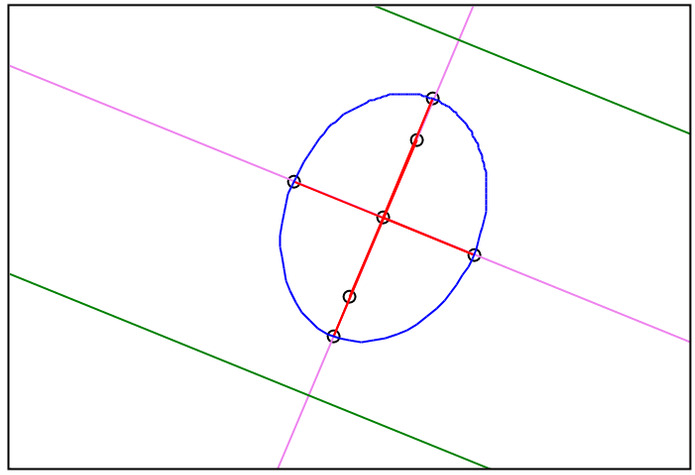
Hyperbola given the general equation coeffs



$\alpha := [-7\ 2\ 3\ 5\ -2\ 1]$

CON(α, "Plot")
CON(α, "Points")
CON(α, "Directrix")
CON(α, "Axis")
CON(α, "Assimptotes")
augment(CON(α, "Points"), "o")

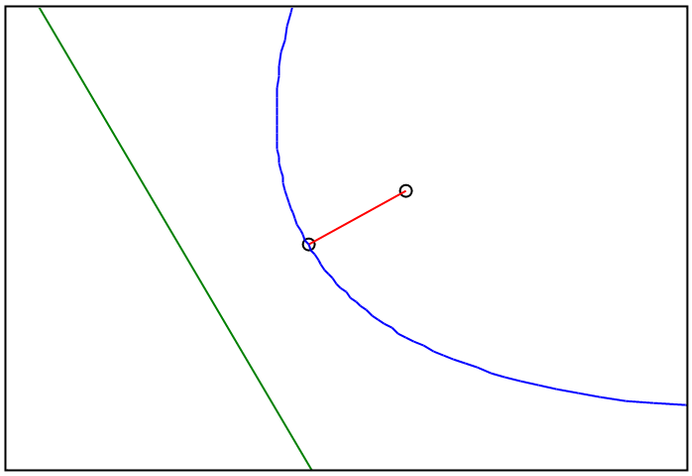
Ellipse given the general equation coeffs



$\alpha := [3\ -1\ 2\ 1\ 4\ -3]$

CON(α, "Plot")
CON(α, "Points")
CON(α, "Directrix")
CON(α, "Axis")
CON(α, "Assimptotes")
augment(CON(α, "Points"), "o")

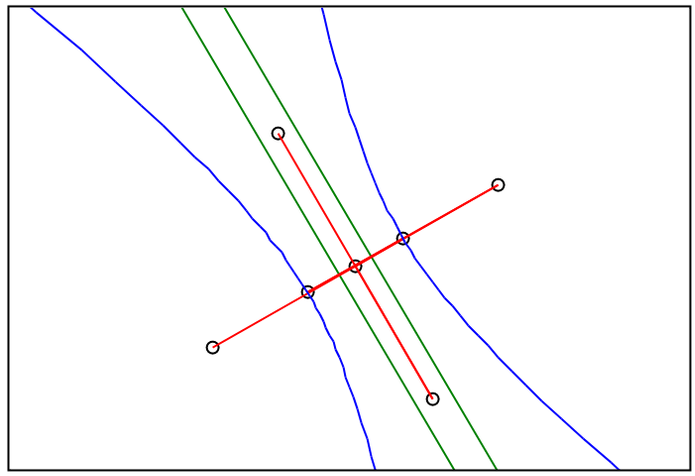
Parabola given the eccentricity, the latus rectum, the rotation angle and the vertex



