Partial fractions decomposition

PartialFrac

$[K\ F] = ParFrac(f, x)$ Returns a vector K and a matrix F with the partial fractions decomposition of a quotient of polynomials f in the variable x. The rational function f could be restored with pf(f,x)

$$f = \frac{a_0 + a_1 \cdot x + a_2 \cdot x^2 + \ldots}{b_0 + b_1 \cdot x + b_2 \cdot x^2 + \ldots} \qquad ParFrac(f, x) = \left[\begin{bmatrix} K_0 \\ K_1 \\ \ldots \end{bmatrix} \begin{bmatrix} A_1 & r_1 & q_1 & o_1 \\ A_2 & r_2 & q_2 & o_2 \\ \ldots & & & \end{bmatrix} \right]$$

Also

$$pf(f, x) = K_0 + K_1 \cdot x + K_2 \cdot x^2 + \ldots + \frac{A_1}{(x - r_1)^{q_1}} + \frac{A_2}{(x - r_2)^{q_2}} + \ldots$$

$$IParFrac(f, x, x_1, x_2) = \int_{x_1}^{x_2} f\, dx$$

References https://www.mathworks.com/help/matlab/ref/residue.html

https://lpsa.swarthmore.edu/BackGround/PartialFraction/PartialFraction.html

Examples

$$f := \frac{6 \cdot s^2 + 3 \cdot s + 2}{2 \cdot s^2 + 14 \cdot s + 20} \qquad ParFrac(f, s) = \left[\begin{bmatrix} 3 \end{bmatrix} \begin{bmatrix} 3.3333 & -2 & 1 & 1 \\ -22.8333 & -5 & 1 & 1 \end{bmatrix} \right]$$

$$pf(f, s) = 3 \cdot s^0 + \frac{3.333333333333}{(s - (-2))^1} + \frac{-22.833333333333}{(s - (-5))^1}$$

$$IParFrac(f, s, 2, 3) = 0.694845039 \qquad \int_2^3 f\, ds = 0.694845039454177$$

$$f := \frac{-4 \cdot s + 8}{s^2 + 6 \cdot s + 8} \qquad ParFrac(f, s) = \left[\begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} 8 & -2 & 1 & 1 \\ -12 & -4 & 1 & 1 \end{bmatrix} \right]$$

$$pf(f, s) = 0 \cdot s^0 + \frac{8}{(s - (-2))^1} + \frac{-12}{(s - (-4))^1}$$

$$IParFrac(f, s, 2, 3) = -0.064659747 \qquad \int_2^3 f\, ds = -0.064659747413126$$

$$f := \frac{2 \cdot s^4 + s}{s^2 + 1} \qquad ParFrac(f, s) = \left[\begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 0.5 - i & i & 1 & 1 \\ 0.5 + i & -i & 1 & 1 \end{bmatrix} \right]$$

$$pf(f, s) = (-2) \cdot s^0 + 0 \cdot s^1 + 2 \cdot s^2 + \frac{0.5 - i}{(s - i)^1} + \frac{0.5 + i}{(s - (-i))^1}$$

$$IParFrac(f, s, 2, 3) = 11.297034366$$

$$\int_2^3 f\, ds = 11.2970343661559$$

$$f := \frac{2 \cdot s^3 + s^2}{s^3 + s + 1}$$

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> f:=(2*s^3+s^2)/(s^3+s+1);
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$$f := \frac{2 s^3 + s^2}{s^3 + s + 1}$$

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> convert(f,parfrac,s,complex);
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$$2 + \frac{0.5354179054 - 1.038991709 I}{s - 0.3411639019 + 1.161541400 I} - \frac{0.07083581160}{s + 0.6823278038} + \frac{0.5354179062 + 1.038991709 I}{s - 0.3411639019 - 1.161541400 I}$$

$$ParFrac(f, s) = \begin{bmatrix} [2] & \begin{bmatrix} -0.070835812033 & -0.682327804 & 1 & 1 \\ 0.535417905934 + 1.038991708586 \cdot i & 0.341163902 + 1.1615414 \cdot i & 1 & 1 \\ 0.535417905973 - 1.038991708595 \cdot i & 0.341163902 - 1.1615414 \cdot i & 1 & 1 \end{bmatrix} \end{bmatrix}$$

IParFrac returns the values of this integral

$$F := \left| \begin{array}{l} \omega := PolyRoots([1\ 1\ 0\ 1]) \\ 2 \cdot s + \sum_{k=1}^{3} \left(\frac{\left(\omega_k^2 - 2 \cdot \omega_k - 2\right) \cdot \ln\left(s - \omega_k\right)}{3 \cdot \omega_k^2 + 1} \right) \end{array} \right.$$

