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Example 12.2 (Page 296) - Soil Mechanics : Principles and Practice by G.E. Barnes

appVersion(4) = "1.0.8348.30405" appVersion(−4) = "1.0.8348.30405"

$t_0 := \text{time}(0)$

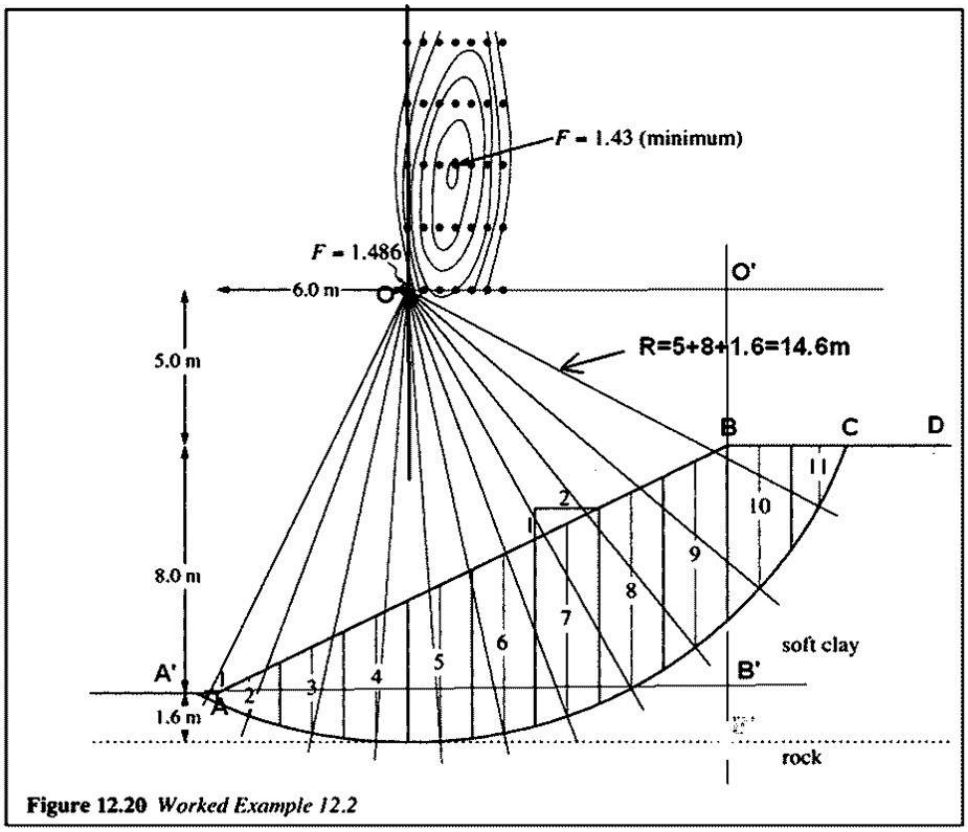


Figure 12.20 *Worked Example 12.2*

A slope is to be cut into a soft clay with undrained shear strength of 30 kN/m2 and unit weight of 18 kN/m3• The slope is 8.0 m high and its inclination is 2: 1 (horizontal:vertical). Determine the factor of safety for the trial circle shown on **Figure 12.20.**

Equation 12.9 is used with values of **b, α and mid-slice height h** determined for each slice. The weight of each slice is obtained from **W = Y*b*h.**

Slip Circle tangential to the ROCK Layer at depth of 1.6 m below the toe.

