⊞— nullspace

⊞— rat

$$TOL := 10^{-9}$$

⊟— Dimensional analysis

$$Dim := \begin{bmatrix} [T] & [L] & [M] & [I] & [\Theta] & [J] & [N] \\ s & m & kg & A & K & cd & mol \end{bmatrix}$$

```
DimAux(u#, k#) := | Clear(dummy#)
                  | ans# := d/(d dummy# := Dim_{2 k#}) UnitsOf(u#)
                  | [ ans#/UnitsOf(ans#) < 0    ans# := (UnitsOf(u#))^{s#} ]
                  | [ s# := (-1)                                          ]
                  | for n# ∈ [1..99]
                  |   if (ans# := d/(d dummy# := Dim_{2 k#}) ans#) = 0
                  |     break
                  |   else
                  |     continue
                  | s#·(n# - 1)
```

```
Dim(x#) := | D# := 1
           | for k# ∈ [1..cols(Dim)]
           |   D# := D#·(Dim_{1 k#})^{DimAux(x#, k#)}
           | D#
```

```
Dim(v#, x#) := | [ D# := 1  K# := 1 ]
               | for λ# ∈ [1..length(x#)]
               |   for k# ∈ [1..cols(Dim)]
               |     D#_{k# λ#} := DimAux(x#_{λ#}, k#)
               | D# := ratint(null(D#))
               | for k# ∈ [1..length(v#)]
               |   K# := K#·(v#_{k#})^{D#_{k#}}
               | K = K#
```

$$Dim(5\ J) = \frac{[M]\ [L]^2}{[T]^2} \qquad Dim(4\ V) = \frac{[M]\ [L]^2}{[I]\ [T]^3} \qquad Dim(6\ W) = \frac{[M]\ [L]^2}{[T]^3} \qquad Dim(h) = \frac{[M]\ [L]^2}{[T]}$$

Speed of free fall

A particle of mass mass m falling at speed v from h in gravity of strength g

$$eq := Dim\left(\left[h\ \ g\ \ v\ \ m\right], \left[m\ \ g_e\ \ \frac{m}{s}\ \ kg\right]\right) = K = \frac{v^2}{h \cdot g} \qquad \text{with} \qquad K = 2 \qquad \text{Free of m}$$

Period of an hamronic oscilator

Period T of a mass m attached to an ideal linear spring of constant k suspended in gravity of strength g

$$eq := Dim\left(\left[\,T\ \ m\ \ k\ \ g\,\right],\left[\,\mathrm{s}\ \ \mathrm{kg}\ \ \frac{\mathrm{kg}}{\mathrm{s}^2}\ \ \mathrm{g_e}\,\right]\right) = K = \frac{m}{T^2 \cdot k}$$

with $K = (2 \cdot \pi)^2$ Free of g

The Trinity Test

Geoffrey Taylor was able to determine how much energy was released for the first atomic bomb explosion just by looking at the photograph doing dimmensional analysis: the time after detonation t, the radius of the explosion R and the density of air ρ

$$eq := Dim\left(\left[\,t\ \ R\ \ \rho\ \ E\,\right],\left[\,\mathrm{s}\ \ \mathrm{m}\ \ \frac{\mathrm{kg}}{\mathrm{m}^3}\ \ \mathrm{J}\,\right]\right) = K = \frac{t^2 \cdot E}{R^5 \cdot \rho}$$

with $K = something$

Black hole temperature

Black body radiation

$$K_1 := K \,\Big|\, Dim\left(\left[\,T\ \ k\ \ \hbar\ \ R\ \ c\,\right],\left[\,\mathrm{K}\ \ \mathrm{k}\ \ \frac{\mathrm{h}}{2 \cdot \pi}\ \ \mathrm{m}\ \ \mathrm{c}\,\right]\right) = \frac{\hbar \cdot c}{T \cdot k \cdot R}$$

Scape speed
(Schwarzschild radius)

$$K_2 := K \,\Big|\, Dim\left(\left[\,G\ \ M\ \ R\ \ c\,\right],\left[\,\mathrm{G_N}\ \ \mathrm{kg}\ \ \mathrm{m}\ \ \mathrm{c}\,\right]\right) = \frac{R \cdot c^2}{G \cdot M}$$

Joining

$$eq := K = K_1 \cdot K_2 = K = \frac{\hbar \cdot c^3}{T \cdot k \cdot G \cdot M}$$

with $K = 8 \cdot \pi$ (Hawking)

Alvaro