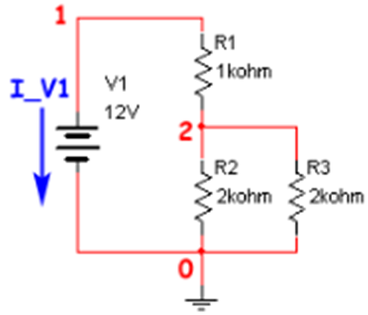
MNA Solver

MNA

Examples from https://www.swarthmore.edu/NatSci/echeeve1/Ref/mna/MNA6.html

Example



$$\chi := \begin{bmatrix} V_1 & 1 & 0 & 12 & \text{V} \\ R_1 & 1 & 2 & 1 & \text{k}\Omega \\ R_2 & 2 & 0 & 2 & \text{k}\Omega \\ R_3 & 2 & 0 & 2 & \text{k}\Omega \end{bmatrix}$$

$$\begin{cases} \chi' := \chi_{[1..\text{rows}(\chi)][1..3]} \\ [A\ b\ x\ V\ I] := MNA(\chi) \\ [A'\ b'\ x\ V'\ I'] := MNA(\chi') \end{cases}$$

Solved variables $x^{T} = \begin{bmatrix} \mathcal{E}_1 & \mathcal{E}_2 & I\_V_1 \end{bmatrix}$

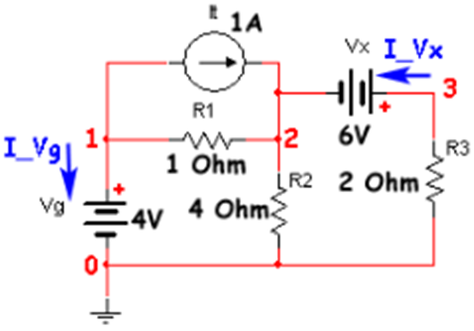
Symbolic

$$V' = \begin{bmatrix} V_1 \\ \dfrac{V_1 \cdot R_3 \cdot R_2}{(R_2 + R_1) \cdot R_3 + R_2 \cdot R_1} \end{bmatrix} \qquad I' = \begin{bmatrix} -\dfrac{(R_3 + R_2) \cdot V_1}{(R_2 + R_1) \cdot R_3 + R_2 \cdot R_1} \end{bmatrix}$$

Numeric

$$V = \begin{bmatrix} 12 \\ 6 \end{bmatrix} \text{V} \qquad I = [-0.006]\ \text{A}$$

Example



$$\chi := \begin{bmatrix} V_g & 1 & 0 & 4 & \text{V} \\ V_x & 3 & 2 & 6 & \text{V} \\ R_1 & 1 & 2 & 1 & \Omega \\ R_2 & 2 & 0 & 4 & \Omega \\ R_3 & 3 & 0 & 2 & \Omega \\ I_t & 1 & 2 & 1 & \text{A} \end{bmatrix}$$

$$\begin{cases} \chi' := \chi_{[1..\text{rows}(\chi)][1..3]} \\ [A\ b\ x\ V\ I] := MNA(\chi) \\ [A'\ b'\ x\ V'\ I'] := MNA(\chi') \end{cases}$$

Solved variables $x^{T} = \begin{bmatrix} \mathcal{E}_1 & \mathcal{E}_2 & \mathcal{E}_3 & I\_V_g & I\_V_x \end{bmatrix}$

Symbolic

$$V' = \begin{bmatrix} V_g \\ \dfrac{R_2 \cdot \left(R_3 \cdot \left(V_g + R_1 \cdot I_t\right) - R_1 \cdot V_x\right)}{R_1 \cdot R_2 + R_3 \cdot (R_2 + R_1)} \\ \dfrac{R_3 \cdot \left(R_2 \cdot \left(V_g + R_1 \cdot I_t\right) + (R_2 + R_1) \cdot V_x\right)}{R_1 \cdot R_2 + R_3 \cdot (R_2 + R_1)} \end{bmatrix}$$

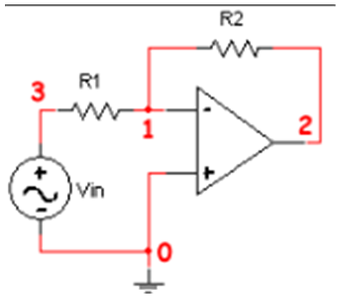
$$I' = \begin{bmatrix} -\dfrac{I_t \cdot \left(R_2 \cdot (R_1 - R_3) + R_3 \cdot (R_2 + R_1)\right) + (R_2 + R_3) \cdot V_g + R_2 \cdot V_x}{R_1 \cdot R_2 + R_3 \cdot (R_2 + R_1)} \\ -\dfrac{R_2 \cdot \left(V_g + R_1 \cdot I_t\right) + V_x \cdot (R_2 + R_1)}{} \end{bmatrix}$$

$$\left[\begin{array}{cc} & R_1 \cdot R_2 + R_3 \cdot (R_2 + R_1) \end{array}\right]$$