Analysis by cordinates and dimensions

Slope Geometry

$H_{bund} := 8\ m$ = "Height of Bund"

$\beta_{slope} := \text{atan}\left(\frac{1}{2}\right)$ = "Angle of slope line AB with Horizontal"

$H_{vc} := 5\ m$ = "Vertical height from Top Level BD to Center"

$H_{hz} := 6\ m$ = "Horizontal distance from B to Center"

$H_{rock} := 1.6\ m$ = "Heigth from Toe to Rock Layer"

$WIDTH := 1\ m$ = "Define width of strips for Stability Analysis"

Define Accuracy for Findroot

$\Delta E := \begin{bmatrix} 10^{-3} \\ 10^{-3} \end{bmatrix} mm$

$AB' := \frac{H_{bund}}{\tan(\beta_{slope})} = 16\ m$ $OO' := AB' - 6\ m = 10\ m$ $AB := \sqrt{H_{bund}^{\ 2} + AB'^{\ 2}} = 17.89\ m$

Define Slope Toe Cordinates

$A := [1\ m\ 1\ m] = [1\ 1]\ m$

Note: Pl do **not change cordinates of A,** as we may have to change the **Center Cordinates** to find the **minimum FS in future development.**

Now define initial **Center** cordinates,
Pl. do not change this, as ACAD values shown in this worksheet have been derived from relevant ACAD drawing.

$Center := [7\ 14]\ m$

Radius of Circle from Center to touch the rock layer

$R := \left(Center_{\ 2} - A_{\ 2}\right) + H_{rock} = 14.6\ m$ (See he ACAD diagram below)

Calculation of other important cordinates

$B := \left[\left(AB' + A_1 \right) \left(H_{bund} + A_2 \right) \right] = [\,17\ 9\,]\ \text{m}$

Line AB (D/S Slope) and Intercept

Slope of D/S profile = $dsl := \tan\left(\beta_{slope}\right) = 0.5$

Intercept of D/S profile using c = y - mx $dsi := A_{12} - dsl \cdot A_{11} = 0.5\ \text{m}$

Initially, find the two cutting points AA and C of the Slip Circle.

Program 1 : Condition for cutting points

$$Cut\_cond(x) := \begin{cases} A_{12} & \text{if } x \le A_{11} \\ dsl \cdot x + dsi & \text{if } A_{11} < x < B_{11} \\ B_{12} & \text{otherwise} \end{cases}$$

Program 2 : To find cutting point of a line and circle of a given center.
Used Program 1. Note how function is passed to this program to use Program 1

$$int\_line\_circle(r, cen, gs\#, \xi\#, FF) := \left| \begin{array}{l} F2\#(r) := \left[\begin{array}{c} FF = 0 \\ \left(cen_{11} - x\right)^2 + \left(cen_{12} - y\right)^2 = r^2 \end{array} \right] \\ \text{FindRoot}\left(F2\#(r), gs\#^{\mathrm{T}}, \xi\#, \xi\#\right) \end{array} \right.$$

Cordinates of cutting point of SLIP CIRCLE with horizontal line thro A

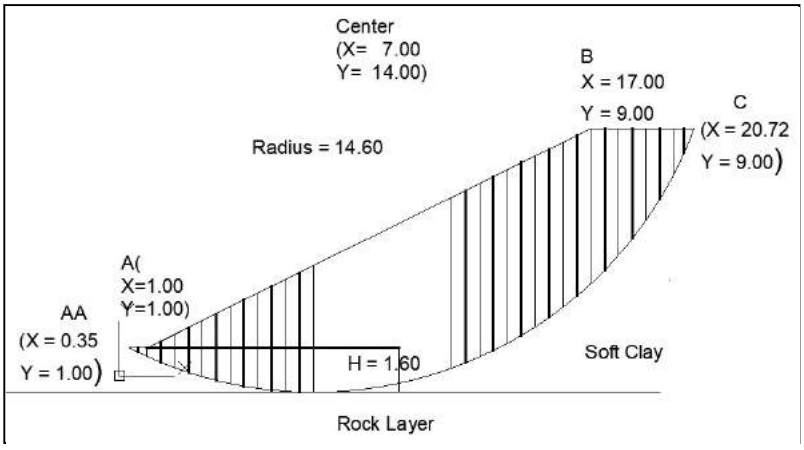
$AA := int\_line\_circle(R, Center, -A, \Delta E, Cut\_cond(x) - y) = \begin{bmatrix} 0.35 \\ 1 \end{bmatrix}\ \text{m}$

