

Point Orthogonal Projection onto a flat or spatial Curve

The cases are considered when the given point is very far from the curve

Calculation procedure

1. We will use Draghilev's method

a) to parameterize the curves and find a good initial guess. We will use the point a_0 , found by the minimum distance from the curve to a given point P .

b) to calculate the unit vector of the tangent $T(x)$ to a point on the curve.

2 To the system of equations we add an equation that expresses the conditions of perpendicularity of the tangent at the point x of the curve and the segment connecting the points P and x . This means that the inner product of the unit tangent vector to the curve $f(x)$ and the unit vector Px is equal to zero. As a result, we obtain the system of equations

$$F(x) := \begin{bmatrix} f(x) \\ T(x) \cdot \frac{Px}{\text{norme}(Px)} \end{bmatrix}$$

3. We solve the system with the initial assumption a_0

Plot

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D(x0, tmax, Δt, P) := [
  f := f(x) n := rows(f) + 1 k := [1..n] u_k := x_k N := tmax/Δt c := [1..n]
  J(x) := Jacobian(f, u) b := (-1) · submatrix(J(x), 1, n-1, n, n)
  A := submatrix(J(x), 1, n-1, 1, n-1)
  sA := submatrix(A, 1, n-1, k+1, n-1) X0 := al_nleqsolve(x0, f(x))
  if n > 2
    [ V_1 := -|augment(b, submatrix(A, 1, n-1, 2, n-1))| ]
    [ V_{n-1} := -|augment(submatrix(A, 1, n-1, 1, n-2), b)| V_n := -|A| ]
    for k := 2, k ≤ n-2, k := k+1
      V_k := -|augment(submatrix(A, 1, n-1, 1, k-1), b, sA)|
    else
      [ V_1 := b_1 V_2 := |A| ]
    [ "Unit Tangent Vector" T(x) := V / norme(V) v(t, x) := T(x) ]
    B := submatrix(rkfixed(X0, 0, tmax, N, v(t, x)), 1, N, 2, n+1)
    [ k := [1..N] dist_k := eval(norme(P - row(B, k))) Px_c := P_c - x_c ]
    [ a0 := eval(row(csort(augment(B, dist), n+1), 1)_c^T ) ]
    [ "Adding orthogonality equation" F(x) := stack( f(x), [ T(x) · Px / norme(Px) ] ) ]
    Pr := al_nleqsolve(a0^T, F(x))^T
    [ Pr B F(Pr) ]
]
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$L(P) := D(x0, tmax, \Delta t, P)$

Example 1

$$P1 := \begin{bmatrix} 565 & 945 \end{bmatrix} \quad P2 := \begin{bmatrix} -1200 & 6000 \end{bmatrix} \quad P3 := \begin{bmatrix} -1 & -20 \end{bmatrix} \quad P4 := \begin{bmatrix} 3.5 & -28 \end{bmatrix}$$

$$f(x) := \left[2 \cdot \left(x_1 \right)^2 + 4 \cdot x_1 \cdot x_2 + 4 \cdot \left(x_2 \right)^3 - 26 \cdot x_2 - 22 + 18 \cdot \sin \left(x_1 \cdot x_2 \right) \right]$$

$$x0 := \begin{bmatrix} 10 & -5 \end{bmatrix}^T \quad \Delta t := 0.1 \quad tmax := 56$$

$$\begin{bmatrix} a1 & B & F(a1) \end{bmatrix} := L(P1) \quad a1 = \begin{bmatrix} 2.1903 & 2.5675 \end{bmatrix}$$

$$F(a1)^T = \begin{bmatrix} 2.1136 \cdot 10^{-8} & 5.396 \cdot 10^{-9} \end{bmatrix}$$

$$\begin{bmatrix} a2 & B & F(a2) \end{bmatrix} := L(P2) \quad a2 = \begin{bmatrix} -4.3664 & 3.3081 \end{bmatrix}$$

