

Finding Distance of a Point

Plot CreateMesh λ γ

$$\begin{aligned}
 \text{Dist}(C, A, x_o, \varepsilon, N) := & \left[\begin{aligned} & a := x_o^T \quad c := [1..C] \quad g := 0 \quad r := 0 \quad s := x_o \quad NA := \text{eval}(\text{norme}(A - x_o)) \\ & L(x) := \sum_{k = \text{Clear}(k)}^C (x_k - A_k)^2 + x_{C+1} \cdot F(x) \\ & \text{if } C = 2 \\ & \quad \left[\begin{aligned} & F(x) := f(x_1, x_2) \\ & \Phi(x) := \left[\frac{d}{dx_1} L(x) \quad \frac{d}{dx_2} L(x) \quad \frac{d}{dx_3} L(x) \right]^T \end{aligned} \right. \\ & \text{else} \\ & \quad \left[\begin{aligned} & F(x) := f(x_1, x_2, x_3) \\ & \Phi(x) := \left[\frac{d}{dx_1} L(x) \quad \frac{d}{dx_2} L(x) \quad \frac{d}{dx_3} L(x) \quad \frac{d}{dx_4} L(x) \right]^T \end{aligned} \right. \\ & \quad \left[\begin{aligned} & \phi(x) := \left[\begin{aligned} & \text{norme}(x - A^T) - NA \cdot \left(1 - \frac{k}{N}\right) \\ & F(x) \end{aligned} \right] \quad g_c := \frac{d}{dx_c} F(x) \quad G(x) := g \end{aligned} \right. \\ & \text{for } k \in [0..N] \\ & \quad \left[\begin{aligned} & a := \text{eval}(\text{al_nleqsolve}(a, \phi(x))) \\ & \text{if } |F(a)| \leq \varepsilon \\ & \quad \left[r := r + 1 \quad s_{rc} := \text{eval}(a_c) \right] \\ & \text{else} \\ & \quad a_c := s_{rc} \end{aligned} \right. \\ & \quad \left[\alpha := s_{rc} \quad \alpha := \text{eval}(\text{al_nleqsolve}(\text{stack}(\alpha^T, 0), \phi(x))) \right] \\ & \quad \left[a := \alpha_c^T \quad G := G(a)^T \quad \delta := \text{norme}(A - a) \right] \\ & \quad \text{eval} \left(\left[a \quad \delta \quad \text{stack} \left(a, a + \frac{\delta}{2} \cdot \frac{G}{\text{norme}(G)} \right) \quad \text{stack}(s, a) \quad \alpha_c \right] \right) \end{aligned} \right]
 \end{aligned}$$

Default iterations and error:

