

## Shape Parametrization Using Fourier Transforms

## Shape data

Reference: Samir Khan,  
<https://www.maplesoft.com/applications/view.aspx?SID=154546>

```
X := stack (558, 533, 512, 509, 519, 538, 527, 501, 485, 493, 499, 499, 465, 406, 413, 441, 430, 406, 397, 393, 392, 397, 403, 406, 397, 365, 356, 354, 326, 311, 336, 35
Y := stack (420, 412, 431, 480, 522, 505, 444, 431, 480, 591, 605, 548, 444, 416, 426, 478, 516, 533, 488, 395, 305, 356, 463, 525, 418, 414, 488, 465, 416, 458, 497, 49
```

$$n := \text{length}(X) = 80 \quad rn := [1..n] \quad m := \lceil 0.5 \cdot n \rceil - 1 \quad k := [1..m]$$

## Fourier Transform

$$FET(z) := \left| \begin{array}{l} z_{rn} := \text{eval} \left( \sqrt{\frac{1}{n}} \cdot \sum_{h=1}^n z_h \cdot e^{-\frac{2 \cdot i \cdot \mathbf{n} \cdot (rn-1) \cdot (h-1)}{n}} \right) \\ \end{array} \right. \quad \begin{array}{l} FX := FET(X)_{1+[1..m]} \\ FY := FET(Y)_{1+[1..m]} \end{array}$$

## Polar forms

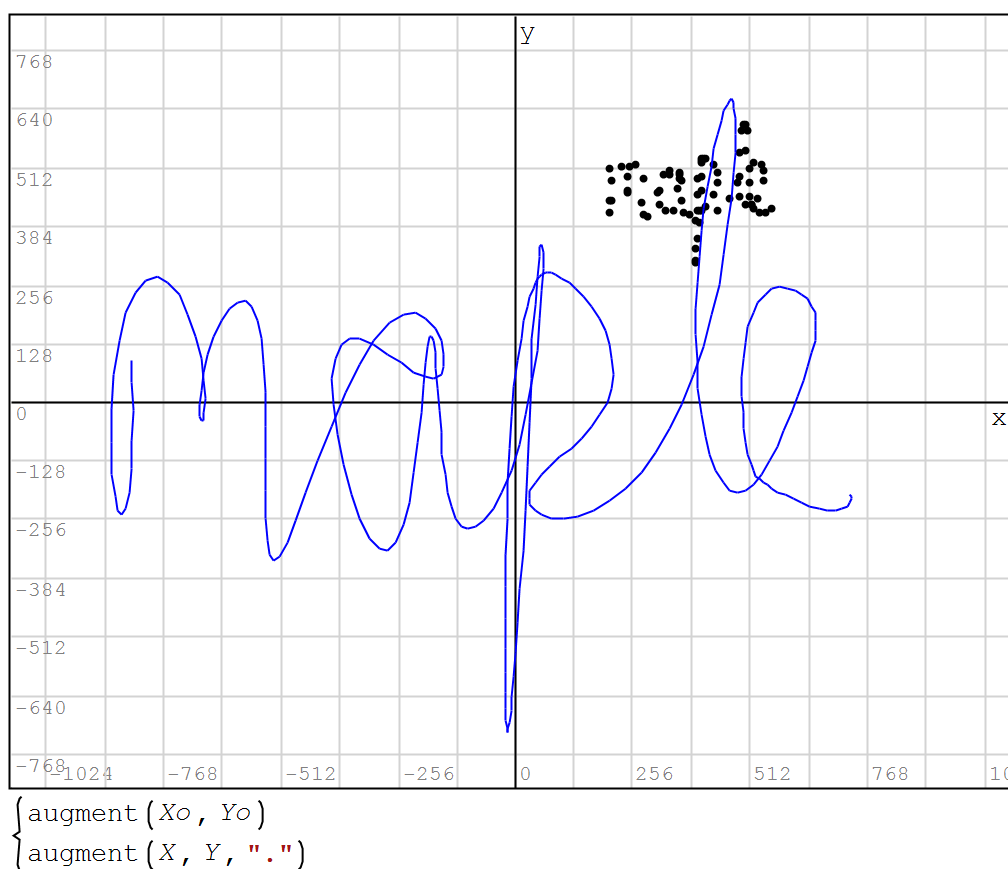
$$[rx \ \varphi x] := \text{eval} \left( \left[ \overrightarrow{|FX|} \overrightarrow{\text{Re}(\arg(FX))} \right] \right) \quad [ry \ \varphi y] := \text{eval} \left( \left[ \overrightarrow{|FY|} \overrightarrow{\text{Re}(\arg(FY))} \right] \right)$$

## Fourier-parametric equations

$$\begin{aligned} finterp(r, \varphi, t) &:= \sum_{h=1}^m r_h \cdot \sin\left(\frac{\pi}{2} - 2 \cdot \mathbf{n} \cdot \frac{h \cdot t}{n} + \varphi_h\right) \\ xf(t) &:= finterp(rx, \varphi_x, t) \\ yf(t) &:= finterp(ry, \varphi_y, t) \end{aligned}$$

## Plot

$$\begin{array}{l} T := \left[ 0, \frac{m+1}{300} .. (m+1) \right] \\ k := [1 .. \text{length}(T)] \\ X_{O_k} := xf \left( \begin{array}{c} T \\ k \end{array} \right) \\ Y_{O_k} := yf \left( \begin{array}{c} T \\ k \end{array} \right) \end{array}$$



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