⊞—d

⊞—equrep

⊞—isol

⊞—Differentials samples

⊟—Lagrangians

Ad hoc function for the square of the magnitude of r'

```
ϑ(r#) := | [ rdot := d(r#)  p# := 0 ]
         | for ρ# ∈ rdot
         |     p# := p# + (ρ#/∂t)^2
         | p#
```

$dSymbol := "\partial"$

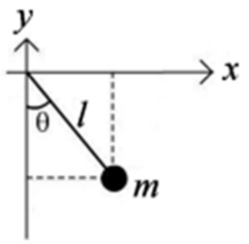
$Clear(a, g, \theta, \theta', \theta'') = 1$

Lagrangian Dynamics

Simple pendulum

Constants $\partial a := 0$ $\quad \partial m := 0$ $\quad \partial g := 0$

Gen Coords $\partial\theta := \theta' \cdot \partial t$ $\quad \partial\theta' := \theta'' \cdot \partial t$



Position $r := \begin{bmatrix} a \cdot \sin(\theta) & -a \cdot \cos(\theta) \end{bmatrix}$

Lagrangian $L := \frac{1}{2} \cdot m \cdot \vartheta(r) - m \cdot g \cdot r_2$

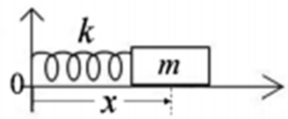
Equations of motion $eq := \left\{ isol\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial(L, \theta')}{\partial\theta'} \right) - \frac{\partial(L, \theta)}{\partial\theta}, \theta'' \right) = \left\{ -\frac{g \cdot \sin(\theta)}{a} \right.\right.$

$Clear(k, m, x, \dot{x}, \ddot{x}) = 1$

Harmonic Oscillator

Constants $\partial k := 0$ $\quad \partial m := 0$

Gen Coords $\partial x := \dot{x} \cdot \partial t$ $\quad \partial\dot{x} := \ddot{x} \cdot \partial t$



Position x

Lagrangian $L := \frac{1}{2} \cdot m \cdot \vartheta(x) - \frac{1}{2} \cdot k \cdot x^2$

Equations of motion $eq := \left\{ isol\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial(L, \dot{x})}{\partial\dot{x}} \right) - \frac{\partial(L, x)}{\partial x}, \ddot{x} \right) = \left\{ \frac{x \cdot k}{m} \right.\right.$

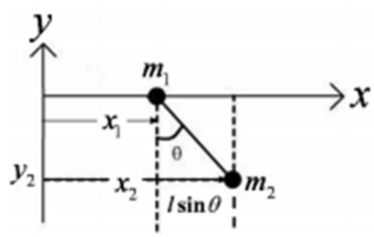
$Clear(a, m_1, m_2, g, \theta, \theta', \theta'', x_1, \dot{x}_1, \ddot{x}_1) = 1$

Pendulum with Horizontal support

Constants $\partial a := 0$ $\quad \partial m_1 := 0$ $\quad \partial m_2 := 0$ $\quad \partial g := 0$

Gen Coords $\partial\theta := \theta' \cdot \partial t$ $\quad \partial\theta' := \theta'' \cdot \partial t$

$\partial x_1 := \dot{x}_1 \cdot \partial t$ $\quad \partial\dot{x}_1 := \ddot{x}_1 \cdot \partial t$



Position $r_1 := \begin{bmatrix} x_1 \\ 0 \end{bmatrix}$ $\quad r_2 := \begin{bmatrix} x_1 + a \cdot \sin(\theta) \\ -a \cdot \cos(\theta) \end{bmatrix}$

Lagrangian
$$L := \frac{1}{2} \cdot m_1 \cdot \vartheta\left(r_1\right) + \frac{1}{2} \cdot m_2 \cdot \vartheta\left(r_2\right) - m_2 \cdot g \cdot {r_2}_2$$

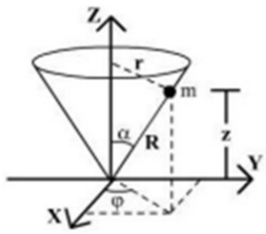
Equations of motion

$$eq := \begin{cases} \text{isol}\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial\left(L, \dot{x}_1\right)}{\partial \dot{x}_1}\right) - \frac{\partial\left(L, x_1\right)}{\partial x_1}, \ddot{x}_1\right) \\ \text{isol}\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial\left(L, \theta'\right)}{\partial \theta'}\right) - \frac{\partial\left(L, \theta\right)}{\partial \theta}, \theta''\right) \end{cases} = \begin{cases} \frac{a \cdot m_2 \cdot \left(-\sin(\theta) \cdot \theta'^2 + \cos(\theta) \cdot \theta''\right)}{m_1 + m_2} \\ -\frac{\sin(\theta) \cdot g + \cos(\theta) \cdot \ddot{x}_1}{a} \end{cases}$$

$$\text{Clear}\left(\alpha, m, g, \varphi, \varphi', \varphi'', r, \dot{r}, \ddot{r}\right) = 1$$