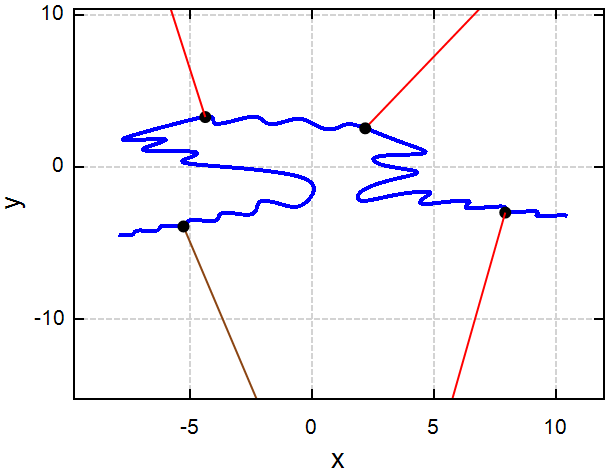
$$F(a2)^T = \begin{bmatrix} 1.028 \cdot 10^{-8} & 3.6598 \cdot 10^{-10} \end{bmatrix}$$

$$\begin{bmatrix} a3 & B & F(a3) \end{bmatrix} := L(P3) \quad a3 = \begin{bmatrix} -5.263 & -3.8824 \end{bmatrix}$$

$$F(a3)^T = \begin{bmatrix} -2.0278 \cdot 10^{-8} & 7.4853 \cdot 10^{-10} \end{bmatrix}$$

$$\begin{bmatrix} a4 & B & F(a4) \end{bmatrix} := L(P4) \quad a4 = \begin{bmatrix} 7.9327 & -2.9704 \end{bmatrix}$$

$$F(a4)^T = \begin{bmatrix} 5.8563 \cdot 10^{-9} & 5.1738 \cdot 10^{-10} \end{bmatrix}$$



Example 2

$$P1 := \begin{bmatrix} 146 & 94 \end{bmatrix} \quad P2 := \begin{bmatrix} 167 & -94 \end{bmatrix} \quad P3 := \begin{bmatrix} 36.7 & -172 \end{bmatrix} \quad P4 := \begin{bmatrix} 43 & 186 \end{bmatrix}$$

$$f(x) := \left[\left(x_1 - 2 \right)^2 + 5 \cdot \left(x_2 \right)^2 + \left(x_1 \right)^2 \cdot \left(x_2 \right)^{10} - 3 \cdot \sin \left(7 \cdot x_1 \cdot x_2 \right) - 9 \right]$$

$$x0 := \begin{bmatrix} 2 & -1 \end{bmatrix}^T \quad \Delta t := 0.01 \quad tmax := 30$$

$$\begin{bmatrix} a1 & B & F(a1) \end{bmatrix} := L(P1) \quad a1 = \begin{bmatrix} 5.3597 & 0.377 \end{bmatrix}$$

$$F(a1)^T = \begin{bmatrix} 1.2273 \cdot 10^{-9} & 7.1325 \cdot 10^{-8} \end{bmatrix}$$

$$\begin{bmatrix} a2 & B & F(a2) \end{bmatrix} := L(P2) \quad a2 = \begin{bmatrix} 5.4024 & -0.291 \end{bmatrix}$$

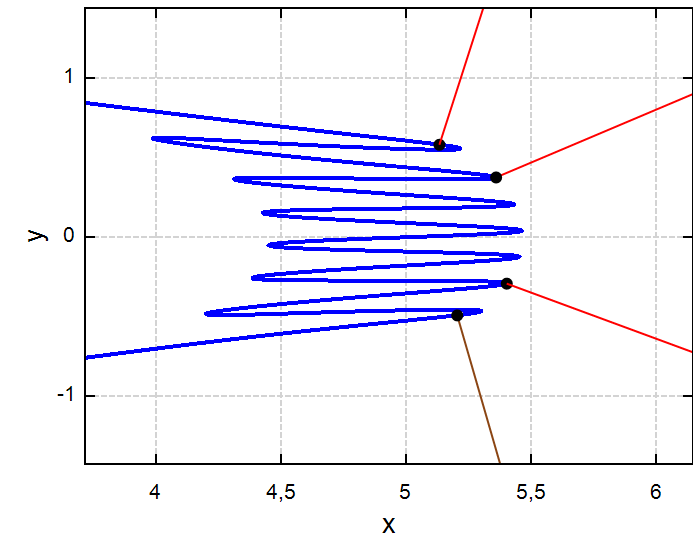
$$F(a2)^T = \begin{bmatrix} -4.2184 \cdot 10^{-10} & 2.365 \cdot 10^{-8} \end{bmatrix}$$

$$\begin{bmatrix} a3 & B & F(a3) \end{bmatrix} := L(P3) \quad a3 = \begin{bmatrix} 5.2044 & -0.4905 \end{bmatrix}$$

$$F(a3)^T = \begin{bmatrix} 2.2694 \cdot 10^{-8} & -3.7245 \cdot 10^{-9} \end{bmatrix}$$

$$\begin{bmatrix} a4 & B & F(a4) \end{bmatrix} := L(P4) \quad a4 = \begin{bmatrix} 5.1331 & 0.5823 \end{bmatrix}$$

$$F(a4)^T = \begin{bmatrix} 2.2382 \cdot 10^{-8} & 4.2928 \cdot 10^{-9} \end{bmatrix}$$



