
 ☒—RK23 For integrals

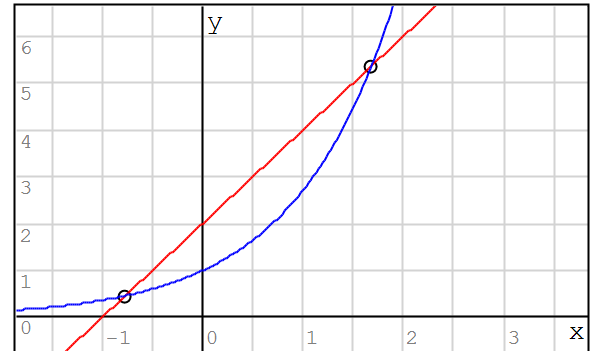
 ☒—GaussLegendre Coefficients

Answer for SteelCat

$$f(x, y) := x^2 + y^2 \quad \begin{cases} a(x) := e^x \\ b(x) := 2 \cdot x + 2 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} := \text{sort}(\text{solve}(b(x) - a(x), x)) = \begin{bmatrix} -0.768 \\ 1.6783 \end{bmatrix}$$

$$I_0 := \text{eval} \left(\int_{x_1}^{x_2} \int_{a(x)}^{b(x)} f(x, y) dy dx \right) = 18.57686864$$



With an ode solver:

$$DInt(f(2), x_1, x_2, Y_1(1), Y_2(1)) := \begin{cases} \text{Clear}(x, u) \\ h(x, u) := f(x, Y_1(x) + u \cdot (Y_2(x) - Y_1(x))) \cdot (Y_2(x) - Y_1(x)) \\ DInt(h(x, u), x_1, x_2, 0, 1, n_x, n_y, \varepsilon_x, \varepsilon_y) \end{cases}$$

$$\varepsilon_x := 10^{-3} \quad \varepsilon_y := 10^{-3} \quad n_x := 15 \quad n_y := 15$$

$$DInt(f(x, y), x_1, x_2, a(x), b(x)) = 18.4681218$$

With Gauss Laguerre method

$$DInt_{GL}(f(2), a, b, c, d, GL) := \begin{cases} Cx := \frac{a+b}{2} \quad hx := \frac{b-a}{2} \quad X := \text{eval}(Cx + hx \cdot \text{UnitsOf}(Cx) \cdot \text{col}(GL, 1)) \\ Cy := \frac{c+d}{2} \quad hy := \frac{d-c}{2} \quad Y := \text{eval}(Cy + hy \cdot \text{UnitsOf}(Cy) \cdot \text{col}(GL, 1)) \\ [N := \text{rows}(GL) \quad \text{Clear}(m, n)] \\ hy \cdot hx \cdot \sum_{m=1}^N \left(GL_{m,2} \cdot \sum_{n=1}^N GL_{n,2} \cdot f(X_n, Y_m) \right) \end{cases}$$

$$DInt(f(2), x_1, x_2, Y_1(1), Y_2(1)) := \begin{cases} \text{Clear}(x, u) \\ h(x, u) := f(x, Y_1(x) + u \cdot (Y_2(x) - Y_1(x))) \cdot (Y_2(x) - Y_1(x)) \\ DInt_{GL}(h(x, u), x_1, x_2, 0, 1, GL_{50}) \end{cases}$$

$$DInt(f(x, y), x_1, x_2, a(x), b(x)) = 18.57686882$$

Alvaro

