⊞ Draghilev method, n=2

⊟ Example: Ellipse

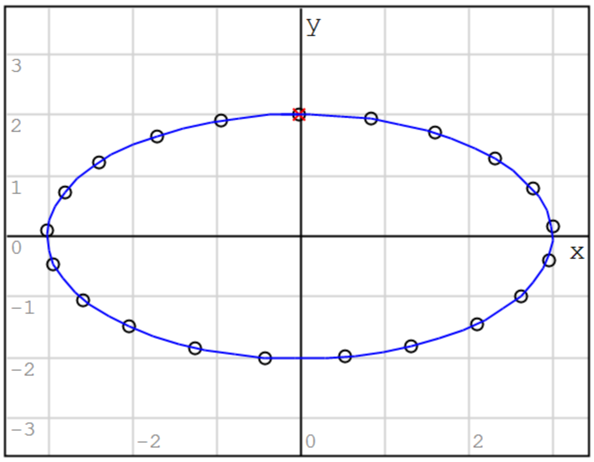
Parametrization of implicit functions by Draghilev method

$f(x, y) := 4 \cdot x^2 + 9 \cdot y^2 - 36$ $\quad ic := [0\ 2]$ $\quad u := [x\ y]$ $\quad T := [0, 0.05 .. 1]$

$U := DM_2(f(x\#, y\#), ic, 0.01, 54)$ Numerical Draghilev method

$[ode\ var] := DM_2(f(x, y), ic, u)$ Symbolic Draghilev method

$[x_s(t)\ y_s(t)] := [-3 \cdot \sin(12 \cdot t)\ 2 \cdot \cos(12 \cdot t)]$ Symbolic solution



$$\begin{cases} U \\ \text{augment}(ic, \text{"x"}, 12, \text{"red"}) \\ \text{augment}\left(\overrightarrow{x_s(T)}, \overrightarrow{y_s(T)}, \text{"o"}\right) \end{cases}$$

Ode for solve

$$ode = \begin{bmatrix} \frac{d}{dt} x(t) = -18 \cdot y(t) \\ \frac{d}{dt} y(t) = 8 \cdot x(t) \\ x(0) = 0 \\ y(0) = 2 \end{bmatrix} \qquad \begin{bmatrix} x' = -\frac{d}{dy} f \\ y' = \frac{d}{dx} f \end{bmatrix}$$

Symbolic solution

$$\begin{cases} x_s(t) \\ y_s(t) \end{cases} = \begin{cases} -3 \cdot \sin(12 \cdot t) \\ 2 \cdot \cos(12 \cdot t) \end{cases}$$

$$\begin{cases} x'_s(t) := \frac{d}{dt} x_s(t) \\ y'_s(t) := \frac{d}{dt} y_s(t) \end{cases}$$

$$\sqrt{x'_s(t)^2 + y'_s(t)^2} = \sqrt{144} \cdot \sqrt{9 \cdot \cos(12 \cdot t)^2 + 4 \cdot \sin(12 \cdot t)^2}$$

⊟ Example: Conic

Parametrization of implicit functions by Draghilev method

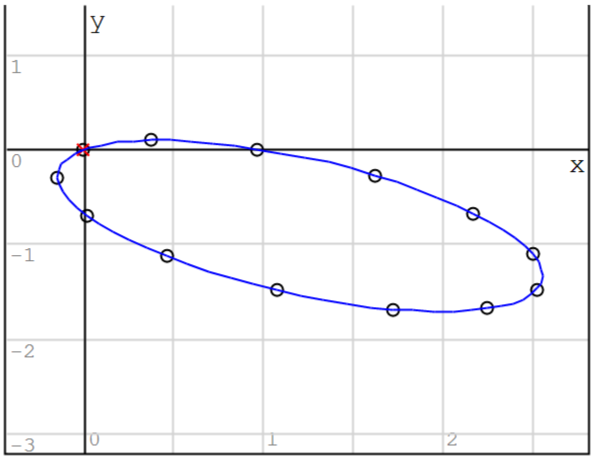
$f(x, y) := 4 \cdot x^2 + 9 \cdot y^2 + 7 \cdot x \cdot y - 4 \cdot x + 6 \cdot y$ $\quad ic := [0\ 0]$ $\quad u := [x\ y]$