$N := 50$

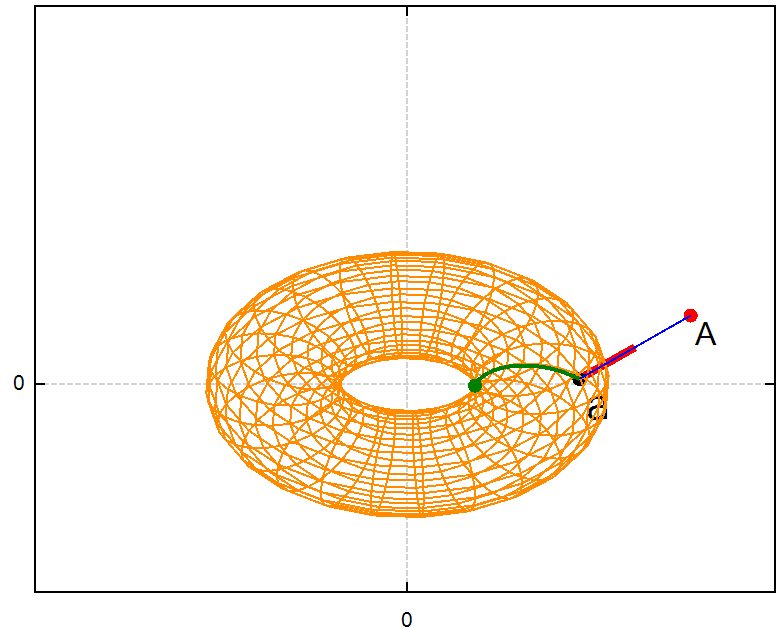
$\varepsilon := 10^{-3}$

 3D
Example 1 $t_o := \text{time}(0)$

```

{ sur := eval (Tore)
  A := rndBox ([ -7 7 ], [ -4 4 ], [ 3 ])
  tmp := (x2 + y2 + z2 + 22 - 12)
  f(x, y, z) := tmp2 - 4 · 22 · (x2 + y2)
  B := Dist (3, A, [ 0 0 A3 ], ε, N)
  [ a δ No s λ ] := B

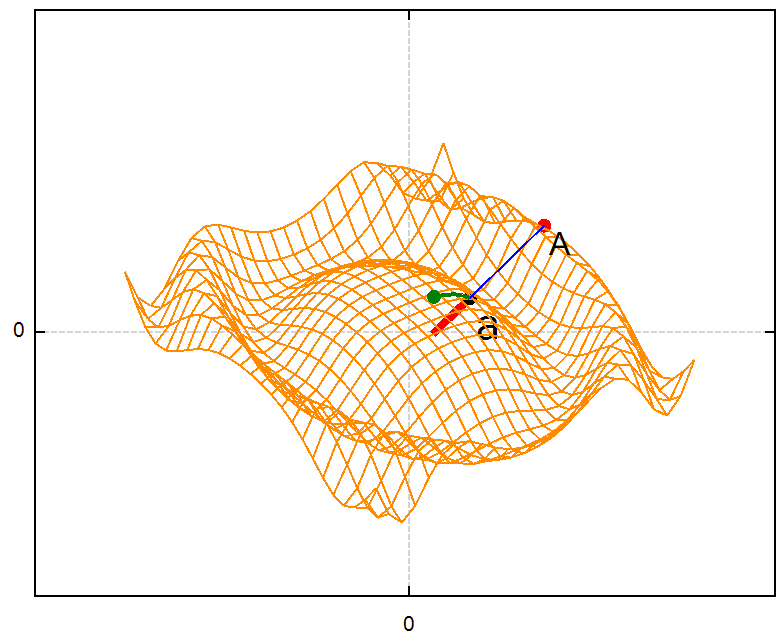
```

 $a = [-1.7849 \ 1.8973 \ 0.7963]$ $A = [-2.9319 \ 3.1165 \ 3]$ $\text{rows}(s) = 20 \quad \delta = 2.7674$ **Example 2**

```

{ sur := eval (fcos)
  A := rndBox ([ -8 8 ], [ -8 8 ], [ 5 ])
  f(x, y, z) := 2 · cos( x2/7 + y2/14 ) - z
  B := Dist (3, A, [ 0 0 A3 ], ε, 2 · N)
  [ a δ No s λ ] := B

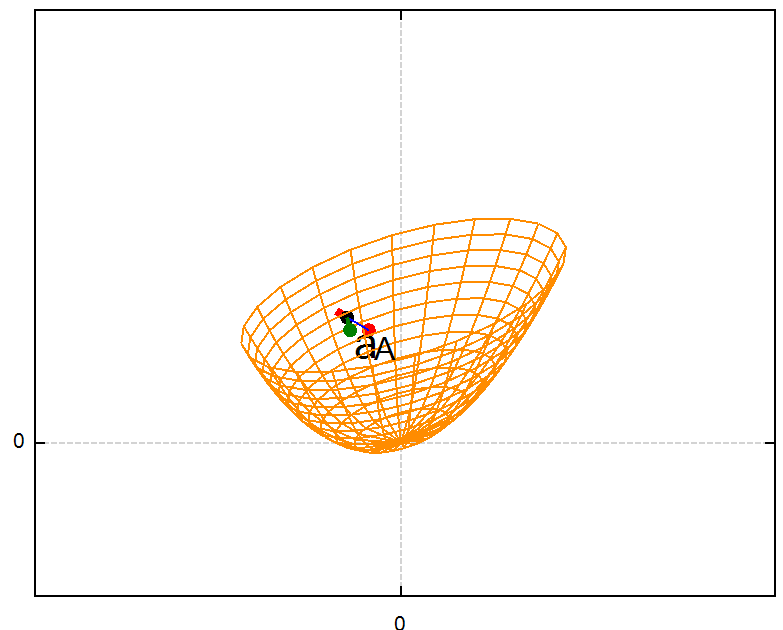
```