Indefinite integral

$$\int \frac{2 s^3 + s^2}{s^3 + s + 1} ds = \sum_{\{\omega : \omega^3 + \omega + 1 = 0\}} \frac{\omega^2 \log(s - \omega) - 2 \omega \log(s - \omega) - 2 \log(s - \omega)}{3 \omega^2 + 1} + 2 s$$

$$IParFrac(f, s, 2, 3) = 1.94913378$$

$$\int_2^3 f\, ds = 1.94913378036444$$

$$F\Big|_2^3 = 1.94913378033686$$

$$f := \frac{s + 3}{s^3 + 7 \cdot s^2 + 10 \cdot s}$$

$$ParFrac(f, s) = \begin{bmatrix} [0] & \begin{bmatrix} 0.3 & 0 & 1 & 1 \\ -0.1667 & -2 & 1 & 1 \\ -0.1333 & -5 & 1 & 1 \end{bmatrix} \end{bmatrix}$$

$$pf(f, s) = 0 \cdot s^0 + \frac{0.3}{(s - 0)^1} + \frac{-0.166666666667}{(s - (-2))^1} + \frac{-0.133333333333}{(s - (-5))^1}$$

$$f := \frac{s + 3}{(s + 5) \cdot \left(s^2 + 4 \cdot s + 5\right)}$$

$$ParFrac(f, s) = \begin{bmatrix} [0] & \begin{bmatrix} 0.1 - 0.2 \cdot i & -2 + i & 1 & 1 \\ 0.1 + 0.2 \cdot i & -2 - i & 1 & 1 \\ -0.2 & -5 & 1 & 1 \end{bmatrix} \end{bmatrix}$$

$$pf(f, s) = 0 \cdot s^0 + \frac{0.1 - 0.2 \cdot i}{(s - (-2 + i))^1} + \frac{0.1 + 0.2 \cdot i}{(s - (-2 - i))^1} + \frac{-0.2}{(s - (-5))^1}$$

Usually, for the cases of repeated poles, we need to increse 'TOL $\quad TOL = 1 \cdot 10^{-10}$

$$y := \frac{x + 3}{x \cdot (x + 2)^2 \cdot (x + 5)}$$

$$ParFrac(y, x) = \begin{bmatrix} [0] & \begin{bmatrix} 0.15 & 0 & 1 & 1 \\ -8.6805565 \cdot 10^5 & -1.9999999 & 1 & 1 \\ 8.6805545 \cdot 10^5 & -2.0000001 & 1 & 1 \\ 0.0444444 & -5 & 1 & 1 \end{bmatrix} \end{bmatrix}$$

$TOL := 10^{-7}$

$$ParFrac(y, x) = \begin{bmatrix} [0] & \begin{bmatrix} 0.15 & 0 & 1 & 1 \\ -0.1944 & -2 & 1 & 2 \\ -0.1667 & -2 & 2 & 2 \\ 0.0444 & -5 & 1 & 1 \end{bmatrix} \end{bmatrix}$$

Tip: increase 'TOL until ParFrac recognize repeated poles.

$$pf(y, x) = 0 \cdot x^0 + \frac{0.15}{(x-0)^1} + \frac{-0.194444444}{(x-(-2))^1} + \frac{-0.166666667}{(x-(-2))^2} + \frac{0.044444444}{(x-(-5))^1}$$

$IParFrac(y, x, 2, 3) = 0.015032$

$$\int_2^3 y \, dx = 0.015032137577468$$

$TOL := 10^{-4}$

$$y := \frac{0.1 \cdot x^8 - 2 \cdot x + 3}{2 \cdot x^2 \cdot (x-4)^3 \cdot (x-6)^2}$$

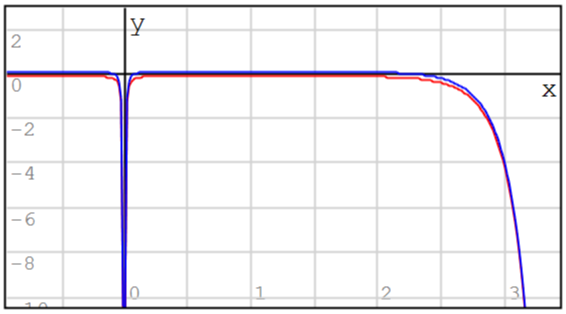
$$ParFrac(y, x) = \begin{bmatrix} \begin{bmatrix} 1.2 \\ 0.05 \end{bmatrix} & \begin{bmatrix} -0.000271 & 0 & 1 & 2 \\ -0.000651 & 0 & 2 & 2 \\ 163.175098 & 4 & 1 & 3 \\ 127.964844 & 4 & 2 & 3 \\ 51.160939 & 4 & 3 & 3 \\ -145.774826 & 6 & 1 & 2 \\ 291.584375 & 6 & 2 & 2 \end{bmatrix} \end{bmatrix}$$

$yp := pf(y, x)$

$$yp = \frac{6}{5} \cdot x^0 + \frac{1}{20} \cdot x^1 + \frac{-\frac{271}{1000000}}{(x-0)^1} + \frac{-\frac{651}{1000000}}{(x-0)^2} + \frac{\frac{81587549}{500000}}{(x-4)^1} + \frac{\frac{31991211}{250000}}{(x-4)^2} + \frac{\frac{51160939}{1000000}}{(x-4)^3} + \frac{-\frac{72887413}{500000}}{(x-6)^1} + \frac{\frac{93307}{320}}{(x-6)^2}$$

$IParFrac(y, x, 2, 3) = -0.747$

$$\int_2^3 y \, dx = -0.747004981643018$$



$$\begin{cases} y + 0.1 \\ yp - 0.1 \end{cases}$$

Alvaro

appVersion(4) = "1.0.8253.4763"