Particle in a Cone

Constants $\quad \partial\alpha := 0 \quad \partial m := 0 \quad \partial g := 0$



Gen Coords $\quad \partial\varphi := \varphi' \cdot \partial t \quad \partial\varphi' := \varphi'' \cdot \partial t$

$\quad \partial\rho := \rho' \cdot \partial t \quad \partial\rho' := \rho'' \cdot \partial t$

Position
$$r := \left[\rho \cdot \cos(\varphi) \;\; \rho \cdot \sin(\varphi) \;\; \rho \cdot \cot(\alpha)\right]$$

Lagrangian
$$L := \frac{1}{2} \cdot m \cdot \vartheta(r) - m \cdot g \cdot z$$

Equations of motion
$$eq := \begin{cases} \text{isol}\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial\left(L, \varphi'\right)}{\partial \varphi'}\right) - \frac{\partial\left(L, \varphi\right)}{\partial \varphi}, \varphi''\right) \\ \text{isol}\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial\left(L, \rho'\right)}{\partial \rho'}\right) - \frac{\partial\left(L, \rho\right)}{\partial \rho}, \rho''\right) \end{cases}$$

$\alpha := 45\ \text{deg}$

$$eq = \begin{cases} -\frac{\left(-\sin(\varphi) \cdot \left(\cos(\varphi) \cdot \rho' - \rho \cdot \sin(\varphi) \cdot \varphi'\right) + \cos(\varphi) \cdot \left(\sin(\varphi) \cdot \rho' + \rho \cdot \cos(\varphi) \cdot \varphi'\right) + \rho \cdot \varphi'\right) \cdot \rho'}{\rho^2} \\ -\frac{\varphi' \cdot \left(-\sin(\varphi) \cdot \left(\cos(\varphi) \cdot \rho' - \rho \cdot \sin(\varphi) \cdot \varphi'\right) + \cos(\varphi) \cdot \left(\sin(\varphi) \cdot \rho' + \rho \cdot \cos(\varphi) \cdot \varphi'\right)\right)}{\cos(\varphi)^2 + \sin(\varphi)^2 + \cot(\alpha)^2} \end{cases}$$

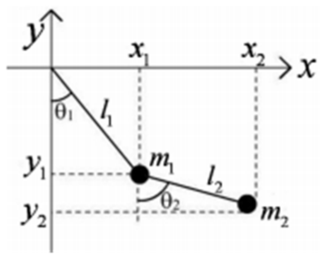
For small angles
$$\text{EquRep}\left(eq, \begin{bmatrix} \sin(a) & 0 \\ \cos(a) & 1 \end{bmatrix}\right) = \begin{cases} -\frac{2 \cdot \varphi' \cdot \rho'}{\rho} \\ -\frac{\varphi'^2 \cdot \rho}{1 + \cot(\alpha)^2} \end{cases}$$

$$\text{Clear}\left(a_1, a_2, m_1, m_2, g, \theta_1, \theta'_1, \theta''_1, \theta_2, \theta''_2\right) = 1$$

Double Pendulum

Constants $\quad \partial m_1 := 0 \quad \partial m_2 := 0 \quad \partial g := 0$

$\quad \partial a_1 := 0 \quad \partial a_2 := 0$



Gen Coords $\quad \partial\theta_1 := \theta'_1 \cdot \partial t \quad \partial\theta'_1 := \theta''_1 \cdot \partial t$

$\quad \partial\theta_2 := \theta'_2 \cdot \partial t \quad \partial\theta'_2 := \theta''_2 \cdot \partial t$

Position
$$r_1 := \begin{bmatrix} a_1 \cdot \sin\left(\theta_1\right) \\ -a_1 \cdot \cos\left(\theta_1\right) \end{bmatrix} \qquad r_2 := \begin{bmatrix} a_1 \cdot \sin\left(\theta_1\right) + a_2 \cdot \sin\left(\theta_2\right) \\ -a_1 \cdot \cos\left(\theta_1\right) - a_2 \cdot \cos\left(\theta_2\right) \end{bmatrix}$$

Lagrangian $\quad L := \frac{1}{2} \cdot m_1 \cdot \vartheta\left(r_1\right) + \frac{1}{2} \cdot m_2 \cdot \vartheta\left(r_2\right) - m_1 \cdot g \cdot {r_1}_2 - m_2 \cdot g \cdot {r_2}_2$

Equations of motion

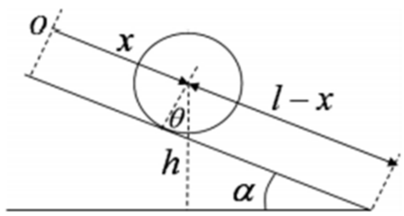
$$eq := \begin{cases} \text{isol}\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial\left(L, \theta'_1\right)}{\partial \theta'_1}\right) - \frac{\partial\left(L, \theta_1\right)}{\partial \theta_1}, \theta''_1\right) \\ \text{isol}\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial\left(L, \theta'_2\right)}{\partial \theta'_2}\right) - \frac{\partial\left(L, \theta_2\right)}{\partial \theta_2}, \theta''_2\right) \end{cases}$$