Numeric

$$V = \begin{bmatrix} 4 \\ 1.1429 \\ 7.1429 \end{bmatrix} \text{V} \qquad I = \begin{bmatrix} -3.8571 \\ -3.5714 \end{bmatrix} \text{A}$$

Example



$$\chi' := \begin{bmatrix} V_{in} & 3 & 0 & \text{NaN} \\ R_1 & 1 & 3 & \text{NaN} \\ R_2 & 2 & 1 & \text{NaN} \\ OA & 0 & 1 & 2 \end{bmatrix} \qquad \chi := \begin{bmatrix} V_{in} & 3 & 0 & 5\ \text{V} \\ R_1 & 1 & 3 & 6\ \Omega \\ R_2 & 2 & 1 & 9\ \Omega \\ OA & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{cases} [A\ b\ x\ V\ I] := MNA(\chi) \\ [A'\ b'\ x\ V'\ I'] := MNA(\chi') \end{cases}$$

Solved variables

$$x^{\mathrm{T}} = \left[\mathcal{E}_1\ \mathcal{E}_2\ \mathcal{E}_3\ I\_V_{in}\ I\_OA \right]$$

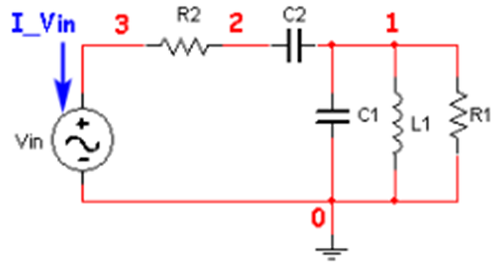
Symbolic

$$V' = \begin{bmatrix} 0 \\ -\dfrac{R_2 \cdot V_{in}}{R_1} \\ V_{in} \end{bmatrix} \qquad I' = \begin{bmatrix} -\dfrac{V_{in}}{R_1} \\ \dfrac{V_{in}}{R_1} \end{bmatrix}$$

Numeric

$$V = \begin{bmatrix} 0 \\ -7.5 \\ 5 \end{bmatrix} \text{V} \qquad I = \begin{bmatrix} -0.8333 \\ 0.8333 \end{bmatrix} \text{A}$$

Example



$$\chi := \begin{bmatrix} V_{in} & 3 & 0 & \text{NaN} \\ R_1 & 0 & 1 & 1\ \text{k}\Omega \\ L_1 & 1 & 0 & 1\ \text{mH} \\ C_1 & 1 & 0 & 1\ \mu\text{F} \\ C_2 & 2 & 1 & 10\ \mu\text{F} \\ R_2 & 2 & 3 & 1\ \text{k}\Omega \end{bmatrix}$$

$$\begin{cases} \chi' := \chi[1..\text{rows}(\chi)][1..3] \\ [A\ b\ x\ V\ I] := MNA(\chi) \\ [A'\ b'\ x\ V'\ I'] := MNA(\chi') \end{cases}$$

Solved variables

$$x^{\mathrm{T}} = \left[\mathcal{E}_1\ \mathcal{E}_2\ \mathcal{E}_3\ I\_V_{in} \right]$$

Symbolic tf

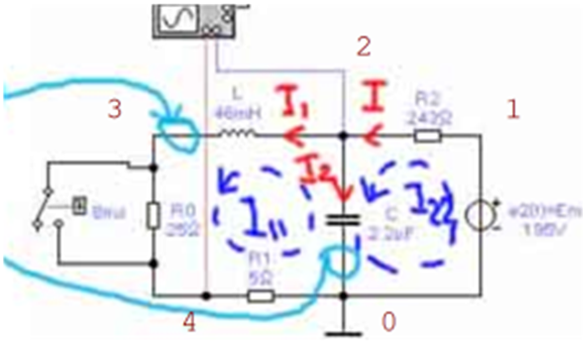
$$\frac{V'_2}{V_{in}} = \frac{R_1 \cdot \left(1 + s^2 \cdot L_1 \cdot \left(C_1 + C_2 \cdot (1 + s \cdot C_2 \cdot R_2) - s \cdot {C_2}^2 \cdot R_2\right)\right) + s \cdot L_1}{-s^3 \cdot {C_2}^2 \cdot L_1 \cdot R_1 \cdot R_2 + \left(R_1 \cdot \left(1 + s^2 \cdot L_1 \cdot (C_1 + C_2)\right) + s \cdot L_1\right) \cdot (1 + s \cdot C_2 \cdot R_2)}$$