$\alpha := CON([1\ 7\ 30\ ^\circ\ [3\ 4]], "\varepsilon l \varphi O")$

CON(α, "Plot")
CON(α, "Points")
CON(α, "Directrix")
augment(CON(α, "Points"), "o")

Hyperbola gieven the eccentricity, the latus rectum, the rotation angle and the center



$\alpha := CON([3\ 7\ 30\ ^\circ\ [3\ 4]], "\varepsilon l \varphi O")$

CON(α, "Plot")
CON(α, "Points")
CON(α, "Directrix")
augment(CON(α, "Points"), "o")

Example: E or H given Φ, Φ', P

Draw the Conic Given by the Foci and one Point

$\Phi := [-4\ -2]$ $\Phi' := [8\ 1]$ $P := [6\ 4]$ $\nu := -1$ Choose ν=1 for an E or ν=-1 for an H

Angle $\varphi := \operatorname{atan}\left(\frac{\Phi'_{2} - \Phi_{2}}{\Phi'_{1} - \Phi_{1}}\right) = 14.0362\ °$

Linear eccentricity $c := \frac{\operatorname{norme}(\Phi - \Phi')}{2} = 6.1847$

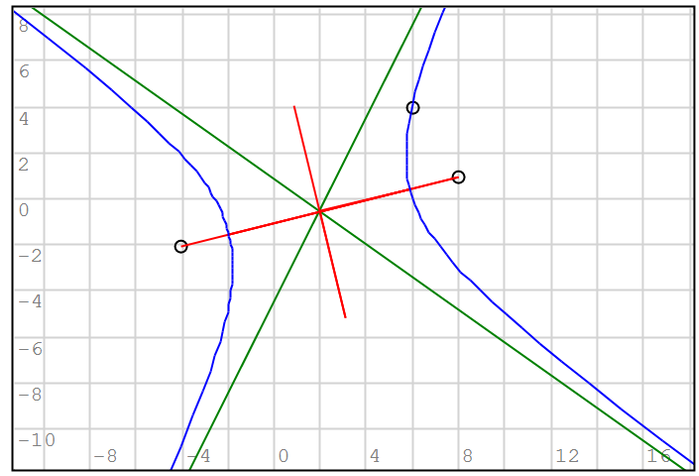
Major Axis Length $a := \frac{\operatorname{norme}(P - \Phi) + \nu \cdot \operatorname{norme}(P - \Phi')}{2} = 4.0282$

Other elements $\varepsilon := \frac{c}{a}$ $l := a \cdot \left|1 - \varepsilon^{2}\right|$ $O := \frac{\Phi + \Phi'}{2}$

$\alpha := CON([\varepsilon\ l\ \varphi\ O], \text{"}\varepsilon l \varphi O\text{"})$

$\Pi := \begin{cases} CON(\alpha, \text{"Plot"}) \\ CON(\alpha, \text{"Points"}) \\ CON(\alpha, \text{"Assimptotes"}) \\ \operatorname{augment}(\operatorname{stack}(\Phi, \Phi', P), \text{"o"}) \end{cases}$

$CON(\alpha, \text{"Focus"}) = \begin{bmatrix} -4 & -2 \\ 8 & 1 \end{bmatrix}$



Example: E or H given Φ, δ, a

Draw the Conic Given by the a Foci, it's directrix and the semi major axis length.