From ACAD
X = 0.35 Y = 1.00

Cordinates of cutting point of SLIP CIRCLE with horizontal line BD

$CC := int\_line\_circle(R, Center, B, \Delta E, y - Cut\_cond(x)) = \begin{bmatrix} 20.72 \\ 9 \end{bmatrix}\ \text{m}$

From ACAD
X = 20.72 Y = 9.00



$A = [\,1\ 1\,]\ \text{m}$

$AA^{\mathrm{T}} = [\,0.35\ 1\,]\ \text{m}$

$B = [\,17\ 9\,]\ \text{m}$

$CC^{\mathrm{T}} = [\,20.72\ 9\,]\ \text{m}$

$Center = [\,7\ 14\,]\ \text{m}$

$R = 14.6\ \text{m}$

Program 3 : To find cutting point of a line and circle of a given center.
Note Function FF is passed to this program from Program 4

$$int\_line\_circle2(r, cen, \xi\#, FF) := \left| \begin{array}{l} F2\#(cen, r) := \left[\begin{array}{c} FF = 0 \\ \left(cen_{11} - x\right)^2 + \left(cen_{12} - y\right)^2 = r^2 \end{array} \right] \\ \text{FindRoot}\left(F2\#(cen, r), \xi\#, \xi\#\right) \end{array} \right.$$

Program 4 : Cutting points with slip circle : Calls Program 3

```
Find_Cords_Bot(Z#, cen#, r) :=
    j := [1 .. rows(Z#)]
    B#_j := int_line_circle2(r, cen#, ΔE, (x − Z#_j_1))_2
    augment(col(Z#, 1), B#_j)
```

Program 5 : Called by Program 6

```
Find_Cords_Hoz(nn, width, aa, aa') :=
    j := [1 .. nn]
        x1_j := (aa_1 + width·(j − 1))
        y1_j := aa_2
        ans1 := augment(x1, y1)
        ans1 := stack(ans1, aa'^T)
```

Program 6 : To find TOP Cordinates of Strips on Horizontal Surfabe: Calls Program 5

```
TC_horiz(width, aa, c) :=
    nn1 := ⌈|(A_1 − aa_1) / width|⌉
    nn2 := ⌈|(c_1 − B_1) / width|⌉
    ans1 := Find_Cords_Hoz(nn1, −width, A, aa)   if (A_1 ≥ aa_1) ∧ (c_1 > B_1)
    ans2 := Find_Cords_Hoz(nn2, width, B, c)
    [ans1; ans2]
    ans1 := Find_Cords_Hoz(nn1, −width, A, aa)   if (A_1 ≥ aa_1) ∧ (c_1 < B_1)
    ans2 := Find_Cords_Hoz(nn2, width, B, c)     otherwise
```

Result may be a **Nested Array** or **not depending on the Center Cords**

$$TC\_hoz := TC\_horiz(WIDTH, AA, CC) = \begin{bmatrix} \begin{bmatrix} 1 & 1 \\ 0.35 & 1 \end{bmatrix} \\ \begin{bmatrix} 17 & 9 \\ 18 & 9 \\ 19 & 9 \\ 20 & 9 \\ 20.72 & 9 \end{bmatrix} \end{bmatrix} \text{ m}$$

