Projection of a curve onto a surface

Computing the projection of a point onto a surface is to find a closest point on the surface, and projection of a curve onto a surface is the locus of all points on the curve project onto the surface.

Proj

$$\gamma := \begin{bmatrix} -0.8232 & -0.4194 & 0.3827 \\ 0.5677 & -0.6187 & 0.543 \\ 0.009 & 0.6643 & 0.7474 \end{bmatrix} \qquad Proj_{\gamma}(x) := \text{eval}(\text{augment}(\text{col}(x\cdot\gamma,1),\ \text{col}(x\cdot\gamma,3)))$$

PMesh returns a maitrx suitable for 3D plots.

$$pMesh(F\#) := \left| \begin{array}{l} [\ F := F\#\ \ nu := \text{rows}(F)-1\ \ nv := \text{cols}(F)-1\ ] \\ [\ np := 5\ \ r_{np} := [1..np]\ \ r_{nv} := [1..(nv-1)]\ ] \\ [\ M := \text{matrix}(0,3)\ \ c := [1..3]\ ] \\ \text{for } y \in [1..nu] \\ \quad \text{for } v \in [1..nv] \\ \quad\quad [\ r := \text{rows}(M)\ \ k := r + r_{np}\ ] \\ \quad\quad M_{k\,c} := F_{[0\ 0\ 1\ 1\ 0]_{k-r} + y\,[0\ 1\ 1\ 0\ 0]_{k-r} + v\ \ c} \\ \quad [\ r := \text{rows}(M)\ \ j := r + r_{nv}\ ] \\ \quad M_{j\,c} := M_{(y-1)\cdot(np\cdot nv + nv - 1) + np\cdot(nv-1) - (j-r-1)\cdot np\ \ c} \\ M \end{array} \right.$$

$$ProjC2S(\lambda(1), b, \varphi(2), B) :=$$

$$:= \left| \begin{array}{l} \left[\ \text{Clear}(u, v, x, S, L, C, a, d)\ \ c := [1..3]\ \ rng(b) := \left[[1..b_3]\ \ b_1 + \dfrac{b_2 - b_1}{b_3 - 1}\cdot[1..b_3] \right] \right] \\ [\ [nx\ X] := rng(\text{row}(B,1))\ \ [ny\ Y] := rng(\text{row}(B,2))\ ] \\ [\ S_{nx\,ny} := \text{eval}(\text{augment}(X_{nx}, Y_{ny}, f(X_{nx}, Y_{ny})))\ \ k := [1..\text{length}(S)]\ ] \\ \left[\ \varphi(x) := f(x_1, x_2) - x_3\ \ \varphi'_c := \dfrac{d}{d\,x_c}\varphi(x)\ \ eN := \dfrac{\varphi'}{\text{norme}(\varphi')}\ \right] \\ [\ [nt\ T] := rng(b)\ \ C_{nt\,c} := \lambda(T_{nt})_c\ ] \\ \text{for } n \in nt \\ \quad \left[\ A := C_{n\,c}\ \ ao := S_{\text{csort}(\text{augment}(d_k := \text{norme}(A - S_k),\ k),\ 1)_2}\ \right] \\ \quad \left[\ F(x) := \begin{bmatrix} \varphi(x) \\ eN_2\cdot(A_3 - x_3) - eN_3\cdot(A_2 - x_2) \\ eN_3\cdot(A_1 - x_1) - eN_1\cdot(A_3 - x_3) \end{bmatrix}\ \ \alpha := \text{al\_nleqsolve}(ao^{T}, F(x))\ \right] \\ \quad [\ a_{n\,c} := \alpha_c\ \ L_n := \text{stack}(\alpha^{T}, A)\ ] \\ [\ C\ \ pMesh(S)\ \ a\ \ L\ ] \end{array} \right.$$

Returns the coordinates of the Curve (C), Surface (S), Projection (a) and Line segments between the point in the curve and the point in the surface (L).

```
pCycleC(A) :=  | [ B := eval(sys2mat(A))  M := [B_1] ]
               | for  k := 2, k ≤ length(B), k := k + 1
               |    | for  n ∈ [1..5]
               |    |    M_(rows(M)+1) := [10^5  10^5]
               |    | M_(rows(M)+1) := B_k
               | eval(mat2sys_1(M))
```

Shorthand for plots

```
Plot(X) := | [ C  S  A  L ] := X
           | { Proj_Y(C)
           |   augment(Proj_Y(A), ".", 12, "green")
           |   ""
           |   pCycleC(→Proj_Y(L))
           |   Proj_Y(S)
```

