Gamma Function

$$_p := \begin{bmatrix} 57.156235665862923517 & -59.597960355475491248 \\ 14.136097974741747174 & -0.491913816097620199 78 \\ 0.33994649984811888699 \cdot 10^{-4} & 0.46523628927048575665 \cdot 10^{-4} \\ (-0.98374475304879564677) \cdot 10^{-4} & 0.15808870322491248884 \cdot 10^{-3} \\ (-0.21026444172410488319) \cdot 10^{-3} & 0.21743961811521264320 \cdot 10^{-3} \\ (-0.16431810653676389022) \cdot 10^{-3} & 0.84418223983852743293 \cdot 10^{-4} \\ (-0.26190838401581408670) \cdot 10^{-4} & 0.36899182659531622704 \cdot 10^{-5} \end{bmatrix}$$

$$\Gamma(a) := \begin{cases} \text{if } \mathrm{Re}(a) < 0.5 \\ \quad S := \dfrac{\pi}{\sin(\pi \cdot a) \cdot \Gamma(1-a)} \\ \text{else} \\ \quad a_T := \mathrm{eval}(a-1) \\ \quad x := 0.99999999999999709182 \\ \quad g := 4.7421875 \\ \quad \text{for } i \in [1..\mathrm{length}(_p)] \\ \quad\quad x := \mathrm{eval}\left(x + \dfrac{_p{}_i}{a_T + i}\right) \\ \quad t := \mathrm{eval}(a_T + g + 0.5) \\ \quad S := \left(t^{\frac{a_T + 0.5}{1.25}} \cdot \mathbf{e}^{-\frac{t}{1.25}} \cdot \left(\sqrt{2 \cdot \pi} \cdot x\right)^{\frac{1}{1.25}}\right)^{1.25} \\ \text{if } a = \mathrm{trunc}(a) \\ \quad S := (a-1)! \\ \text{else} \\ \quad S \end{cases}$$

Lower Incomplete Gamma Function

$$\gamma(a; x) := x^a \cdot \exp(-x) \cdot \sum_{k=0}^{50} \frac{x^k}{\prod_{j=0}^{k} (a+j)}$$

Upper Incomplete Gamma Function

$$\Gamma(a; x) := \Gamma(a) - \gamma(a; x)$$

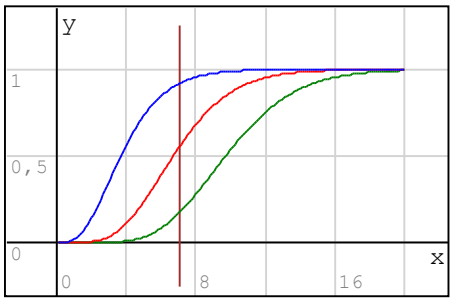
Example_1: Multistage XFR

$T := 1$ $\quad s := 1$

$$\lambda(x) := \begin{cases} 1 & \text{if } (0 \le x) \wedge (x \le 20) \\ "" & \text{otherwise} \end{cases} \qquad time := 7$$

$$f(n; t) := \frac{\Gamma(n+1) - \Gamma\left(n+1; \frac{t}{T}\right)}{s \cdot \Gamma(n+1)}$$

Laplace XFR multistage system
'n' stages, 'T' time constant



$$\begin{bmatrix} f(3; time) \\ f(6; time) \\ f(9; time) \end{bmatrix} = \begin{bmatrix} 0.92 \\ 0.55 \\ 0.17 \end{bmatrix}$$

Example_2: Fitting a data set.

$$XY := \begin{bmatrix} 0 & 127.42 \\ 0.1 & 123.469 \\ 0.2 & 118.008 \\ 0.3 & 111.187 \\ 0.4 & 102.811 \\ 0.5 & 93.554 \\ 0.6 & 84.168 \\ 0.7 & 74.818 \\ 0.8 & 66.373 \\ 0.9 & 58.587 \\ 1 & 51.678 \\ 1.1 & 45.739 \\ 1.2 & 40.675 \\ 1.3 & 36.404 \\ 1.4 & 32.74 \\ 1.5 & 29.581 \\ 1.6 & 26.848 \\ 1.7 & 24.472 \\ 1.8 & 22.395 \\ 1.9 & 20.573 \\ 2 & 18.97 \\ 2.1 & 17.555 \\ 2.2 & 16.3 \\ 2.3 & 15.184 \\ 2.4 & 14.188 \\ 2.5 & 13.295 \\ 2.6 & 12.492 \\ 2.7 & 11.768 \\ 2.8 & 11.12 \\ 2.9 & 10.547 \\ 3 & 10.028 \end{bmatrix}$$

Fitting a data set Γ(a,x) ... data Mathsoft

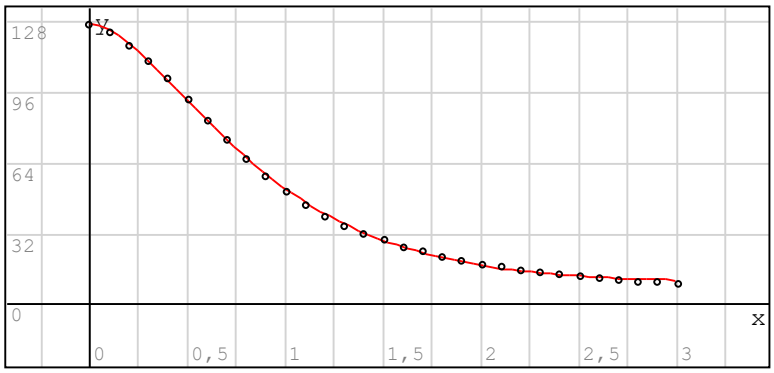
$$plot(data; char; size; clr) := \left| \begin{array}{l} \text{for } k \in [1..\text{rows}(data)] \\ \quad \left[r3_k := char \;\; r4_k := size \;\; r5_k := clr \right] \\ \text{augment}(data; r3; r4; r5) \end{array} \right.$$

$$pts := plot(XY; "o"; 5; "black")$$

$$[yo := 10.028 \;\; a := 143.017 \;\; b := 1.7666 \;\; n := 2.486] \qquad \varepsilon := 10^{-3}$$

$$f(x) := yo + a \cdot \Gamma(b; x)^{n}$$

$$u := [\varepsilon; 0.05..3] \qquad \Omega := \text{augment}\left(u; \overrightarrow{f(u)}\right)$$



$$\begin{cases} pts \\ \Omega \end{cases}$$

$$[yo := 10.028 \;\; a := 143.01681 \;\; b := 1.766652 \;\; n := 2.48644]$$