$\operatorname{Clear}(x, y) = 1$

$\Phi := [-5\ \ 6]$ $\delta := 2 \cdot x + 3 \cdot y - 1$ $a := 2$ $\nu := -1$ Choose ν = (-1, 1) for (H, E)

Directrix coeffs $\delta(x, y) := \delta$

$\left[a_\delta\ b_\delta\ c_\delta\right] := \left[\frac{d}{dx}\delta\ \frac{d}{dy}\delta\ \delta(0, 0)\right] = [2\ 3\ -1]$

Angle $\varphi := \operatorname{atan}\left(\frac{b_\delta}{a_\delta}\right) = 56.3099\ °$

Axis slope = -1/δ slope

$R := \begin{bmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{bmatrix}$

Focal Parameter $p := \left|\frac{\delta(\Phi_1, \Phi_2)}{\sqrt{a_\delta^{2} + b_\delta^{2}}}\right| = 1.9415$

$Distance(\Phi, \delta)$

Linear Eccentricity $Distance(O, \delta)$ equals $\frac{a^{2}}{c} = c + \nu \cdot p$ then

$c := \operatorname{polyroots}\left(\left[-a^{2}\ \nu \cdot p\ 1\right]^{T}\right) = \begin{bmatrix} -1.2524 \\ 3.1939 \end{bmatrix}$

$c := \begin{cases} \min\left(\overrightarrow{|c|}\right) = 3.1939 & \text{if } \nu = 1 \\ \max\left(\overrightarrow{|c|}\right) & \text{else} \end{cases}$

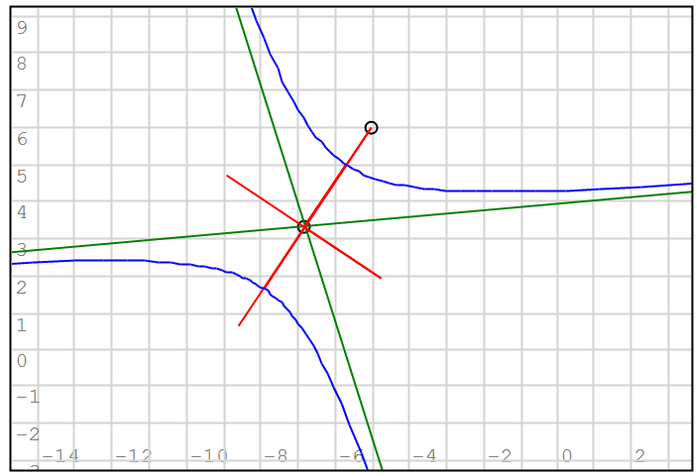
Other elements $\varepsilon := \frac{c}{a}$ $l := p \cdot \varepsilon$ $O := \Phi + [\nu \cdot c\ \ 0] \cdot R$

$$\alpha := CON([\varepsilon\ l\ \varphi\ O], \text{"}\varepsilon l\varphi O\text{"})$$

$$\Pi := \begin{cases} CON(\alpha, \text{"Plot"}) \\ CON(\alpha, \text{"Points"}) \\ CON(\alpha, \text{"Assimptotes"}) \\ \text{augment}(\text{stack}(O, \Phi), \text{"o"}) \end{cases}$$

$$CON(\alpha, \text{"Focus"}) = \begin{bmatrix} -8.5433 & 0.6851 \\ -5 & 6 \end{bmatrix}$$

$$CON(\alpha, \text{"a"}) = 2$$



Example: P, E or H given O, Φ, ε

Draw the Conic Given by the Center (or the Vertex, if ε=1), a Foci and the eccentricity

$O := [-1\ 3]$ $\Phi := [2\ 4]$ $\varepsilon := 0.4$

Angle

$$\varphi := \text{atan}\left(\frac{O_2 - \Phi_2}{O_1 - \Phi_1}\right) = 18.4349\ °$$

Linear ecc. and latus rectum

$c := \text{norme}(O - \Phi) = 3.1623$

$$l := \text{if } \varepsilon = 1 \quad 2 \cdot c \quad \text{else} \quad c \cdot \frac{\left|1 - \varepsilon^2\right|}{\varepsilon} = 6.6408$$