 $a = [-2.2126 \ 0.6208 \ 1.4944]$ $A = [-5.1582 \ 1.034 \ 5]$ $\text{rows}(s) = 14 \quad \delta = 4.5974$ **Example 3**

```

{ sur := eval (par)
  A := rndBox ([ -4 4 ], [ -4 4 ], [ 6 ])
  f(x, y, z) := x2/2 + y2/1 - z
  B := Dist (3, A, [ 0 0 A3 ], ε, N)
  [ a δ No s λ ] := B

```

 $a = [0.5176 \ -2.3867 \ 5.7631]$ $A = [0.4563 \ -1.256 \ 6]$ $\text{rows}(s) = 8 \quad \delta = 1.1568$ 

Example 4

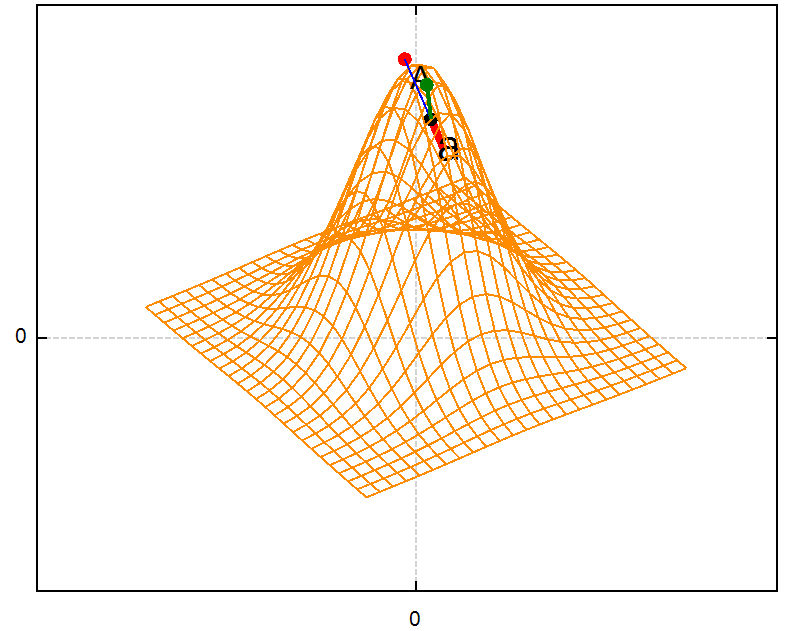
```

sur := eval(fexp)
A := rndBox([-2 2], [-2 2], [6])

$$f(x, y, z) := 8 \cdot \exp\left(-\frac{x^2 + (2 \cdot y)^2}{2}\right) - z$$

B := Dist(3, A, [0 0 A_3], ε, N)
[a δ No s λ] := B

```

 $a = [-0.4435 \ -0.3315 \ 5.8205]$
 $A = [-0.9068 \ -1.7166 \ 6]$
 $\text{rows}(s) = 14 \quad \delta = 1.4716$
 $\text{time}(0) - t_o = 75.038 \text{ s}$