For small angles

$$\text{EquRep}\left(eq, \begin{bmatrix} \sin(a) & 0 \\ \cos(a) & 1 \end{bmatrix}\right) = \begin{cases} -\frac{m_2 \cdot a_2 \cdot \theta''_2}{a_1 \cdot \left(m_1 + m_2\right)} \\ -\frac{a_1 \cdot \theta''_1}{a_2} \end{cases}$$

$\text{Clear}(m, a, R, \alpha, g, x, \dot{x}, \ddot{x}) = 1$

Hoop rolling without sliding down an inclined plane



Constants $\quad \partial m := 0 \quad \partial a := 0 \quad \partial g := 0$

$\partial R := 0 \quad \partial \alpha := 0$

Gen Coords $\quad \partial x := \dot{x} \cdot \partial t \qquad \partial \dot{x} := \ddot{x} \cdot \partial t$

Restrictions $\quad \theta' := \frac{\dot{x}}{R} \qquad$ Non sliding condition.

Position $\quad x$

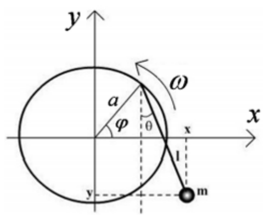
Lagrangian $\quad L := \frac{1}{2} \cdot m \cdot \vartheta(x) + \frac{1}{2} \cdot m \cdot R^2 \cdot \theta'^2 - m \cdot g \cdot (a - x) \cdot \sin(\alpha)$

Equations of motion

$$eq := \left\{ \text{isol}\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial(L, \dot{x})}{\partial \dot{x}}\right) - \frac{\partial(L, x)}{\partial x}, \ddot{x}\right) = \left\{ -\frac{g \cdot \sin(\alpha)}{2} \right.\right.$$

$\text{Clear}(\partial a, \partial \alpha, \partial m, \partial R, \partial g, \theta, \theta', \theta'') = 1$

Pendulum whose support rotates with constant angular speed



Constants $\quad \partial m := 0 \quad \partial a := 0 \quad \partial g := 0$

$\partial R := 0 \quad \partial \omega := 0$

Gen Coords $\quad \partial \theta := \theta' \cdot \partial t \qquad \partial \theta' := \theta'' \cdot \partial t$

Position

$$r := \begin{bmatrix} R \cdot \cos(\omega \cdot t) + a \cdot \sin(\theta) \\ R \cdot \sin(\omega \cdot t) - a \cdot \cos(\theta) \end{bmatrix}$$

Lagrangian $\quad L := \frac{1}{2} \cdot m \cdot \vartheta(r) - m \cdot g \cdot r_2$

Equations of motion

$$eq := \left\{ \text{isol}\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial(L, \theta')}{\partial \theta'}\right) - \frac{\partial(L, \theta)}{\partial \theta}, \theta''\right) \right.$$

$$eq = \left\{ -\frac{-\omega^2 \cdot R \cdot (\cos(\omega \cdot t) \cdot \cos(\theta) + \sin(\omega \cdot t) \cdot \sin(\theta)) + g \cdot \sin(\theta)}{a} \right.$$

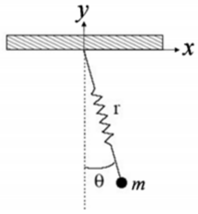
For zero angular speed, we recover the simple pendulum

$$\text{EquRep}(eq, \omega, 0) = \left\{ -\frac{g \cdot \sin(\theta)}{a} \right.$$

$\text{Clear}(m, a, k, g, \theta, \theta', \theta'') = 1$

Pendulum whose support rotates with constant angular speed

Constants $\partial m := 0 \quad \partial a := 0 \quad \partial g := 0$



Gen Coords $\partial\theta := \theta' \cdot \partial t \quad \partial\theta' := \theta'' \cdot \partial t$

Position $r := [\rho \cdot \cos(\theta) \ \ -\rho \cdot \sin(\theta)]$

Lagrangian $L := \frac{1}{2} \cdot m \cdot \vartheta(r) - m \cdot g \cdot y - \frac{1}{2} \cdot k \cdot (\rho - a)^2$

Equations of motion $eq := \left\{ \text{isol}\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial(L, \theta')}{\partial\theta'} \right) - \frac{\partial(L, \theta)}{\partial\theta}, \theta'' \right) \right.$

$$eq = \left\{ \frac{(-\sin(\theta) \cdot (-\rho \cdot \sin(\theta) \cdot \theta' + \cos(\theta) \cdot \rho') + \cos(\theta) \cdot (\rho \cdot \cos(\theta) \cdot \theta' + \sin(\theta) \cdot \rho') + \theta' \cdot \rho) \cdot \rho'}{\rho^2} \right.$$

For small angles $\text{EquRep}\left(eq, \begin{bmatrix} \sin(a) & 0 \\ \cos(a) & 1 \end{bmatrix} \right) = \left\{ \frac{2 \cdot \theta' \cdot \rho'}{\rho} \right.$

Alvaro