Example 3

$P1 := [96 \ 94] \qquad P2 := [- 67 \ - 94] \qquad P3 := [36.7 \ - 172] \qquad P4 := [- 43 \ 186]$

$$\vec{f}\left(x\right):=\left[\left(x_1\right)^2\cdot x_2-\sin\left(x_1+x_2\right)\right]$$

$$\left[x0:=[- 5 \ 0]^T \ \Delta t:=- 0.1 \ tmax:=- 20 \right]$$

$$\left[a1 \ B \ F(a1)\right]:=L(P1) \qquad a1=[1.2346 \ 0.6283]$$

$$F(a1)^T=\left[7.8431\cdot10^{-10} \ - 4.2628\cdot10^{-11}\right]$$

$$\left[a2 \ B \ F(a2)\right]:=L(P2) \qquad a2=[- 1.4258 \ - 0.4667]$$

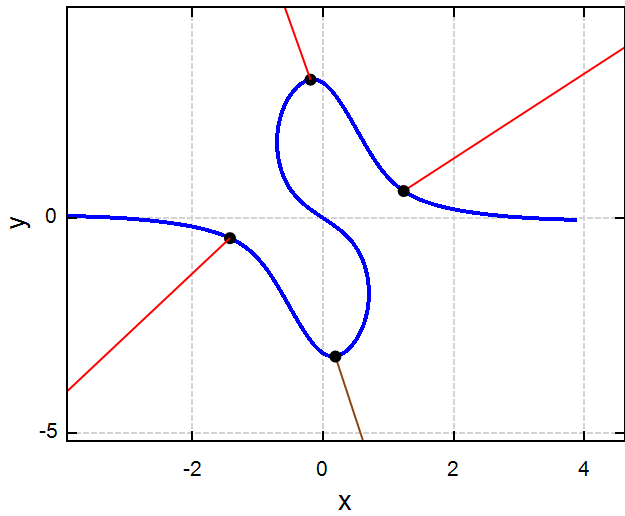
$$F(a2)^T=\left[1.0585\cdot10^{-8} \ 3.1513\cdot10^{-8}\right]$$

$$\left[a3 \ B \ F(a3)\right]:=L(P3) \qquad a3=[0.1891 \ - 3.2155]$$

$$F(a3)^T=\left[- 1.5088\cdot10^{-9} \ - 5.363\cdot10^{-9}\right]$$

$$\left[a4 \ B \ F(a4)\right]:=L(P4) \qquad a4=[- 0.1919 \ 3.2148]$$

$$F(a4)^T=\left[3.1089\cdot10^{-10} \ - 2.6101\cdot10^{-9}\right]$$



Example 4

$P1 := [217 \ 273 \ 129] \qquad P2 := [354 \ - 376 \ 481] \qquad P3 := [- 348 \ 677 \ - 697] \qquad P4 := [- 385 \ - 347 \ - 467]$

$$f(x) := \begin{bmatrix} \left(x_1\right)^2 + \left(x_2\right)^2 + \left(x_3\right)^2 - 9 \\ x_1 + \sin\left(9 \cdot x_2\right) + \cos\left(2 \cdot x_3\right) \end{bmatrix}$$

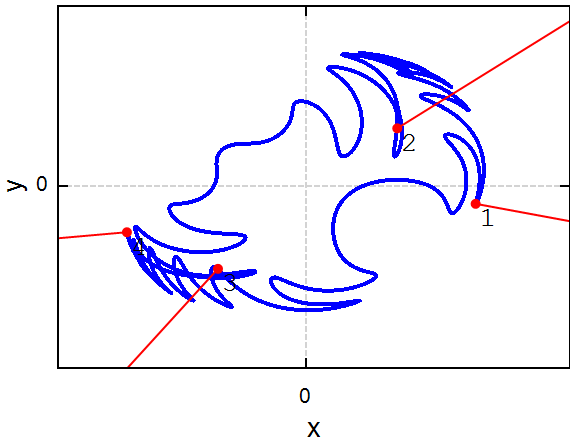
$$\left[x0 := \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}^T \quad \Delta t := 0.05 \quad tmax := 74 \right]$$

$$\left[Pr1 \ B \ F(Pr1)\right] := L(P1) \quad Pr1 = \begin{bmatrix} 1.8308 & 1.9608 & 1.3429 \end{bmatrix}$$