From ACAD

X = 1.00 Y = 1.00
X = 0.35 Y = 1.00
X = 17.00 Y = 9.00
X = 19.00 Y = 9.00
X = 20.72 Y = 9.00

$(A_1 \geq AA_1) \wedge (CC_1 \geq B_1) = 1$

$(A_1 \geq AA_1) \wedge (CC_1 < B_1) = 0$

$(A_1 < AA_1) \wedge (CC_1 \geq B_1) = 0$

Program 7 : Top Cords on Sloping Surface AB : Called by Program 8

```
Find_AB_Cords(width, nn, aa, bb, ββ#, rem#) :=
    j := [1 .. (nn)]
        x1_j := (aa_1 + width·j)                          if rem# = 0
        x1 := stack(aa_1, x1)
        y1_j := (aa_2 + width·j·tan(ββ#))
        y1 := stack(aa_2, y1)
        augment(x1, y1)
        x1_j := aa_1 + width·j                            otherwise
        x1 := stack(aa_1, x1, bb_1)
        y1_j := (aa_2 + width·j·tan(ββ#))
        y1 := stack(aa_2, y1, bb_2)
        augment(x1, y1)
```

Program : 8 Top Cords of strips on Slope AB : Calls Program 7

$$Find\_TC\_AB(width\#, a, b, \beta\#) := \begin{bmatrix} \Delta X := \left|\left(a_1 - b_1\right)\right| \\ nn := \left(\lfloor \Delta X \rfloor_{width\#}\right) \\ rem\# := \Delta X - nn \\ nn1 := \frac{nn}{width\#} \\ Find\_AB\_Cords(width\#, nn1, a, b, \beta\#, rem\#) \end{bmatrix}$$

$$\left(A_1 \geq AA_1\right) \wedge \left(CC_1 \geq B_1\right) = 1$$

$$\left(A_1 \geq AA_1\right) \wedge \left(CC_1 < B_1\right) = 0$$

$$\left(A_1 < AA_1\right) \wedge \left(CC_1 \geq B_1\right) = 0$$

Program 9 : To find TOP Cordinates of Strips on Sloping Surface AB Calls Program 8

$$TC\_AB(width, aa, c, \beta\#) := \begin{cases} Find\_TC\_AB(width, A, B, \beta\#) & \text{if } \left(A_1 \geq aa_1\right) \wedge \left(c_1 \geq B_1\right) \\ Find\_TC\_AB(width, A, c, \beta\#) & \text{if } \left(A_1 \geq aa_1\right) \wedge \left(c_1 < B_1\right) \\ Find\_TC\_AB(width, aa, B, \beta\#) & \text{otherwise} \end{cases}$$

Top Cords of Strips on Slope AB Program 9.

$$TC\_AB := TC\_AB\left(WIDTH, AA, CC, \beta_{slope}\right)$$

From ACAD

$$TC\_AB = \begin{bmatrix} 1 & 1 \\ 2 & 1.5 \\ 3 & 2 \\ 4 & 2.5 \\ 5 & 3 \\ 6 & 3.5 \\ 7 & 4 \\ 8 & 4.5 \\ 9 & 5 \\ 10 & 5.5 \\ 11 & 6 \\ 12 & 6.5 \\ 13 & 7 \\ 14 & 7.5 \\ 15 & 8 \\ 16 & 8.5 \\ 17 & 9 \end{bmatrix} \text{ m}$$

X = 1.00 Y = 1.00
X = 2.00 Y = 1.50
X = 5.00 Y = 3.00
X = 16.00 Y = 8.50
X = 17.00 Y = 9.00

To obtain BC_AB, directly use Find_Cords_Bot(() function. Program 4.

$$BC\_AB := Find\_Cords\_Bot\left(TC\_AB, Center, R\right)$$

From ACAD

$$BC\_AB = \begin{bmatrix} 1 & 0.69 \\ 2 & 0.28 \\ 3 & -0.04 \\ 4 & -0.29 \\ 5 & -0.46 \\ 6 & -0.57 \\ 7 & -0.6 \\ 8 & -0.57 \\ 9 & -0.46 \\ 10 & -0.29 \\ 11 & -0.04 \\ 12 & 0.28 \\ 13 & 0.69 \\ 14 & 1.19 \\ 15 & 1.79 \\ 16 & 2.5 \\ 17 & 3.36 \end{bmatrix} \text{ m}$$