2D Examples

Example 1

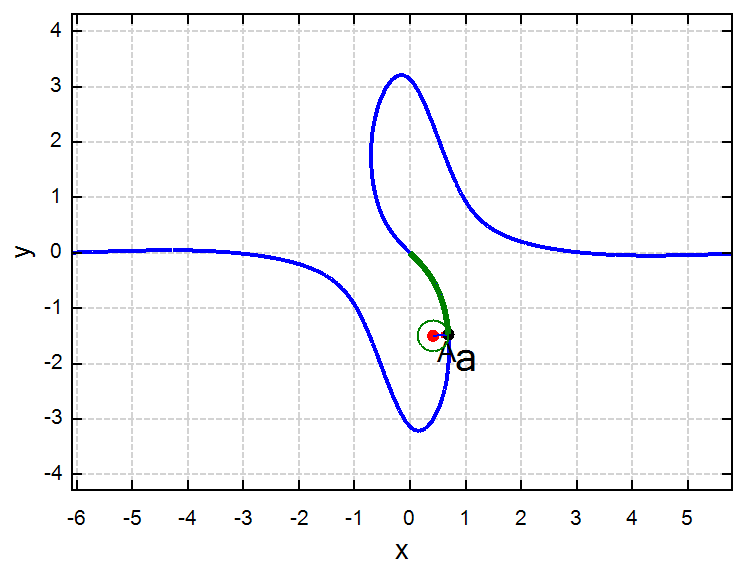
```

A := rndBox([-6 6], [-4 4])

$$f(x, y) := x^2 \cdot y - \sin(x + y)$$

B := Dist(2, A, [0 0], ε, N)
[a δ No s λ] := B

```

 $a = [0.6915 \ -1.4731]$
 $A = [0.4154 \ -1.4963]$
 $\text{rows}(s) = 43 \quad \delta = 0.2771$
 $t_o := \text{time}(0)$
**Example 2**

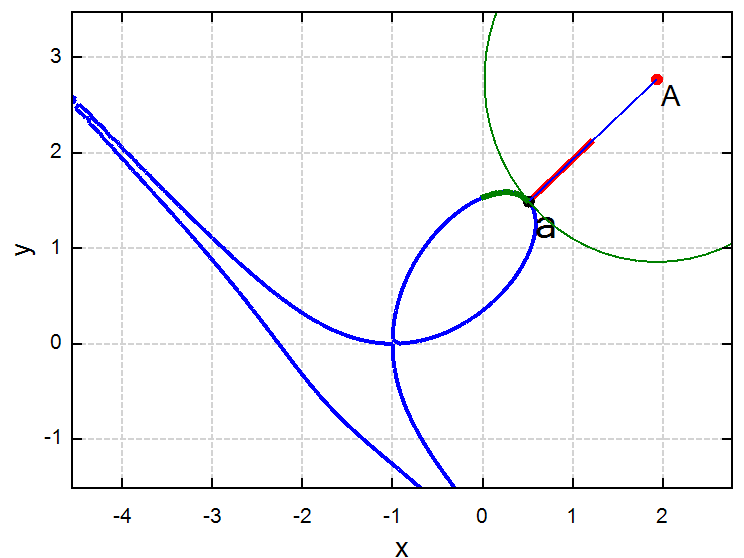
```

A := rndBox([-4 2], [-1 3])

$$f(x, y) := \ln\left|\left|1 + (1+x)^3 - 3 \cdot y \cdot (1+x) + y^3\right|\right|$$

B := Dist(2, A, [0 1.5], ε, N)
[a δ No s λ] := B

```

 $a = [0.5096 \ 1.4898]$
 $A = [1.9313 \ 2.7689]$
 $\text{rows}(s) = 10 \quad \delta = 1.9123$