$$\left[Pr2 \ B \ F(Pr2)\right] := L(P2) \quad Pr2 = \begin{bmatrix} 1.4942 & -1.6229 & 2.0331 \end{bmatrix}$$

$$\left[Pr3 \ B \ F(Pr3)\right] := L(P3) \quad Pr3 = \begin{bmatrix} -0.9231 & 1.6137 & -2.3545 \end{bmatrix}$$

$$\left[Pr4 \ B \ F(Pr4)\right] := L(P4) \quad Pr4 = \begin{bmatrix} -1.203 & -1.2273 & -2.459 \end{bmatrix}$$



$$F(Pr1) = \begin{bmatrix} -1.7949 \cdot 10^{-11} \\ -1.6162 \cdot 10^{-9} \\ 1.11 \cdot 10^{-9} \end{bmatrix}$$

$$F(Pr2) = \begin{bmatrix} -3.5147 \cdot 10^{-10} \\ -6.0093 \cdot 10^{-9} \\ -5.1002 \cdot 10^{-11} \end{bmatrix}$$

$$F(Pr3) = \begin{bmatrix} -1.6984 \cdot 10^{-11} \\ -2.4526 \cdot 10^{-9} \\ -8.2835 \cdot 10^{-9} \end{bmatrix}$$

$$F(Pr4) = \begin{bmatrix} -1.6984 \cdot 10^{-11} \\ -2.4526 \cdot 10^{-9} \\ -8.2835 \cdot 10^{-9} \end{bmatrix}$$

Example 5

$$f(x) := \begin{bmatrix} \sin\left(\frac{\left(x_1\right)^2}{4} + \frac{\left(x_2\right)^2}{4}\right) - x_3 - 0.7097 \\ \left(x_1 - 2\right)^2 + \left(x_2 - 2\right)^2 + \left(x_3\right)^2 - 9 \end{bmatrix}$$

$$\left[x0 := \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}^T \quad \Delta t := 0.1 \quad tmax := 74 \right]$$

$$\left[Pr1 \ B \ F(Pr1)\right] := L(P1) \quad Pr1 = \begin{bmatrix} 3.7602 & 4.426 & 0.1278 \end{bmatrix}$$

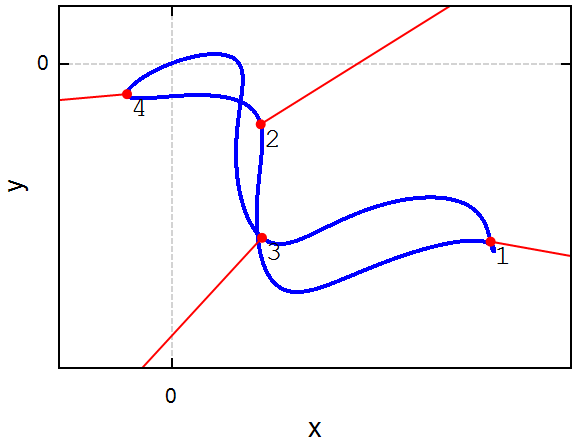
$$\left[Pr2 \ B \ F(Pr2)\right] := L(P2) \quad Pr2 = \begin{bmatrix} 2.7074 & -0.9097 & 0.1825 \end{bmatrix}$$

$$\left[Pr3 \ B \ F(Pr3)\right] := L(P3) \quad Pr3 = \begin{bmatrix} 0.9079 & 4.212 & -1.7072 \end{bmatrix}$$

$$\left[Pr4 \ B \ F(Pr4)\right] := L(P4) \quad Pr4 = \begin{bmatrix} -0.1159 & -0.006 & -0.7063 \end{bmatrix}$$

$$F(Pr1) = \begin{bmatrix} -6.0736 \cdot 10^{-11} \\ 1.7983 \cdot 10^{-10} \\ -9.956 \cdot 10^{-11} \end{bmatrix}$$

$$F(Pr2) = \begin{bmatrix} 1.8386 \cdot 10^{-9} \\ -2.089 \cdot 10^{-9} \\ -9.0759 \cdot 10^{-10} \end{bmatrix}$$



$$F(Pr3)=\begin{bmatrix} 1.4826\cdot10^{-10} \\ -6.1922\cdot10^{-9} \\ 1.0655\cdot10^{-8} \end{bmatrix}$$

$$F(Pr4)=\begin{bmatrix} 1.4826\cdot10^{-10} \\ -6.1922\cdot10^{-9} \\ 1.0655\cdot10^{-8} \end{bmatrix}$$