Worksheet : 07-PLACAS.sm
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PLATES

Academic Worksheet - Structural Computational Models

OBSERVATIONS

A node "n" is related to the unknowns "2*n-1" for x-displacement and "2*n" for y -displacement.

Follow the counter-clockwise direction to assign nodes to an element.

The δ.x y δ.y values in the NODE DATA matrix represent restrictions. If a displacement is restricted, then this value equals 1, else, 0.

EXAMPLE 6.7

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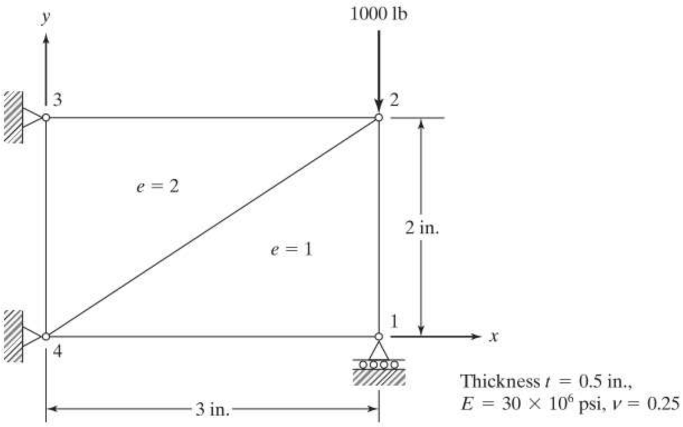
For the two-dimensional loaded plate shown in Fig. E6.7, determine the displacements of nodes 1 and 2 and the element stresses using plane stress conditions. Body force may be neglected in comparison with the external forces.



SOLUTION

MATERIAL

$$\begin{cases} E := 30 \cdot 10^{6} \\ \nu := 0.25 \end{cases}$$

CONNECTIVITY TABLE

$$C := \begin{bmatrix} \text{"Element"} & \text{"Node 1"} & \text{"Node 2"} & \text{"Node 3"} \\ \text{"1"} & 1 & 2 & 4 \\ \text{"2"} & 3 & 4 & 2 \end{bmatrix}$$

PLATE THICKNESS

$t := 0.5$

NODE DATA

$$N := \begin{bmatrix} \text{"Node"} & \text{"x"} & \text{"y"} & \text{"F.x"} & \text{"F.y"} & \text{"δ.x"} & \text{"δ.y"} \\ \text{"1"} & 3 & 0 & 0 & 0 & 0 & 1 \\ \text{"2"} & 3 & 2 & 0 & -1000 & 0 & 0 \\ \text{"3"} & 0 & 2 & 0 & 0 & 1 & 1 \\ \text{"4"} & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

MATERIAL MATRIX

$$D=\begin{bmatrix}320 & 80 & 0\\ 80 & 320 & 0\\ 0 & 0 & 120\end{bmatrix}10^{5}$$

STRAIN-DISPLACEMENT MATRICES

$$B=\begin{bmatrix}\begin{bmatrix}33.3333 & 0 & 0 & 0 & -33.3333 & 0\\ 0 & -50 & 0 & 50 & 0 & 0\\ -50 & 33.3333 & 50 & 0 & 0 & -33.3333\end{bmatrix}\\ \begin{bmatrix}-33.3333 & 0 & 0 & 0 & 33.3333 & 0\\ 0 & 50 & 0 & -50 & 0 & 0\\ 50 & -33.3333 & -50 & 0 & 0 & 33.3333\end{bmatrix}\end{bmatrix}10^{-2}$$

ELEMENT STIFFNESS MATRICES

$$k_e=\begin{bmatrix}\begin{bmatrix}98.3333 & -50 & -45 & 20 & -53.3333 & 30\\ -50 & 140 & 30 & -120 & 20 & -20\\ -45 & 30 & 45 & 0 & 0 & -30\\ 20 & -120 & 0 & 120 & -20 & 0\\ -53.3333 & 20 & 0 & -20 & 53.3333 & 0\\ 30 & -20 & -30 & 0 & 0 & 20\end{bmatrix}\\ \begin{bmatrix}98.3333 & -50 & -45 & 20 & -53.3333 & 30\\ -50 & 140 & 30 & -120 & 20 & -20\\ -45 & 30 & 45 & 0 & 0 & -30\\ 20 & -120 & 0 & 120 & -20 & 0\\ -53.3333 & 20 & 0 & -20 & 53.3333 & 0\\ 30 & -20 & -30 & 0 & 0 & 20\end{bmatrix}\end{bmatrix}10^{5}$$

AREAS

$$A_e=\begin{bmatrix}3.0000\\ 3.0000\end{bmatrix}$$

GLOBAL STIFFNESS MATRIX

$$K=\begin{bmatrix}0.983 & -0.5 & -0.45 & 0.2 & 0 & 0 & -0.533 & 0.3\\ -0.5 & 1.4 & 0.3 & -1.2 & 0 & 0 & 0.2 & -0.2\\ -0.45 & 0.3 & 0.983 & 0 & -0.533 & 0.2 & 0 & -0.5\\ 0.2 & -1.2 & 0 & 1.4 & 0.3 & -0.2 & -0.5 & 0\\ 0 & 0 & -0.533 & 0.3 & 0.983 & -0.5 & -0.45 & 0.2\\ 0 & 0 & 0.2 & -0.2 & -0.5 & 1.4 & 0.3 & -1.2\\ -0.533 & 0.2 & 0 & -0.5 & -0.45 & 0.3 & 0.983 & 0\\ 0.3 & -0.2 & -0.5 & 0 & 0.2 & -1.2 & 0 & 1.4\end{bmatrix}10^{7}$$

REDUCED STIFFNESS MATRIX

$$K_r=\begin{bmatrix}0.983 & -0.450 & 0.200\\ -0.450 & 0.983 & 0.000\\ 0.200 & 0.000 & 1.400\end{bmatrix}10^{7}$$

GLOBAL F

$$F=\begin{bmatrix}0\\ 0\\ 0\\ -1000\\ 0\\ 0\\ 0\\ 0\end{bmatrix}$$

REDUCED F

$$F_r=\begin{bmatrix}0\\ 0\\ -1000\end{bmatrix}$$

GLOBAL Q

$$Q=\begin{bmatrix}0.0019\\ 0\\ 0.0009\\ -0.0074\\ 0\\ 0\\ 0\\ 0\end{bmatrix}10^{-2}$$

REDUCED Q

$$Q_r=\begin{bmatrix}0.0019\\ 0.0009\\ -0.0074\end{bmatrix}10^{-2}$$

EXTERNAL FORCES

$$K\cdot Q=\begin{bmatrix}0\\ 820.7\\ 0\\ -1000\\ -269\\ 165.8\\ 269\\ 13.6\end{bmatrix}$$

ELEMENT DISPLACEMENTS

$$q^{T}=\begin{bmatrix}\begin{bmatrix}0.0019\\ 0\\ 0.0009\\ -0.0074\\ 0\\ 0\end{bmatrix} & \begin{bmatrix}0\\ 0\\ 0\\ 0\\ 0.0009\\ -0.0074\end{bmatrix}\end{bmatrix}10^{-2}$$

ELEMENT STRESSES

$$\sigma^{T}=\begin{bmatrix}\begin{bmatrix}-93.12\\ -1135.59\\ -62.08\end{bmatrix} & \begin{bmatrix}93.12\\ 23.28\\ -296.62\end{bmatrix}\end{bmatrix}$$