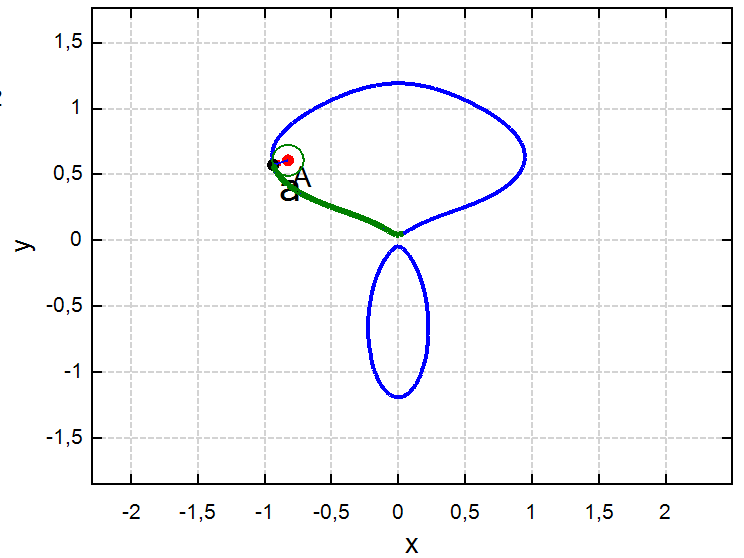
Example 3

$$\left\{ \begin{array}{l} A := \text{rndBox}([[-2, 2], [-1.5, 1.5]]) \\ \alpha o := 0.842 \\ f(x, y) := \frac{1}{(x^2 + y^2 - \alpha o^2)^2 + (2 \cdot x \cdot (y - \alpha o))^2} - 2 \\ B := \text{Dist}(2, A, [0, 0], \varepsilon, N) \\ [a \ \delta \ No \ s \ \lambda] := B \end{array} \right.$$

$$a = [-0.9362 \ 0.5725]$$

$$A = [-0.8248 \ 0.6076]$$

$$\text{rows}(s) = 46 \quad \delta = 0.1168$$

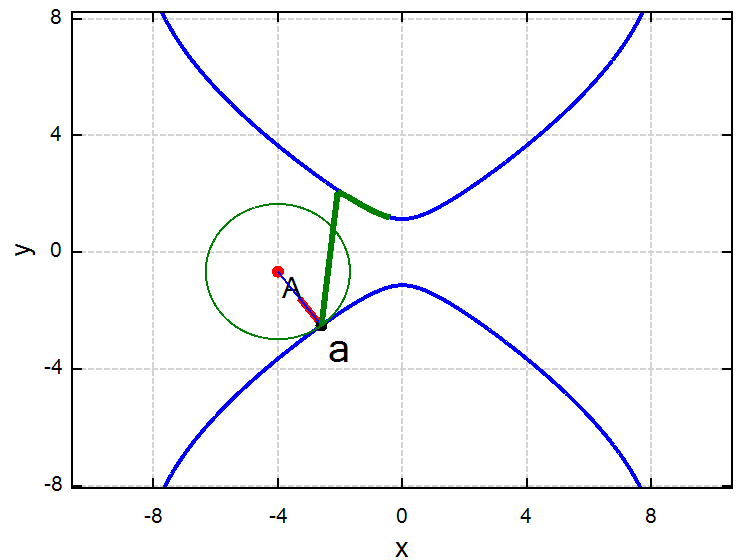
**Example 4**

$$\left\{ \begin{array}{l} A := \text{rndBox}([[-4, -4], [-2, 2]]) \\ \alpha o := \text{eval}(81 \cdot \sqrt{6 \cdot \pi}) \\ f(x, y) := e^{-\frac{1}{3} \cdot \sqrt{x^2 + y^2}} \cdot \frac{x^2 - 2 \cdot y^2}{\alpha o} + 0.005 \\ B := \text{Dist}(2, A, [0, 0], \varepsilon, N) \\ [a \ \delta \ No \ s \ \lambda] := B \end{array} \right.$$