$U := DM_2(f(x\#, y\#), ic, 0.01, 66)$ Numerical Draghilev method

$[ode\ var] := DM_2(f(x\#, y\#), ic, u)$ Symbolic Draghilev method

$$\begin{bmatrix} x_s(t) \\ y_s(t) \end{bmatrix} := \begin{bmatrix} -\frac{6 \cdot \left(\sqrt{95} \cdot \left(-1 + \cos\left(t \cdot \sqrt{95}\right)\right) + 5 \cdot \sin\left(t \cdot \sqrt{95}\right)\right)}{5 \cdot \sqrt{95}} \\ -\frac{4 \cdot \left(5 \cdot \sin\left(t \cdot \sqrt{95}\right) + \sqrt{95} \cdot \left(1 - \cos\left(t \cdot \sqrt{95}\right)\right)\right)}{5 \cdot \sqrt{95}} \end{bmatrix}$$ Symbolic solution

End point: $t_2 := \text{roots}(y_s(t), t, 0.7) = 0.6446$ $\quad T := [0, 0.05 .. t_2]$



$$\begin{cases} U \\ \text{augment}(ic, \text{"x"}, 12, \text{"red"}) \\ \text{augment}\left(\overrightarrow{x_s(T)}, \overrightarrow{y_s(T)}, \text{"o"}\right) \end{cases}$$

Ode for solve

$$ode = \begin{bmatrix} \frac{d}{d\,t} x(t) = -(7 \cdot x(t) + 6 \cdot (1 + 3 \cdot y(t))) \\ \frac{d}{d\,t} y(t) = 4 \cdot (-1 + x(t) + x(t)) + 7 \cdot y(t) \\ x(0) = 0 \\ y(0) = 0 \end{bmatrix}$$

The symbolic solution for the ode is the parametric equation for the function

$$\begin{cases} x_s(t) \\ y_s(t) \end{cases} = \begin{cases} -\dfrac{6 \cdot \left(\sqrt{95} \cdot \left(-1 + \cos\left(t \cdot \sqrt{95}\right)\right) + 5 \cdot \sin\left(t \cdot \sqrt{95}\right)\right)}{5 \cdot \sqrt{95}} \\ -\dfrac{4 \cdot \left(5 \cdot \sin\left(t \cdot \sqrt{95}\right) + \sqrt{95} \cdot \left(1 - \cos\left(t \cdot \sqrt{95}\right)\right)\right)}{5 \cdot \sqrt{95}} \end{cases}$$

Draghilev application: Curve length and area

$$A := \int_0^{t_2} x_s(t) \cdot \left(\frac{d}{d\,t} y_s(t)\right) d\,t = 3.0943$$

$$L := \int_0^{t_2} \sqrt{\left(\frac{d}{d\,t} x_s(t)\right)^2 + \left(\frac{d}{d\,t} y_s(t)\right)^2}\, d\,t = 6.979$$

area	$\dfrac{48 \sqrt{\frac{1}{95}(243 - 26\sqrt{74})}\,\pi}{65 - 5\sqrt{74}} \approx 3.09428$
circumference	$-\dfrac{16 \sqrt{\frac{57}{13+\sqrt{74}}}\, E\left(\frac{2}{95}(-74 + 13\sqrt{74})\right)}{\sqrt{74} - 13} \approx 6.97902$

$$\begin{cases} x'_s(t) := \frac{d}{d\,t} x_s(t) \\ y'_s(t) := \frac{d}{d\,t} y_s(t) \end{cases}$$

$$Test := \text{maple}\left(simplify\left(\sqrt{\left(x'_s(t)\right)^2 + \left(y'_s(t)\right)^2}, symbolic\right)\right)$$

$$Test = \frac{2 \cdot \sqrt{13 \cdot \left(19 - 14 \cdot \cos\left(t \cdot \sqrt{95}\right)^2\right) - 10 \cdot \sqrt{95} \cdot \sin\left(t \cdot \sqrt{95}\right) \cdot \cos\left(t \cdot \sqrt{95}\right)}}{\sqrt{5}}$$

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