The following three figures give the equations for the fission yield calculations.

Figure 1:

$$\frac{dE}{dt} = \left(\frac{N_o V_{core} E_f}{\tau}\right) e^{(\alpha/\tau)t} \qquad \text{(B.22)}$$

This was easily integrated analytically in the analytic approximation because α and τ were assumed to be constant. Here, they are time-varying, so this must be integrated numerically.

Again, from Section 7.3, we relate the kinetic energy of the expanding core to the pressure as follows. Recall that $P(t) = \gamma U(t)$ and $U(t) = E(t)/V$

$$P(t) = \frac{\gamma E(t)}{V(t)_{core}}$$

As before in Section 7.3, the thermodynamic work energy theorem provides.

$$P(t)\frac{dV_{core}}{dt} = \frac{dK_{\text{core}}}{dt}$$

$$K_{\text{core}} = \frac{1}{2}Mv^2$$

$$\frac{dK_{core}}{dt} = \frac{1}{2}M2\text{v}\frac{dv}{dt}$$

To approximate the retarding effect of the tamper, we take $M_{total} = M_{core} + M_{tamper}$. So, this results in the following.

$$\frac{\gamma E(t)}{V(t)_{\text{core}}}\left(\frac{dV_{\text{core}}}{dt}\right) = \frac{dK_{\text{total}}}{dt}$$

$$\Rightarrow \frac{\gamma E(t)}{V(t)_{\text{core}}}\left(4\pi r^2\frac{dr}{dt}\right) = \frac{1}{2}M_{\text{total}}\left(2v\frac{dv}{dt}\right)$$

And, noting that $dr/dt = v$, we solve for the differential equation for v versus time t.

$$\frac{dv(t)}{dt} = \left(\frac{4\pi r^2 \gamma E(t)}{V(t)_{\text{core}}\, M_{\text{total}}}\right) \qquad \text{(B.24)}$$

The time between fissions is also a time-varying quantity because of the expansion of the core which in turn increases $\lambda_f$ in the following definition of τ.

$$\tau(t) = \frac{\lambda_f(t)}{V_{\text{neut}}} \qquad \text{(B.25)}$$

Where $V_{neut}$ is the velocity of the fission neutrons and is not to be confused with the velocity v(t) of the expanding core. The velocity of the fission neutrons is constant in time; for an assumed energy of the fission neutrons of 2MeV, we have:

$$V_{neut} = \sqrt{\frac{2(2MeV)}{Neutron\ Mass}} = 1.955x10^9\frac{cm}{sec}$$

Figure 2:

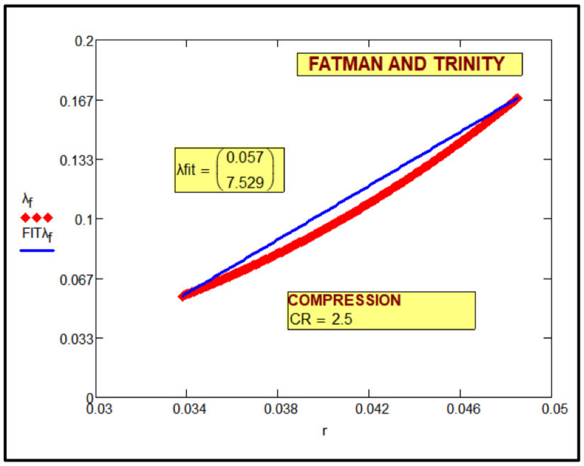


Figure 21. Fat Man Fission Mean-Free-Path vs. Expanding Radius

As with α(t), $\lambda_f(t)$ is included in the time evolution of yield through an expanding core radius r(t). It turns out that $\lambda_f(r)$ is also close to a linear relationship with an expanding r and can be approximated with a linear fit.

From the above, we have a set of coupled differential equations that can be solved by the matrix methods of implicit finite difference as shown in (B.26).

$\alpha_1$ and $\alpha_0$ are the coefficients of the linear fit to α(r). $\lambda_1$ and $\lambda_0$ are the coefficients of the linear fit to $\lambda_f(r)$.

Notice that the core's expanding radius is determined by the expansion velocity defined in (B.24).

$$\begin{bmatrix} r_j \\ \alpha_j \\ \tau_j \\ v_j \\ V_{core_j} \\ E_j \end{bmatrix} = \begin{bmatrix} r_{j-1} + v_{j-1}\Delta t \\ \alpha_1\left(\frac{r_{j-1}-r_0}{cm}\right) + \alpha_0 \\ \frac{\lambda_1(r_{j-1}-r_0)+\lambda_0\cdot m}{V_{neut}} \\ v_{j-1} + \frac{4\pi r_{j-1}^2 \gamma E_{j-1}}{V_{core_{j-1}} M_{total}}\Delta t \\ \frac{4}{3}\pi\left(r_{j-1}\right)^3 \\ E_{j-1} + \left(\frac{N_0 V_{core_{j-1}}}{\tau_{j-1}} E_f\right) e^{\left(\frac{\alpha_{j-1}}{\tau_{j-1}}\right)t_{j-1}} \end{bmatrix} \quad \text{(B.26)}$$