$$a = [-2.5981 \ -2.5114]$$

$$A = [-4 \ -0.6667]$$

$$\text{rows}(s) = 11 \quad \delta = 2.317$$

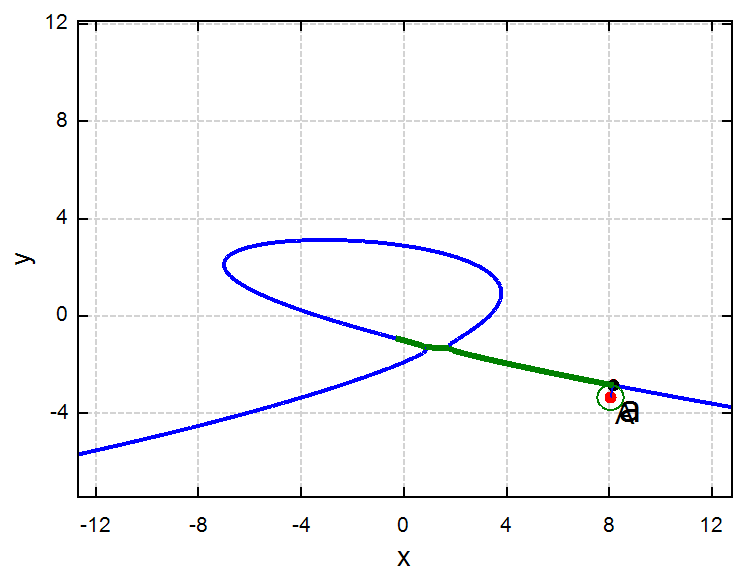
**Example 5**

$$\left\{ \begin{array}{l} A := \text{rndBox}([[-12, 12], [-4, 10]]) \\ f(x, y) := 2 \cdot x^2 + 4 \cdot x \cdot y + 4 \cdot y^3 - 26 \cdot y - 22 \\ B := \text{Dist}(2, A, [0, 0], \varepsilon, N) \\ [a \ \delta \ No \ s \ \lambda] := B \end{array} \right.$$

$$a = [8.1439 \ -2.8434]$$

$$A = [8.0414 \ -3.3442]$$

$$\text{rows}(s) = 45 \quad \delta = 0.5112$$



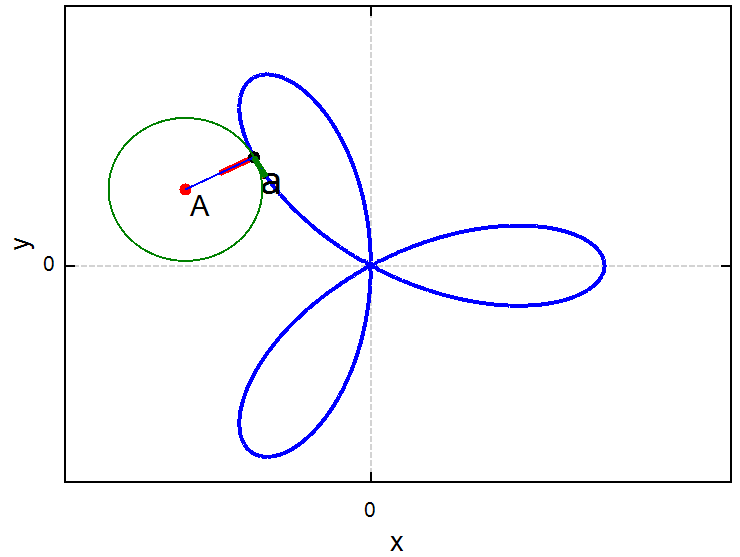
Example 6

$$\begin{cases} A := \text{rndBox}([[-1 \ 1], [-1 \ 1]]) \\ f(x, y) := (x^2 + y^2)^2 - (x^3 - 3 \cdot x \cdot y^2) \\ B := \text{Dist}\left(2, A, \begin{bmatrix} A \\ 1 \end{bmatrix} 0, \varepsilon, 3 \cdot N\right) \\ [a \ \delta \ No \ s \ \lambda] := B \end{cases}$$

