Worksheet : 07-PLACAS.sm
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Version : 1.0.1
Revision : 19-11-2020

Date : 30-05-2021
Hour : 08:20:34

PLATES

Academic Worksheet - Structural Computational Models

OBSERVATIONS

A node "n" is related to the unknowns "2*n-1" for x-displacement and "2*n" for y -displacement.

Follow the counter-clockwise direction to assign nodes to an element.

The δ.x y δ.y values in the NODE DATA matrix represent restrictions. If a displacement is restricted, then this value equals 1, else, 0.

EXAMPLE 6.7

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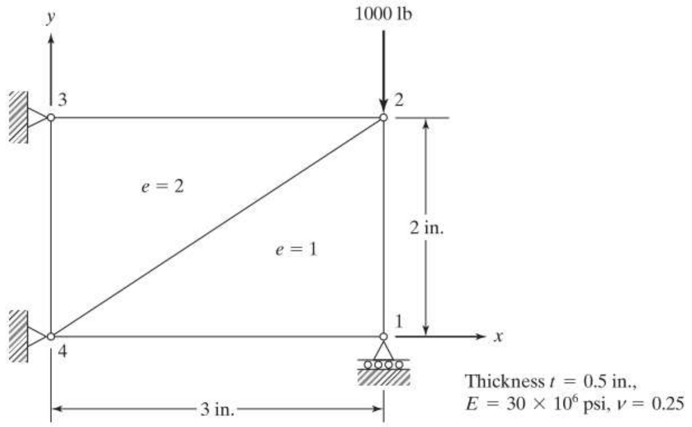
For the two-dimensional loaded plate shown in Fig. E6.7, determine the displacements of nodes 1 and 2 and the element stresses using plane stress conditions. Body force may be neglected in comparison with the external forces.



SOLUTION

MATERIAL

$$\begin{cases} E := 30 \cdot 10^{6} \\ \nu := 0.25 \end{cases}$$

CONNECTIVITY TABLE

$$C := \begin{bmatrix} \text{"Element"} & \text{"Node 1"} & \text{"Node 2"} & \text{"Node 3"} \\ \text{"1"} & 1 & 2 & 4 \\ \text{"2"} & 3 & 4 & 2 \end{bmatrix}$$

PLATE THICKNESS

$t := 0.5$

NODE DATA

$$N := \begin{bmatrix} \text{"Node"} & \text{"x"} & \text{"y"} & \text{"F.x"} & \text{"F.y"} & \text{"δ.x"} & \text{"δ.y"} \\ \text{"1"} & 3 & 0 & 0 & 0 & 0 & 1 \\ \text{"2"} & 3 & 2 & 0 & -1000 & 0 & 0 \\ \text{"3"} & 0 & 2 & 0 & 0 & 1 & 1 \\ \text{"4"} & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

matrix to vector

$$\text{mat2vec}(\text{mat}) := \begin{array}{|l} \text{for } i \in [1..\text{length}(\text{mat})] \\ \quad \text{ans}_i := \text{mat}_i \\ \text{ans} \end{array}$$

delete row

$$\text{delrow}(\text{mat}, r) := \begin{array}{|l} m := \text{rows}(\text{mat}) \\ n := \text{cols}(\text{mat}) \\ M_1 := \text{if } r = 1 \\ \qquad \text{matrix}(0, n) \\ \quad \text{else} \\ \qquad \text{submatrix}(\text{mat}, 1, r-1, 1, n) \\ M_2 := \text{if } r = m \\ \qquad \text{matrix}(0, n) \\ \quad \text{else} \\ \qquad \text{submatrix}(\text{mat}, r+1, m, 1, n) \\ \text{stack}(M_1, M_2) \end{array}$$

delete column

$$\text{delcol}(\text{mat}, c) := \begin{array}{|l} m := \text{rows}(\text{mat}) \\ n := \text{cols}(\text{mat}) \\ M_1 := \text{if } c = 1 \\ \qquad \text{matrix}(m, 0) \\ \quad \text{else} \\ \qquad \text{submatrix}(\text{mat}, 1, m, 1, c-1) \\ M_2 := \text{if } c = n \\ \qquad \text{matrix}(m, 0) \\ \quad \text{else} \\ \qquad \text{submatrix}(\text{mat}, 1, m, c+1, n) \\ \text{augment}(M_1, M_2) \end{array}$$

delete row and column

$$\text{delrc}(\text{mat}, rc) := \begin{array}{|l} M_1 := \text{delrow}(\text{mat}, rc) \\ M_2 := \text{delcol}(M_1, rc) \end{array}$$