Numeric tf

$$\left.\frac{V_2}{V_{in}}\right|_{s = s\ \text{Hz}} = \frac{100 \cdot \left(10 \cdot \left(100 \cdot (1000000 + s) + s^2\right) + s^2\right)}{-10 \cdot s^3 + \left(10 \cdot \left(100 \cdot (1000000 + s) + s^2\right) + s^2\right) \cdot (100 + s)}$$

Numerical Inverse Laplace Transform

Example



$$\chi := \begin{bmatrix} V_m & 1 & 0 \\ R_0 & 3 & 4 \\ R_1 & 0 & 4 \\ R_2 & 1 & 2 \\ C_1 & 0 & 2 \\ L_1 & 2 & 3 \\ V_{C0} & 3 & 4 \end{bmatrix}$$

$[A\ b\ x\ V\ I] := MNA(\chi)$

Solved variables

$$x^T = \left[\varepsilon_1\ \varepsilon_2\ \varepsilon_3\ \varepsilon_4\ I\_V_m\ I\_V_{C0} \right]$$

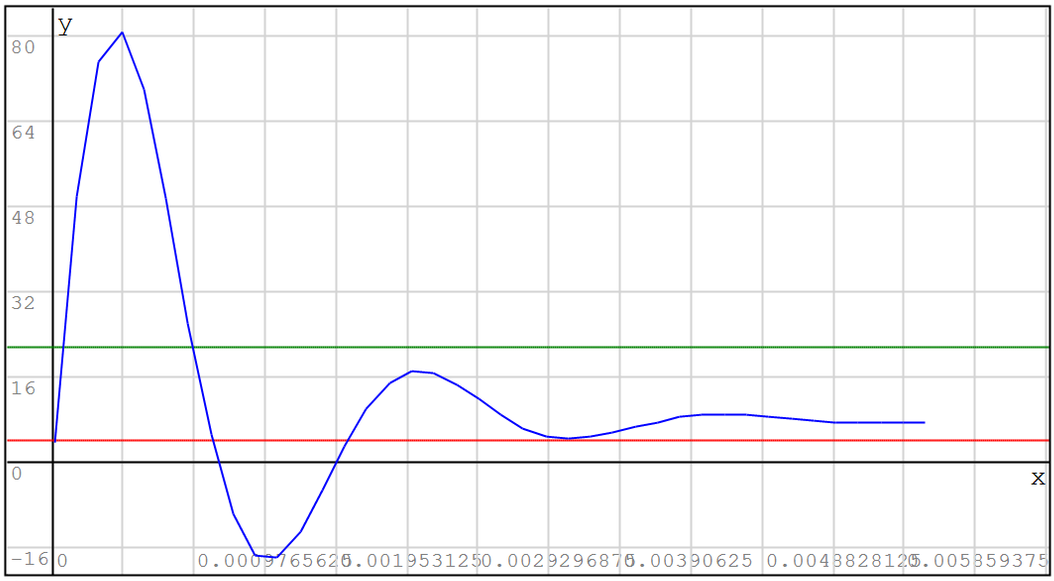
Symbolic tf

$$H(s) := \frac{V_2}{1 \cdot s} = \frac{\left(L_1 \cdot \left(s \cdot L_1 \cdot \left(\left(R_2 + \left(1 + s \cdot C_1 \cdot R_2\right) \cdot s \cdot L_1\right) \cdot R_1 + R_0{}^2 \cdot \left(1 + s \cdot C_1 \cdot R_2\right)\right) - \left(R_1 + R_0\right.\right.\right.}{}$$

$R_0 := 25 \quad R_1 := 5 \quad R_2 := 243 \quad V_m := 195 \quad C_1 := 2.2 \cdot 10^{-6} \quad L_1 := 46 \cdot 10^{-3}$

$$V_{C0} := \frac{V_m \cdot R_1}{R_1 + R_2} = 3.9315 \qquad V_{C1} := \frac{V_m \cdot \left(R_0 + R_1\right)}{R_0 + R_1 + R_2} = 21.4286$$

$u_C := ILap\left(H(s), 0, 0.006, 40, [6\ 20\ 19]\right)$



u_C
V_{C0}
V_{C1}