$$a = [-0.4991 \ 0.4983]$$

$$A = [-0.7928 \ 0.3505]$$

$$\text{rows}(s) = 12 \quad \delta = 0.3288$$

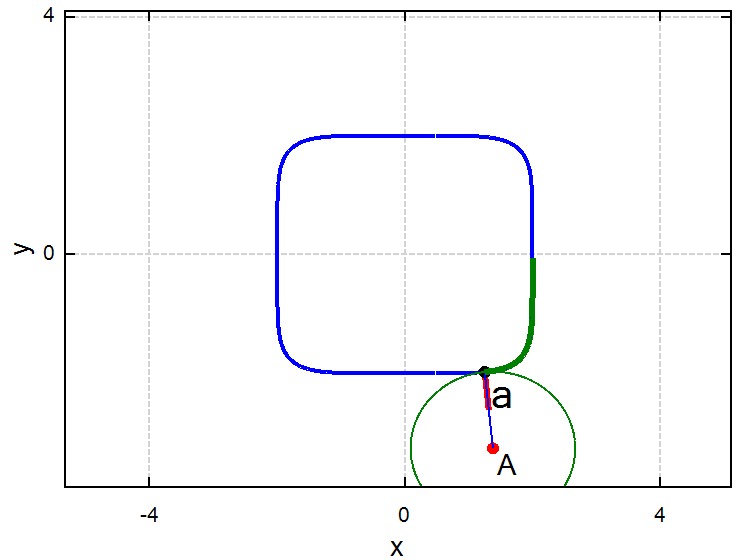
**Example 7**

$$\begin{cases} A := \text{rndBox}([[-4 \ 4], [-4 \ 4]]) \\ f(x, y) := x^6 + y^6 - 2^6 \\ B := \text{Dist}\left(2, A, \begin{bmatrix} A \\ 1 \end{bmatrix} 0, \varepsilon, N\right) \\ [a \ \delta \ No \ s \ \lambda] := B \end{cases}$$

$$a = [1.2502 \ -1.9796]$$

$$A = [1.3792 \ -3.2632]$$

$$\text{rows}(s) = 35 \quad \delta = 1.2901$$

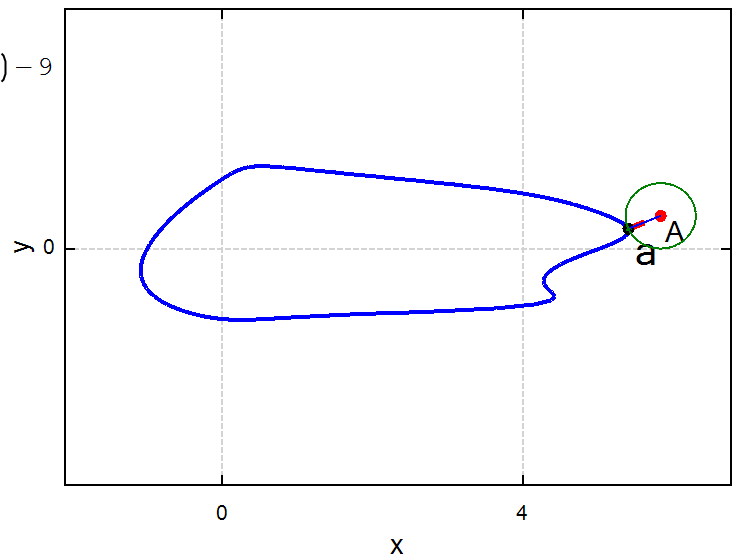
**Example 8**

$$\begin{cases} A := \text{rndBox}([[-1 \ 6], [-2 \ 2]]) \\ f(x, y) := (x-2)^2 + 5 \cdot y^2 + x^2 \cdot y^{10} - 3 \cdot \sin(x \cdot y) - 9 \\ B := \text{Dist}\left(2, A, \begin{bmatrix} A \\ 1 \end{bmatrix} 0, \varepsilon, 3 \cdot N\right) \\ [a \ \delta \ No \ s \ \lambda] := B \end{cases}$$