Number of points:

$nt := 50$

$nx := 30$

$ny := 30$

Curve given by the parametric equations $\quad \lambda(t) = [\, x(t) \;\; y(t) \;\; z(t) \,]$ and $box := [\, t1 \;\; t2 \;\; nt \,]$

Surface given by the explicit equation $\quad z = f(x, y)$ and $Box = \begin{bmatrix} x1 & x2 & nx \\ y1 & y2 & ny \end{bmatrix}$

Curve

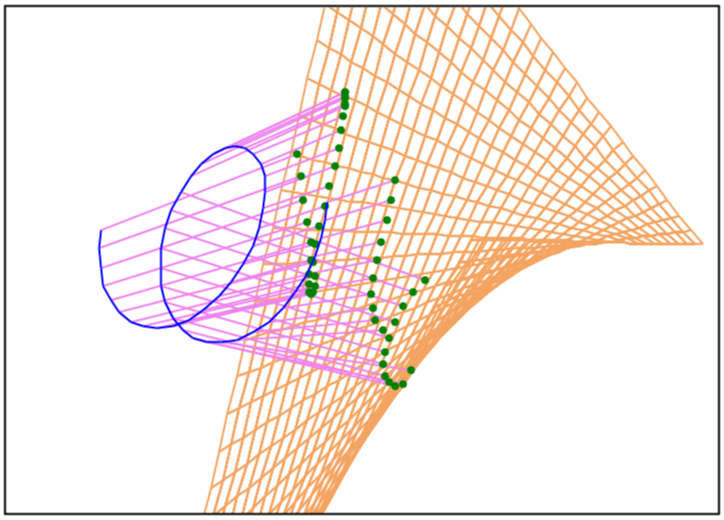
$\lambda(t) := [\, 0.2 \cdot t \;\; 2 \cdot \sin(t) \;\; 4 \cdot \cos(t) \,]$

$Bt := [\, 20 \;\; 30 \;\; nt \,]$

Surface

$f(x, y) := x \cdot y - 4 \cdot x$

$Bxy := \begin{bmatrix} -1 & 6 & nx \\ -1 & 6 & ny \end{bmatrix}$



$Plot(ProjC2S(\lambda(t), Bt, f(x, y), Bxy))$

Curve

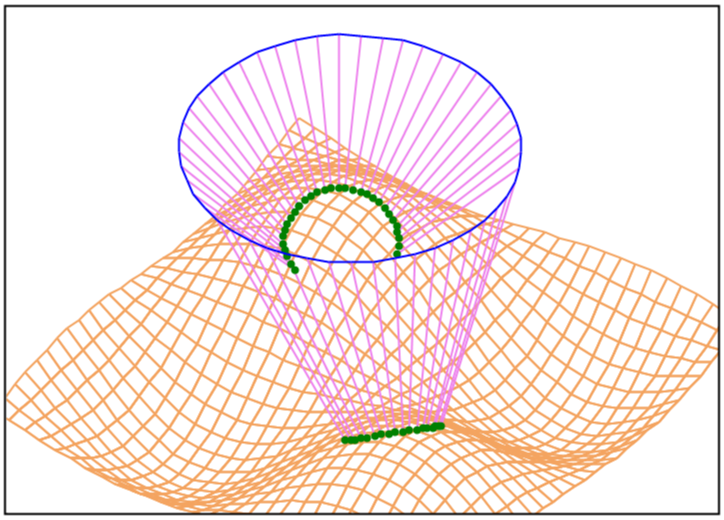
$\lambda(t) := [\, 2 \cdot \cos(t) + 0.7 \;\; 2 \cdot \sin(t) + 0.5 \;\; 3 \,]$