X = 1.00 Y = 0.69
X = 3.00 Y = -0.04
X = 5.00 Y = -0.46
X = 7.00 Y = -0.60
X = 13.00 Y = 0.69
X = 15.00 Y = 1.79
X = 17.00 Y = 3.36

Program 10 : To find Bottom Cordinates of Top Strip Lines cutting the Slip Circle - Calls Program 4

$$Find\_All\_BC\_Hoz2(M, cen, r, a2, c) := \begin{cases} \begin{bmatrix} Z1\# := Find\_Cords\_Bot\left(M_1, cen, r\right) \\ Z2\# := Find\_Cords\_Bot\left(M_2, cen, r\right) \end{bmatrix} & \text{if } \left(A_1 \geq a2_1\right) \wedge \left(c_1 \geq B_1\right) \\ Ans\# := \begin{bmatrix} Z1\# \\ Z2\# \end{bmatrix} \\ Ans\# := Find\_Cords\_Bot(M, cen, r) & \text{if } \left(A_1 \geq a2_1\right) \wedge \left(c_1 < B_1\right) \\ Ans\# := Find\_Cords\_Bot(M, cen, r) & \text{otherwise} \end{cases}$$

$$\left(A_1 \geq AA_1\right) \wedge \left(CC_1 \geq B_1\right) = 1$$

$$\left(A_1 \geq AA_1\right) \wedge \left(CC_1 < B_1\right) = 0$$

$$\left(A_1 < AA_1\right) \wedge \left(CC_1 \geq B_1\right) = 0$$

Result may be a **Nested Array** or **not depending on the Center Cords** Calls **Program 10**

$$BC\_HOZ := Find\_All\_BC\_Hoz2(TC\_hoz, Center, R, AA, CC) = \begin{bmatrix} \begin{bmatrix} 1 & 0.69 \\ 0.35 & 1 \end{bmatrix} \\ \begin{bmatrix} 17 & 3.36 \\ 18 & 4.4 \\ 19 & 5.68 \\ 20 & 7.35 \\ 20.72 & 9 \end{bmatrix} \end{bmatrix} \text{ m}$$

From ACAD

X = 1.00 Y = 0.69
X = 0.35 Y = 1.00
X = 17.00 Y = 3.36
X = 19.00 Y = 5.68
X = 20.72 Y = 9.00

Program 11 : To Find Mid Cordinates

$$Find\_Mid\_Cords(ZZ\#) := \left| \begin{array}{l} j := [1..(\text{rows}(ZZ\#) - 1)] \\ x\#_j := 0.5 \cdot (ZZ\#_{j,1} + ZZ\#_{(j+1),1}) \\ y\#_j := 0.5 \cdot (ZZ\#_{j,2} + ZZ\#_{(j+1),2}) \\ \text{augment}(x\#, y\#) \end{array} \right.$$

Program 12 : To Find Mid Cordinates between vertical strip lines on Horizontal Surface - Calls Program 9

$$Find\_TMC\_Hoz(M, a2, c) := \left\{ \begin{array}{ll} Z1\# := Find\_Mid\_Cords(M_1) & \text{if } (A_1 \geq a2_1) \wedge (c_1 \geq B_1) \\ Z2\# := Find\_Mid\_Cords(M_2) & \\ Ans\# := \begin{bmatrix} Z1\# \\ Z2\# \end{bmatrix} & (A_1 \geq AA_1) \wedge (CC_1 \geq B_1) = 1 \\ Ans\# := Find\_Mid\_Cords(M) & \text{if } (A_1 \geq a2_1) \wedge (c_1 < B_1) \\ & (A_1 \geq AA_1) \wedge (CC_1 < B_1) = 0 \\ Ans\# := Find\_Mid\_Cords(M) & \text{otherwise} \\ & (A_1 < AA_1) \wedge (CC_1 \geq B_1) = 0 \end{array} \right.$$