$$a = [5.4025 \ 0.2909]$$

$$A = [5.8321 \ 0.4757]$$

$$\text{rows}(s) = 6 \quad \delta = 0.4677$$



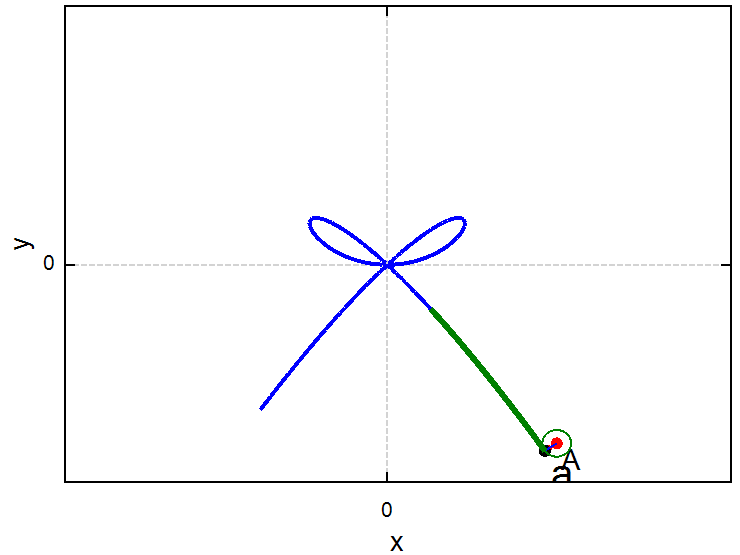
Example 9

$$\begin{cases} A := \text{rndBox}([[-1 \ 1], [-1 \ 1]]) \\ f(x, y) := x^4 - x^2 \cdot y + y^3 \\ B := \text{Dist}\left(2, A, \begin{bmatrix} A_1 & 0 \end{bmatrix}, \varepsilon, N\right) \\ [a \ \delta \ No \ s \ \lambda] := B \end{cases}$$

$$a = [0.7762 \ -0.9853]$$

$$A = [0.8347 \ -0.9455]$$

$$\text{rows}(s) = 48 \quad \delta = 0.0708$$

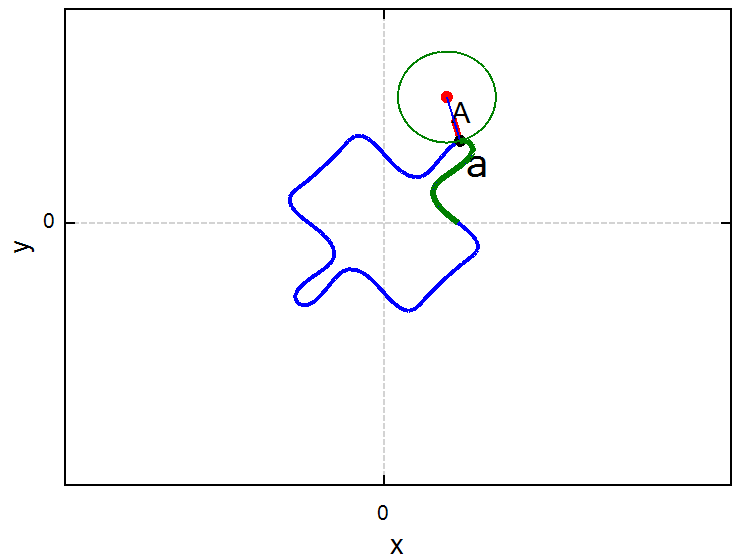
**Example 10**

$$\begin{cases} A := \text{rndBox}([[-1.5 \ 1.5], [-1.5 \ 1.5]]) \\ g(a, b) := (a + 0.125 \cdot \sin(10 \cdot b))^4 \\ f(x, y) := g(x, y) + g(y, x) - 0.4^4 \\ B := \text{Dist}\left(2, A, \begin{bmatrix} A_1 & 0 \end{bmatrix}, \varepsilon, 3 \cdot N\right) \\ [a \ \delta \ No \ s \ \lambda] := B \end{cases}$$

$$a = [0.4088 \ 0.4732]$$

$$A = [0.3374 \ 0.7251]$$

$$\text{rows}(s) = 98 \quad \delta = 0.2618$$



$$\text{time}(0) - t_o = 298.431 \text{ s}$$