$Bt := [\, 0 \;\; 2 \cdot \pi \;\; nt \,]$

Surface

$f(x, y) := \sin(x) \cdot \sin(y)$

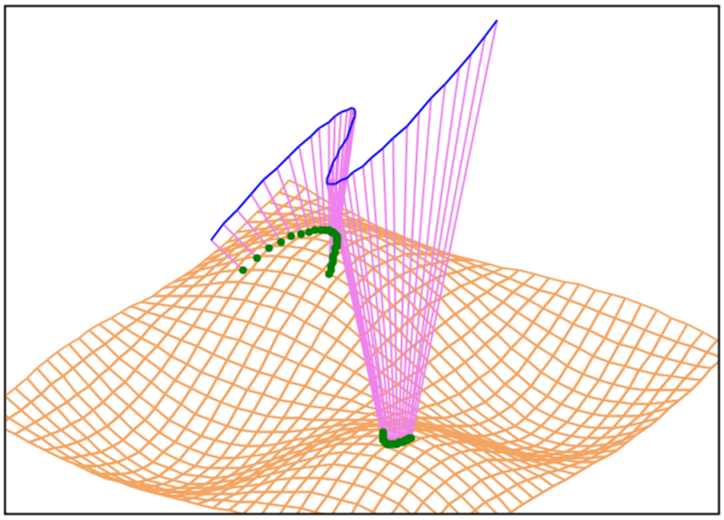
$Bxy := \begin{bmatrix} -\pi & \pi & nx \\ -\pi & \pi & ny \end{bmatrix}$



$Plot(ProjC2S(\lambda(t), Bt, f(x, y), Bxy))$

Same example with

$\lambda(t) := [2 \cdot \cos(0.5 \cdot t) + 0.7 \;\; 2 \cdot \sin(t) + 0.5 \;\; t + 1]$



$Plot(ProjC2S(\lambda(t), Bt, f(x, y), Bxy))$

Curve

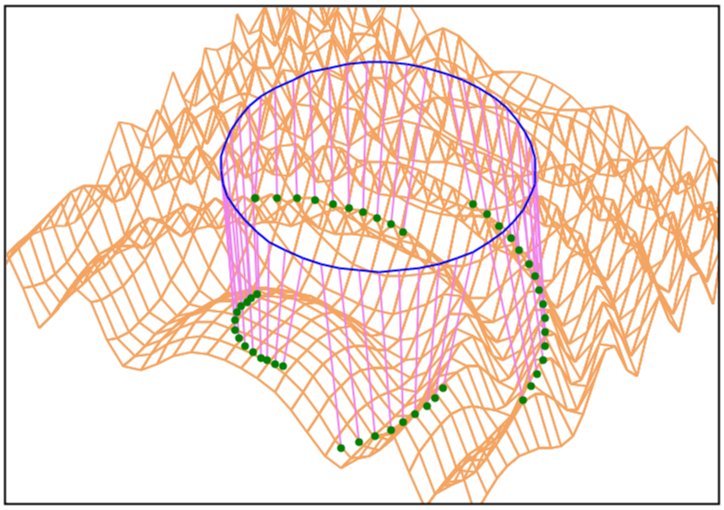
$\lambda(t) := [2 \cdot \cos(t) - 1 \;\; 2 \cdot \sin(t) + 1 \;\; 3]$

$Bt := [0 \;\; 2 \cdot \pi \;\; nt]$

Surface

$f(x, y) := 0.5 \cdot \cos\left(\frac{x^2}{1} + \frac{y^2}{1}\right)$

$Bxy := \begin{bmatrix} -4 & 3.5 & nx \\ -1 & 6 & ny \end{bmatrix}$

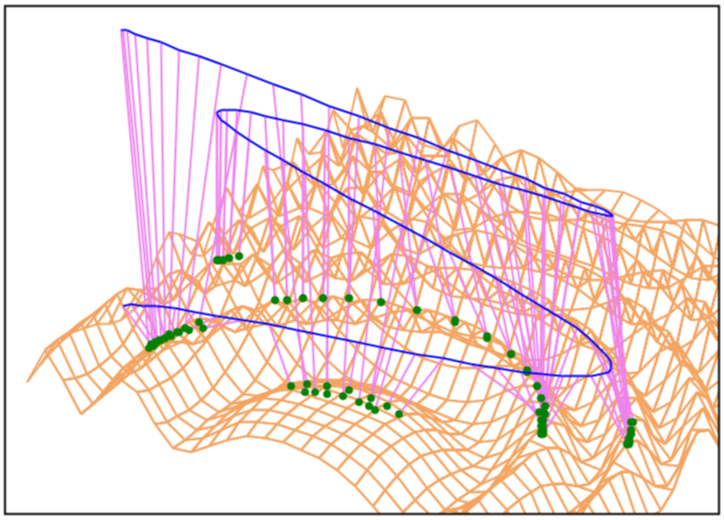


$Plot(ProjC2S(\lambda(t), Bt, f(x, y), Bxy))$

Same example with

$\lambda(t) := \left[3 \cdot \cos(2 \cdot t) \;\; 2 \cdot \sin(0.5 \cdot t) \;\; \sqrt{t^2 + 1}\right]$

$Bt := [0 \;\; 2 \cdot \pi \;\; 2 \cdot nt]$



$Plot(ProjC2S(\lambda(t), Bt, f(x, y), Bxy))$