By plotting $E_j$ versus $t_j$, we have $E_j \Longrightarrow E(t)$, the evolution of the device yield versus time. $E(t)$ plots for the Uranium bomb and the Plutonium bomb are provided in Section 3.2

**NOTE:** A word of caution about the confidence in the accuracy of these yield calculations. Calculation of yield is notoriously difficult. This is especially true for these approximate models. These models are extremely sensitive to the input parameters. For example, changing the initial number for neutron density from $1/cm^3$ to $10/cm^3$ increases the predicted yield for Little Boy from 14 KT to 14.7 KT. Also, the choice for Δt in B.26 has a strong impact on the resulting yield. The time step is chosen based on the following logic. It is chosen to be significantly smaller than the rate of increase of neutrons in the core. Recall that the number density of neutrons in the core versus time is given by $N(t) = N_o e^{(\alpha/\tau)t}$, where $N_o$ is the initial neutron density.

$$\frac{dN(t)}{dt} = N_o\left(\frac{\alpha}{\tau}\right)e^{\left(\frac{\alpha}{\tau}\right)t}$$

$$\frac{\frac{dN(t)}{dt}}{N(t)} = \left(\frac{\alpha}{\tau}\right)$$

Figure 3:



$$\frac{\Delta N(t)}{N(t)} = \left(\frac{\alpha}{\tau}\right)\Delta t$$

$$\Delta t = \left(\frac{\tau}{\alpha}\right)\frac{\Delta N(t)}{N(t)}$$

Therefore, we want the change in the neutron density over Δt to be much smaller than the neutron density for our time step Δt to resolve the change in neutron density. We have arbitrarily chosen the following ratio to accomplish this.

$$\frac{\Delta N(t)}{N(t)} \le \frac{1}{20}$$

The time-dependent yield calculation is sensitive to this value. For example, 1/100 for the ratio changes the Little Boy yield from 14 KT to 13 KT. Changing the ratio to 1/10 increases the yield to 15.4 KT.

NOTE: This is where MATHCAD's speed is superior. In the subroutine for the neutron, diffusion equation solution, I have used only 50 times steps. In MATHCAD, I run 200 steps and that reduces the yield to about 18 kt instead of 20. To use 200 times steps in SMATH the runtime is excessive; however, if you are patient it will give the same answer.

The next figure gives the coupled transmission line equations for the EMP calculations.

[4] MATHCAD was used to solve the coupled, non-linear transmission line equation given by

$$\begin{aligned} \frac{\partial V(z,t)}{\partial z} + \frac{\partial}{\partial t}\big(L(z,t)I(z,t)\big) + R(z,t)I(z,t) &= E(z,t) \\ \frac{\partial I(z,t)}{\partial z} + \frac{\partial}{\partial t}\big(C(z,t)V(z,t)\big) + G(z,t)V(z,t) &= 0 \end{aligned}$$

Where V(z,t) = induced voltage on the cable, I(z,t) = induced current on the cable, L(z,t) = cable inductance per unit length, R(z,t) = cable resistance per unit length, C(z,t) = cable capacitance per unit length, and G(z,t) = cable conductance per unit length. E(z,t) = the horizontal driving E-field along the cable.

This figure gives the equations for the SIR epidemiology model.

**[4]** The contagion models used herein are standard SIR models that solve three coupled non-linear differential equations for three variables: the cumulative number of infected cases, I(t), the remaining susceptible population, S(t), and the number removed, R(t), by either recovery or death. The mathematical solutions for the differential equations are obtained from the PC software application, MATHCAD.

$$\frac{d}{dt}S(t) = -\frac{\beta I(t)\,S(t)}{N}$$

$$\frac{d}{dt}I(t) = \frac{\beta I(t)\,S(t)}{N} - \gamma I(t)$$

$$\frac{d}{dt}R(t) = \gamma I(t)$$

$$N = S(t) + I(t) + R(t) = \textbf{Total Population}$$

$$R_o = \frac{\beta}{\gamma}$$

N is the total population, β is the expected number of people an infected person infects per unit time. It is given by the average number of contacts per unit time of the infected person with uninfected people multiplied by the probability to infect a healthy person. The fraction of susceptible people left to infect is S/N, which is the probability that the contacts are with susceptible people left to infect. The probability of an infected individual recovering in any time interval dt is simply γdt. If an individual is infectious for an average time period D, then γ=1/D. The recovery time for COVID-19 is estimated to be 14 days; therefore, γ = 1/14. The Basic Reproduction Rate $R_0$= β/γ. $R_0$ has been adjusted in the model to give the best fit to the cumulative case data. The model fit has been broken into multiple sections of the data because the case rates have varied significantly over the course of the pandemic. This is observed by the different slopes of the case curve. Each fit section of the model has its separate $R_0$ to best fit the data in that section.