delete a set of rows and columns

$$\text{delsrc}(\text{mat}, \text{src}) := \begin{array}{|l} \text{tmp} := \text{mat} \\ \text{for } i \in [1..\text{length}(\text{src})] \\ \quad \text{tmp} := \text{delrc}\left(\text{tmp}, \text{src}_i - (i-1)\right) \\ \text{tmp} \end{array}$$

material matrix

$$D := \frac{E}{1-\nu^2} \cdot \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

coordinates

$$\begin{cases} x := N_{[2..\text{rows}(N)]\,2} \\ y := N_{[2..\text{rows}(N)]\,3} \end{cases}$$

number of elements

$$\text{nel} := \text{rows}(C) - 1$$

number of unknows

$$n_i := 2 \cdot \text{length}(x)$$

restrictions vector

$$R := \text{mat2vec}\left(N_{[2..\text{rows}(N)]\,[6..7]}\right)$$

unknowns linked to each element

$$\begin{array}{|l} \text{for } el \in [1..\text{nel}] \\ \quad \begin{array}{|l} \text{nd} := \text{findrows}(C, \text{num2str}(el), 1)_{[2..4]} \\ \text{for } j \in [1..3] \\ \quad \begin{array}{|l} \text{in}_{2 \cdot j - 1} := 2 \cdot \text{nd}_j - 1 \\ \text{in}_{2 \cdot j} := 2 \cdot \text{nd}_j \end{array} \\ p_{el} := \text{in} \end{array} \end{array}$$

set of rows and columns to reduce

$$\begin{array}{|l} \text{src} := \text{matrix}(0, 1) \\ \text{snrc} := \text{matrix}(0, 1) \\ \text{for } i \in [1..n_i] \\ \quad \text{if } R_i \\ \qquad \text{src} := \text{stack}(\text{src}, i) \\ \quad \text{else} \\ \qquad \text{snrc} := \text{stack}(\text{snrc}, i) \end{array}$$

element stiffness matrices

$kij(k, i, j) := k_i - k_j$

for $el \in [1..nel]$

$nd := findrows(C, num2str(el), 1)_{[2..4]}$

$y_{23} := kij(y, nd_2, nd_3)$

$y_{31} := kij(y, nd_3, nd_1)$

$y_{12} := kij(y, nd_1, nd_2)$

$y_{13} := kij(y, nd_1, nd_3)$

$x_{32} := kij(x, nd_3, nd_2)$

$x_{13} := kij(x, nd_1, nd_3)$

$x_{21} := kij(x, nd_2, nd_1)$

$x_{23} := kij(x, nd_2, nd_3)$

$J := \begin{bmatrix} x_{13} & y_{13} \\ x_{23} & y_{23} \end{bmatrix}$

$A_{e_{el}} := \frac{1}{2} \cdot |J|$

$\left[N_1 := \frac{1}{3} \quad N_2 := \frac{1}{3} \quad N_3 := \frac{1}{3} \right]$

$B_{el} := \frac{1}{|J|} \cdot \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$

$k_{e_{el}} := t \cdot A_{e_{el}} \cdot B_{el}^{T} \cdot D \cdot B_{el}$

global stiffness matrix

$K_{n_i n_i} := 0$

for $el \in [1..2]$
  for $i \in [1..6]$
    for $j \in [1..6]$
      $K_{\left(p_{el}\right)_i \left(p_{el}\right)_j} := K_{\left(p_{el}\right)_i \left(p_{el}\right)_j} + \left(k_{e_{el}}\right)_{i\,j}$

reduced stiffness matrix

$K_r := delsrc(K, src)$

forces

$F := mat2vec\left(N_{[2..rows(N)][4..5]}\right)$

$F_r := F_{snrc}$

displacements

$Q_r := K_r^{-1} \cdot F_r$

$Q_{n_i} := 0$

$j := 0$

for $i \in snrc$
  $j := j + 1$
  $Q_i := Q_{r_j}$

element displacements and stresses

for $el \in [1..nel]$
  $q_{el} := Q_{p_{el}}$
  $\sigma_{el} := D \cdot B_{el} \cdot q_{el}$

MATERIAL MATRIX

$$D = \begin{bmatrix} 320 & 80 & 0 \\ 80 & 320 & 0 \\ 0 & 0 & 120 \end{bmatrix} 10^{5}$$