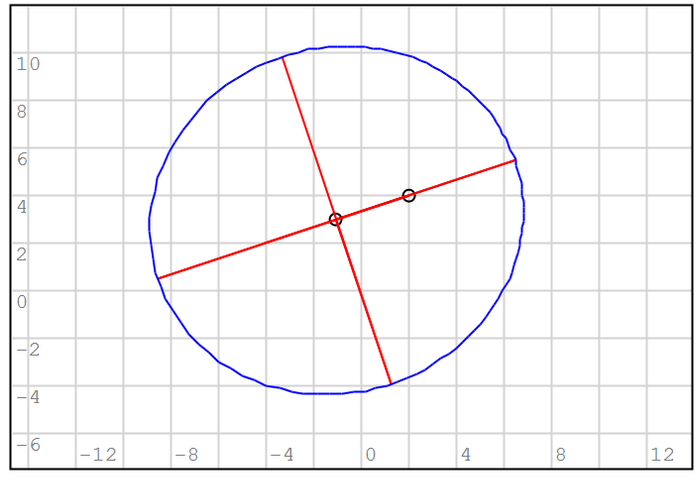
$$\alpha := CON([\varepsilon\ l\ \varphi\ O], \text{"}\varepsilon l\varphi O\text{"})$$

$$\Pi := \begin{cases} CON(\alpha, \text{"Plot"}) \\ CON(\alpha, \text{"Points"}) \\ CON(\alpha, \text{"Assimptotes"}) \\ \text{augment}(\text{stack}(O, \Phi), \text{"o"}) \end{cases}$$

$$CON(\alpha, \text{"Center"}) = [-1\ 3]$$

$$CON(\alpha, \text{"Focus"}) = \begin{bmatrix} -4 & 2 \\ 2 & 4 \end{bmatrix}$$

$$CON(\alpha, \text{"}\varepsilon\text{"}) = 0.4$$



Example: P given Φ, δ

Draw the Parabola given by the a Foci and directrix.

$\text{Clear}(x, y) = 1$

$\Phi := [-8\ 6]$ $\delta := 7 \cdot x + 3 \cdot y - 1$ $\varepsilon := 1$

Directrix coeffs

$\delta(x, y) := \delta$

$$\begin{bmatrix} a_\delta & b_\delta & c_\delta \end{bmatrix} := \begin{bmatrix} \frac{d}{dx}\delta & \frac{d}{dy}\delta & \delta(0, 0) \end{bmatrix} = [7\ 3\ -1]$$

Angle

$$\varphi := \text{atan}\left(\frac{b_\delta}{a_\delta}\right) = 23.1986\ °$$

$$R := \begin{bmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{bmatrix}$$

Latus rectum
and Vertice

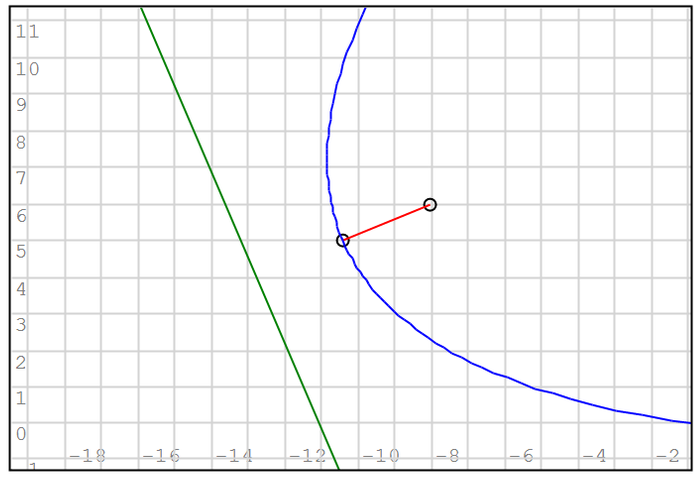
$$l := \left| \frac{\delta(\Phi_1, \Phi_2)}{\sqrt{a_\delta^2 + b_\delta^2}} \right| = 5.121 \qquad V := \Phi - \left[\frac{l}{2} \quad 0 \right] \cdot R = [-10.3534 \quad 4.9914]$$

