⊞ svd nr

⊞ MPinv

⊟ MPInv Examples

Pseudoinverse Summary	**Dim**	**Rank**	**PseudoInverse**
	$A_{n\,n}$	$\mathrm{rank}(A) = n$	$A^* = A^{-1}$
	$A_{m\,n}$	$\mathrm{rank}(A) = m$	$A^* = A^T \cdot (A \cdot A^T)^{-1}$
	$A_{m\,n}$	$\mathrm{rank}(A) = n$	$A^* = (A^T \cdot A)^{-1} \cdot A^T$
	$A_{m\,n}$	$\mathrm{rank}(A) = r$	$A^* = G^* \cdot F^*$ with $A = F \cdot G$ any full rank factorization of A

Examples Setup $TOL := 10^{-12}$

$A := [2]$

$A^* := A^{-1} = [0.5]$ $\qquad MRank(A) = 1$

$G := MPInv(A) = [0.5]$ $\qquad \max\left(\overrightarrow{|A^* - G|}\right) = 0$

$A := [1\ 2]$

$A^* := A^T \cdot (A \cdot A^T)^{-1} = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}$ $\qquad MRank(A) = 1$

$G := MPInv(A) = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}$ $\qquad \max\left(\overrightarrow{|A^* - G|}\right) = 3.4 \cdot 10^{-15}$

$A := \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$A^* := (A^T \cdot A)^{-1} \cdot A^T = [0.2\ 0.4]$ $\qquad MRank(A) = 1$

$G := MPInv(A) = [0.2\ 0.4]$ $\qquad \max\left(\overrightarrow{|A^* - G|}\right) = 3.8 \cdot 10^{-15}$

$A := \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$A^* := A^{-1} = \begin{bmatrix} -2 & 1.5 \\ 1 & -0.5 \end{bmatrix}$ $\qquad MRank(A) = 2$

$G := MPInv(A) = \begin{bmatrix} -2 & 1.5 \\ 1 & -0.5 \end{bmatrix}$ $\qquad \max\left(\overrightarrow{|A^* - G|}\right) = 3.3 \cdot 10^{-14}$

$A := \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$

$A^* := MPInv_{dn}(A) = \begin{bmatrix} 0 & 0 \\ 0.5 & 0 \end{bmatrix}$ $\qquad MRank(A) = 1$

$G := MPInv(A) = \begin{bmatrix} 0 & 0 \\ 0.5 & 0 \end{bmatrix}$ $\qquad \max\left(\overrightarrow{|A^* - G|}\right) = 0$

$A := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$A^* := MPInv_{dn}(A) = \begin{bmatrix} -0.6389 & -0.1667 & 0.3056 \\ -0.0556 & 1.7283 \cdot 10^{-16} & 0.0556 \\ 0.5278 & 0.1667 & -0.1944 \end{bmatrix}$ $\qquad MRank(A) = 2$

$G := MPInv(A) = \begin{bmatrix} -0.6389 & -0.1667 & 0.3056 \\ -0.0556 & 8.2757 \cdot 10^{-17} & 0.0556 \\ 0.5278 & 0.1667 & -0.1944 \end{bmatrix}$ $\qquad \max\left(\overrightarrow{|A^* - G|}\right) = 6 \cdot 10^{-15}$

$$A := \begin{bmatrix} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 4 \\ 7 & 8 & 3 & 7 \\ 6 & 7 & 8 & 6 \\ 4 & 4 & 4 & 4 \end{bmatrix}$$

$$A* := MPInv_{dn}(A) = \begin{bmatrix} -0.3818 & -0.1085 & -0.0833 & 0.0736 & 0.3643 \\ 0.6667 & 0.1667 & 0.3333 & -0.1667 & -0.6667 \\ 0.0039 & 0.062 & -0.1667 & 0.1008 & 0.0775 \\ -0.3818 & -0.1085 & -0.0833 & 0.0736 & 0.3643 \end{bmatrix} \qquad MRank(A) = 3$$

$$G := MPInv(A) = \begin{bmatrix} -0.3818 & -0.1085 & -0.0833 & 0.0736 & 0.3643 \\ 0.6667 & 0.1667 & 0.3333 & -0.1667 & -0.6667 \\ 0.0039 & 0.062 & -0.1667 & 0.1008 & 0.0775 \\ -0.3818 & -0.1085 & -0.0833 & 0.0736 & 0.3643 \end{bmatrix} \qquad \max\left(\overrightarrow{|A* - G|}\right) = 1.8 \cdot 10^{-14}$$

Rank The code could be used for calculate the rank of a matrix.

$n := 28$ $\quad [a \; b] := [100 \; 200]$ $\quad c := n - 10$ $\quad A := b \cdot MRand(n, n) - a$ $\quad P := \mathrm{dn\_LinAlgSVD}(A)_2$

$\lambda := b \cdot MRand(c, 1) - a$ $\quad \lambda_n := 0$

Matrix M is nxn, have rank c
and eigenvalues λ

$$M := P \cdot \mathrm{diag}(\lambda) \cdot P^{\mathrm{T}} \qquad M = \begin{bmatrix} -8.9511 & -3.0757 & \cdots \\ -3.0757 & 9.7087 & \\ 2.0139 & -8.8169 & \\ -7.8875 & 3.167 & \\ & \vdots & \ddots \end{bmatrix}$$

SMath's rank have several bugs.
For instance, can't calculate this

$\mathrm{rank}(M) = \blacksquare$ $\quad \boxed{MRank(M) = 18}$ $\quad \boxed{MRank_{dn}(M) = 18}$

Timing

$[m \; n] := [30 \; 40]$ $\quad A := 200 \cdot MRand(m, n) - 100$ $\quad A* := MPInv_{dn}(A)$

$\mathrm{Id} := \mathrm{eval}(\mathrm{identity}(\min([m \; n])))$

$t_o := \mathrm{time}(0)$ $\quad G := MPInv(A)$ $\quad \boxed{\mathrm{time}(0) - t_o = 50.5 \text{ s}}$ $\quad \max\left(\overrightarrow{|A* - G|}\right) = 2.2 \cdot 10^{-15}$

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Id := if m > n
        G·A
      else
        A·G
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$\max\left(\overrightarrow{|Id - \mathrm{Id}|}\right) = 6.9 \cdot 10^{-13}$

$Ido := \mathrm{eval}\left(\overrightarrow{Id \cdot (Id > \sqrt{\mathrm{TOL}})}\right)$

$\sum Ido = 30$

equal to min(m,n)

$$Ido = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ & & & & & & & \vdots & & & & & & & & \ddots \end{bmatrix}$$

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