STRAIN-DISPLACEMENT MATRICES

$$B = \begin{bmatrix} \begin{bmatrix} 33.3333 & 0 & 0 & 0 & -33.3333 & 0 \\ 0 & -50 & 0 & 50 & 0 & 0 \\ -50 & 33.3333 & 50 & 0 & 0 & -33.3333 \end{bmatrix} \\ \begin{bmatrix} -33.3333 & 0 & 0 & 0 & 33.3333 & 0 \\ 0 & 50 & 0 & -50 & 0 & 0 \\ 50 & -33.3333 & -50 & 0 & 0 & 33.3333 \end{bmatrix} \end{bmatrix} 10^{-2}$$

ELEMENT STIFFNESS MATRICES

$$k_e = \begin{bmatrix} \begin{bmatrix} 98.3333 & -50 & -45 & 20 & -53.3333 & 30 \\ -50 & 140 & 30 & -120 & 20 & -20 \\ -45 & 30 & 45 & 0 & 0 & -30 \\ 20 & -120 & 0 & 120 & -20 & 0 \\ -53.3333 & 20 & 0 & -20 & 53.3333 & 0 \\ 30 & -20 & -30 & 0 & 0 & 20 \end{bmatrix} \\ \begin{bmatrix} 98.3333 & -50 & -45 & 20 & -53.3333 & 30 \\ -50 & 140 & 30 & -120 & 20 & -20 \\ -45 & 30 & 45 & 0 & 0 & -30 \\ 20 & -120 & 0 & 120 & -20 & 0 \\ -53.3333 & 20 & 0 & -20 & 53.3333 & 0 \\ 30 & -20 & -30 & 0 & 0 & 20 \end{bmatrix} \end{bmatrix} 10^{5}$$

AREAS

$$A_e = \begin{bmatrix} 3.0000 \\ 3.0000 \end{bmatrix}$$

GLOBAL STIFFNESS MATRIX

$$K = \begin{bmatrix} 0.983 & -0.5 & -0.45 & 0.2 & 0 & 0 & -0.533 & 0.3 \\ -0.5 & 1.4 & 0.3 & -1.2 & 0 & 0 & 0.2 & -0.2 \\ -0.45 & 0.3 & 0.983 & 0 & -0.533 & 0.2 & 0 & -0.5 \\ 0.2 & -1.2 & 0 & 1.4 & 0.3 & -0.2 & -0.5 & 0 \\ 0 & 0 & -0.533 & 0.3 & 0.983 & -0.5 & -0.45 & 0.2 \\ 0 & 0 & 0.2 & -0.2 & -0.5 & 1.4 & 0.3 & -1.2 \\ -0.533 & 0.2 & 0 & -0.5 & -0.45 & 0.3 & 0.983 & 0 \\ 0.3 & -0.2 & -0.5 & 0 & 0.2 & -1.2 & 0 & 1.4 \end{bmatrix} 10^{7}$$

REDUCED STIFFNESS MATRIX

$$K_r = \begin{bmatrix} 0.983 & -0.450 & 0.200 \\ -0.450 & 0.983 & 0.000 \\ 0.200 & 0.000 & 1.400 \end{bmatrix} 10^{7}$$

GLOBAL F

$$F = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1000 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

REDUCED F

$$F_r = \begin{bmatrix} 0 \\ 0 \\ -1000 \end{bmatrix}$$

GLOBAL Q

$$Q = \begin{bmatrix} 0.0019 \\ 0 \\ 0.0009 \\ -0.0074 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} 10^{-2}$$

REDUCED Q

$$Q_r = \begin{bmatrix} 0.0019 \\ 0.0009 \\ -0.0074 \end{bmatrix} 10^{-2}$$

EXTERNAL FORCES

$$K \cdot Q = \begin{bmatrix} 0 \\ 820.7 \\ 0 \\ -1000 \\ -269 \\ 165.8 \\ 269 \\ 13.6 \end{bmatrix}$$

ELEMENT DISPLACEMENTS

$$q^{T} = \begin{bmatrix} \begin{bmatrix} 0.0019 \\ 0 \\ 0.0009 \\ -0.0074 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.0009 \\ -0.0074 \end{bmatrix} \end{bmatrix} 10^{-2}$$

ELEMENT STRESSES

$$\sigma^{T} = \begin{bmatrix} \begin{bmatrix} -93.12 \\ -1135.59 \\ -62.08 \end{bmatrix} & \begin{bmatrix} 93.12 \\ 23.28 \\ -296.62 \end{bmatrix} \end{bmatrix}$$