$$\alpha := CON([\varepsilon \quad l \quad \varphi \quad V], \text{"}\varepsilon l \varphi O\text{"})$$

$$\Pi := \begin{cases} CON(\alpha, \text{"Plot"}) \\ CON(\alpha, \text{"Points"}) \\ CON(\alpha, \text{"Directrix"}) \\ \text{augment}(\text{stack}(V, \Phi), \text{"o"}) \end{cases}$$

$$CON(\alpha, \text{"Focus"}) = [-8 \quad 6]$$

$$CON(\alpha, \text{"Vertex"}) = [-10.3534 \quad 4.9914]$$



Example: Five Points Conic

Enter Five Points

$$P_1 := \begin{bmatrix} -1 & 3 \\ 2 & -2 \\ -3 & 2 \\ 4 & 1 \\ 3 & 3 \end{bmatrix}$$

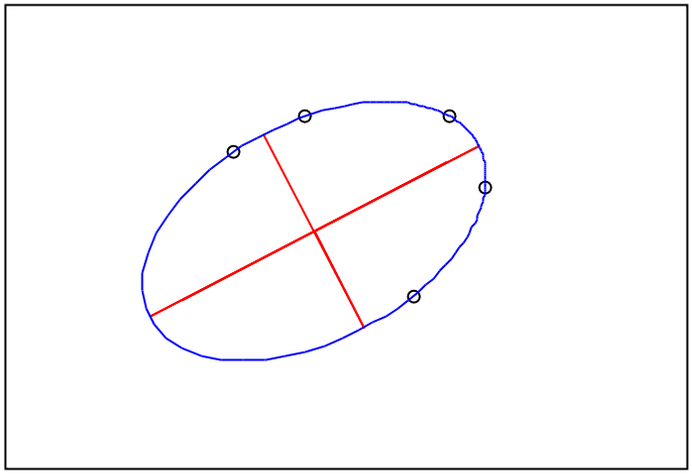
$$\alpha := CON(P_1, \text{"5PCon"})$$

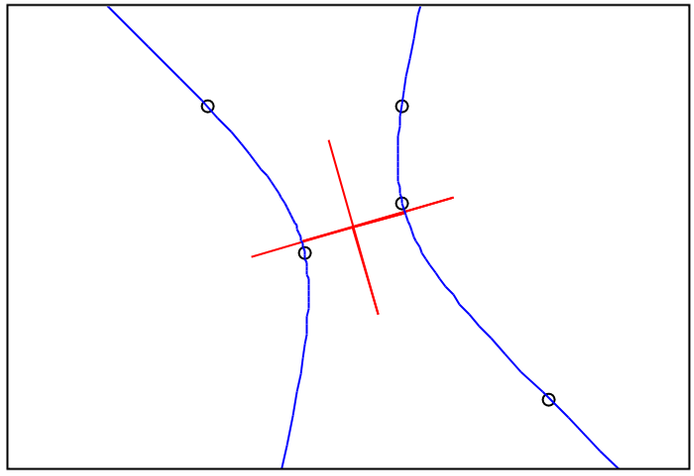
$$\Pi_1 := \begin{cases} CON(\alpha, \text{"Plot"}) \\ CON(\alpha, \text{"Points"}) \\ \text{augment}(P_1, \text{"o"}) \end{cases}$$

$$P_2 := \begin{bmatrix} 1 & 2 \\ -3 & 4 \\ 4 & -2 \\ 1 & 4 \\ -1 & 1 \end{bmatrix}$$

$$\alpha := CON(P_2, \text{"5PCon"})$$

$$\Pi_2 := \begin{cases} CON(\alpha, \text{"Plot"}) \\ CON(\alpha, \text{"Points"}) \\ \text{augment}(P_2, \text{"o"}) \end{cases}$$





Alvaro