MPinv

MPInv Examples

Pseudoinverse Summary

Dim	Rank	PseudoInverse
$A_{n\,n}$	$\mathrm{rank}(A)=n$	$A^* = A^{-1}$
$A_{m\,n}$	$\mathrm{rank}(A)=m$	$A^* = A^{T}\cdot\left(A\cdot A^{T}\right)^{-1}$
$A_{m\,n}$	$\mathrm{rank}(A)=n$	$A^* = \left(A^{T}\cdot A\right)^{-1}\cdot A^{T}$
$A_{m\,n}$	$\mathrm{rank}(A)=r$	$A^* = G^*\cdot F^*$ with $A = F\cdot G$ any full rank factorization of A

Examples

Setup $\mathrm{TOL} := 10^{-9}$ $MEval(x) := x$

$A := \begin{bmatrix} 2 \end{bmatrix}$

$A^* := A^{-1} = \begin{bmatrix} 0.5 \end{bmatrix}$

$G := MPInv(A) = \begin{bmatrix} 0.5 \end{bmatrix}$

$MRank(A) = 1$

$\max\left(\overrightarrow{|A^* - G|}\right) = 0$

$A := \begin{bmatrix} 1 & 2 \end{bmatrix}$

$A^* := A^{T}\cdot\left(A\cdot A^{T}\right)^{-1} = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}$

$G := MPInv(A) = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}$

$MRank(A) = 1$

$\max\left(\overrightarrow{|A^* - G|}\right) = 0$

$A := \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$A^* := \left(A^{T}\cdot A\right)^{-1}\cdot A^{T} = \begin{bmatrix} 0.2 & 0.4 \end{bmatrix}$

$G := MPInv(A) = \begin{bmatrix} 0.2 & 0.4 \end{bmatrix}$

$MRank(A) = 1$

$\max\left(\overrightarrow{|A^* - G|}\right) = 0$

$A := \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$A^* := A^{-1} = \begin{bmatrix} -2 & 1.5 \\ 1 & -0.5 \end{bmatrix}$

$G := MPInv(A) = \begin{bmatrix} -2 & 1.5 \\ 1 & -0.5 \end{bmatrix}$

$MRank(A) = 2$

$\max\left(\overrightarrow{|A^* - G|}\right) = 7.2\cdot 10^{-14}$

$A := \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$

$A^* := MPInv_{dn}(A) = \begin{bmatrix} 0 & 0 \\ 0.5 & 0 \end{bmatrix}$

$G := MPInv(A) = \begin{bmatrix} 0 & 0 \\ 0.5 & 0 \end{bmatrix}$

$MRank(A) = 1$

$\max\left(\overrightarrow{|A^* - G|}\right) = 0$

$A := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$A^* := MPInv_{dn}(A) = \begin{bmatrix} -0.6389 & -0.1667 & 0.3056 \\ -0.0556 & 1.7283\cdot 10^{-16} & 0.0556 \\ 0.5278 & 0.1667 & -0.1944 \end{bmatrix}$

$G := MPInv(A) = \begin{bmatrix} -0.6389 & -0.1667 & 0.3056 \\ -0.0556 & -6.4056\cdot 10^{-15} & 0.0556 \\ 0.5278 & 0.1667 & -0.1944 \end{bmatrix}$

$MRank(A) = 2$

$\max\left(\overrightarrow{|A^* - G|}\right) = 3.1\cdot 10^{-13}$

$$A := \begin{bmatrix} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 4 \\ 7 & 8 & 3 & 7 \\ 6 & 7 & 8 & 6 \\ 4 & 4 & 4 & 4 \end{bmatrix} \qquad A^* := MPInv_{dn}(A) = \begin{bmatrix} -0.3818 & -0.1085 & -0.0833 & 0.0736 & 0.3643 \\ 0.6667 & 0.1667 & 0.3333 & -0.1667 & -0.6667 \\ 0.0039 & 0.062 & -0.1667 & 0.1008 & 0.0775 \\ -0.3818 & -0.1085 & -0.0833 & 0.0736 & 0.3643 \end{bmatrix} \qquad MRank(A) = 3$$

$$G := MPInv(A) = \begin{bmatrix} -0.3818 & -0.1085 & -0.0833 & 0.0736 & 0.3643 \\ 0.6667 & 0.1667 & 0.3333 & -0.1667 & -0.6667 \\ 0.0039 & 0.062 & -0.1667 & 0.1008 & 0.0775 \\ -0.3818 & -0.1085 & -0.0833 & 0.0736 & 0.3643 \end{bmatrix} \qquad \max\left(\overrightarrow{|A^* - G|}\right) = 1.7 \cdot 10^{-13}$$

Rank The code could be used for calculate the rank of a matrix.

$n := 28$ $[a\ b] := [100\ 200]$ $c := n - 10$ $A := b \cdot MRand(n, n) - a$ $P := dn\_LinAlgSVD(A)_2$

$\lambda := b \cdot MRand(c, 1) - a$ $\lambda_n := 0$

Matrix M is nxn, have rank c
and eigenvalues λ

$M := P \cdot diag(\lambda) \cdot P^T$ $M = \begin{bmatrix} 1.7294 & 2.1048 & \cdots \\ 2.1048 & -14.9403 & \\ 6.3433 & 4.0771 & \\ -5.2002 & 4.6515 & \\ \vdots & & \ddots \end{bmatrix}$

SMath's rank have several bugs.
For instance, can't calculate this

$rank(M) =$ $\boxed{MRank(M) = 18}$

$\boxed{MRank_{dn}(M) = 18}$

Timing

$[m\ n] := [60\ 40]$ $A := 200 \cdot MRand(m, n) - 100$ $A^* := MPInv_{dn}(A)$

$Id := eval(identity(min([m\ n])))$

Slow but more accurate $MEval(x) := x$

$t_o := time(0)$ $G := MPInv(A)$ $\boxed{time(0) - t_o = 26.3\ s}$ $\max\left(\overrightarrow{|A^* - G|}\right) = 2.4 \cdot 10^{-16}$

$Id :=$ if $m > n$ $G \cdot A$ else $A \cdot G$ $\max\left(\overrightarrow{|Id - Id|}\right) = 2.3 \cdot 10^{-13}$

$Ido := eval\left(\overrightarrow{Id \cdot \left(Id > \sqrt{TOL}\right)}\right)$

$\sum Ido = 40$

equal to min(m,n)

$Ido = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ \vdots & & & & & & & & & & & & & \ddots \end{bmatrix}$

Faster version $MEval(x) := eval(x)$

$t_o := time(0)$ $G := MPInv(A)$ $\boxed{time(0) - t_o = 8.8\ s}$ $\max\left(\overrightarrow{|A^* - G|}\right) = 1.1 \cdot 10^{-12}$

$Id :=$ if $m > n$ $G \cdot A$ else $A \cdot G$ $\max\left(\overrightarrow{|Id - Id|}\right) = 7.5 \cdot 10^{-10}$

Reference: https://arxiv.org/ftp/arxiv/papers/0804/0804.4809.pdf

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