TMC_HOZ (Top Mid Cords on Horizontal lines AA / BC)

$$TMC\_HOZ := Find\_TMC\_Hoz(TC\_hoz, AA, CC)$$

$$TMC\_HOZ = \begin{bmatrix} \begin{bmatrix} 0.68 & 1 \end{bmatrix} \\ \begin{bmatrix} 17.5 & 9 \end{bmatrix} \\ \begin{bmatrix} 18.5 & 9 \end{bmatrix} \\ \begin{bmatrix} 19.5 & 9 \end{bmatrix} \\ \begin{bmatrix} 20.36 & 9 \end{bmatrix} \end{bmatrix} \text{ m}$$

From ACAD

X = 0.68 Y = 1.00
X = 17.50 Y = 9.00
X = 19.50 Y = 9.00
X = 20.36 Y = 9.00

BMC_HOZ (Bottom Mid Cords on Horizontal lines AA / BC))

$$BMC\_HOZ := Find\_All\_BC\_Hoz2(TMC\_HOZ, Center, R, AA, CC)$$

$$BMC\_HOZ = \begin{bmatrix} \begin{bmatrix} 0.68 & 0.84 \end{bmatrix} \\ \begin{bmatrix} 17.5 & 3.86 \end{bmatrix} \\ \begin{bmatrix} 18.5 & 5.01 \end{bmatrix} \\ \begin{bmatrix} 19.5 & 6.46 \end{bmatrix} \\ \begin{bmatrix} 20.36 & 8.11 \end{bmatrix} \end{bmatrix} \text{ m}$$

From ACAD

X = 0.68 Y = 0.84
X = 17.50 Y = 3.86
X = 18.50 Y = 5.01
X = 19.50 Y = 6.46
X = 20.36 Y = 8.11

TMC_AB (Top Mid Cords on Slope AB)

$$TMC\_AB := Find\_Mid\_Cords(TC\_AB)$$

$$TMC\_AB = \begin{bmatrix} 1.5 & 1.25 \\ 2.5 & 1.75 \\ 3.5 & 2.25 \\ 4.5 & 2.75 \\ 5.5 & 3.25 \\ 6.5 & 3.75 \\ 7.5 & 4.25 \\ 8.5 & 4.75 \\ 9.5 & 5.25 \\ 10.5 & 5.75 \\ 11.5 & 6.25 \\ 12.5 & 6.75 \\ 13.5 & 7.25 \\ 14.5 & 7.75 \\ 15.5 & 8.25 \\ 16.5 & 8.75 \end{bmatrix} \text{ m}$$

From ACAD

X = 1.50 Y = 1.25
X = 3.50 Y = 2.25
X = 5.50 Y = 3.25
X = 12.50 Y = 6.75
X = 14.50 Y = 7.75
X = 16.50 Y = 8.75

BMC_AB (Bottom Mid Cords on Slope AB)

$$BMC\_AB := Find\_Cords\_Bot(TMC\_AB, Center, R)$$

$$BMC\_AB = \begin{bmatrix} 1.5 & 0.48 \\ 2.5 & 0.11 \\ 3.5 & -0.17 \\ 4.5 & -0.38 \\ 5.5 & -0.52 \\ 6.5 & -0.59 \\ 7.5 & -0.59 \\ 8.5 & -0.52 \\ 9.5 & -0.38 \\ 10.5 & -0.17 \\ 11.5 & 0.11 \\ 12.5 & 0.48 \\ 13.5 & 0.93 \\ 14.5 & 1.47 \\ 15.5 & 2.13 \\ 16.5 & 2.91 \end{bmatrix} \text{ m}$$

From ACAD

X = 1.50 Y = 0.48
X = 3.50 Y = -0.17
X = 5.50 Y = -0.52
X = 12.50 Y = 0.48
X = 14.50 Y = 1.47
X = 16.50 Y = 2.91

$$t_0 - \text{time}(0) = -0.22 \text{ s}$$