# Linkage bars animations

⊞— Linkage Bars

⊟— Examples

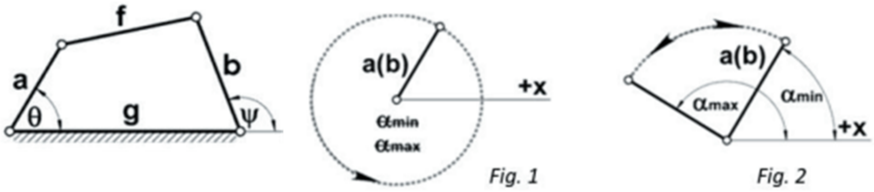
Clear$(N, t, \tau, x, y, z, \theta, \omega) = 1$

⊟— 4-bar

## Four Bar Mechanism

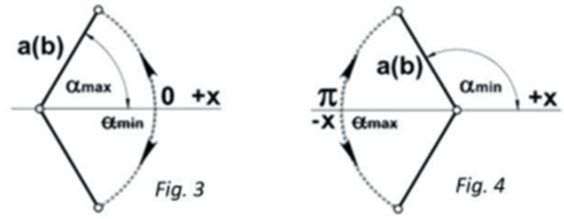
Reference https://www.researchgate.net/publication/325172892_Classification_geometrical_and_kinematic_analysis_of_four-bar_linkages/link/5bce3cdc299bf1a43d9a38ff/download

There are 3^3 = 27 different configuration types considerating movement performed by input link a and output link b. If a link performs a complete rotation it is called a crank and a rocker if not, with both limits αmin and αmax for the angle.




The case 0-rocker is when not exist αmin and the link oscilates about 0 with amplitude ± αmax.

The case π-rocker is when not exist αmax and the link oscilates about π with amplitude π ± αmax.




$Bar4(a, b, f, g)$ returns the classification and the angles.

### Double crank

- In this example N + 1 is the numbers of frames in the animation, n the variable for it,
- The two first rows in E are the node numbers and the third the lenght for each bar.
- Po are the x,y,z coordinates of the fixes nodes and Pg the guess values for the coordinates of the movible nodes.
- θo is a vector with N + 1 values for the rotating angle.

$$N := 40 \quad n := [1..(N+1)] \quad \theta o := pR(0°, 360°, N)$$

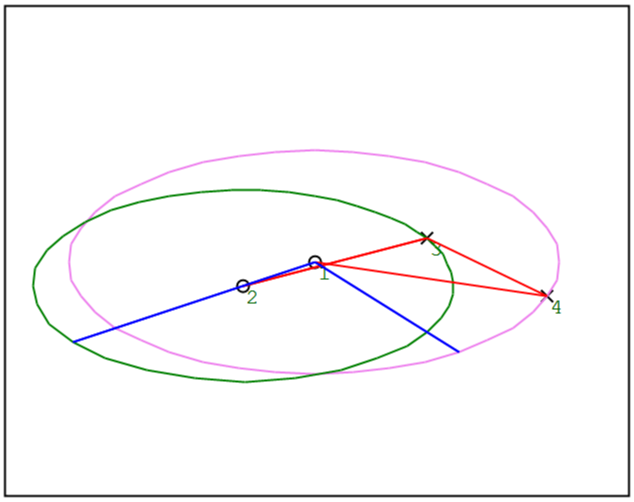
$$a := 1.4 \quad b := 1.2 \quad f := 1 \quad g := 0.5$$

$$Po := \begin{bmatrix} 0 & 0 & 0 \\ g & 0 & 0 \end{bmatrix} \quad Pg := \begin{bmatrix} 0 & b & 0 \\ f & b & 0 \end{bmatrix}$$

$$E := \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 1 \\ b & f & a \end{bmatrix}$$

$$BarE(\theta, x, y, z) := \begin{bmatrix} BarR(\theta, x_4, y_4) \\ z_3 \\ z_4 \end{bmatrix}$$

$$[P \; W] := BarPW(\theta o, E, Po, Pg)$$




BarPlot(t, Po, P, W)

$$Bar4(a, b, f, g) = \begin{bmatrix} \text{"Inp link a"} & \text{"θmin"} & \text{"θmax"} & \text{"Out link b"} & \text{"ψmin"} & \text{"ψmax"} \\ \text{"crank"} & \text{"∄"} & \text{"∄"} & \text{"crank"} & \text{"∄"} & \text{"∄"} \end{bmatrix}$$

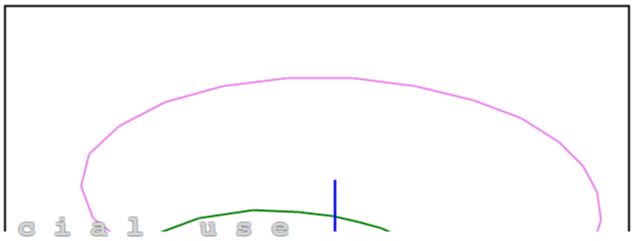
We can change the speed of θo adding a mapping expression in pR. Also can change the z-values, by fixed values or using τ, ranged between 0 and 1

$$N := 40 \quad n := [1..(N+1)] \quad \theta o := pR\left(0, 2 \cdot \pi, N, x^2\right)$$

$$a := 1.4 \quad b := 1 \quad f := 1.2 \quad g := 0.4 \quad zo := 0.5$$

$$Po := \begin{bmatrix} 0 & 0 & 0 \\ g & 0 & -zo \end{bmatrix} \quad Pg := \begin{bmatrix} g & -b & 0 \\ a & 0 & 0 \end{bmatrix}$$

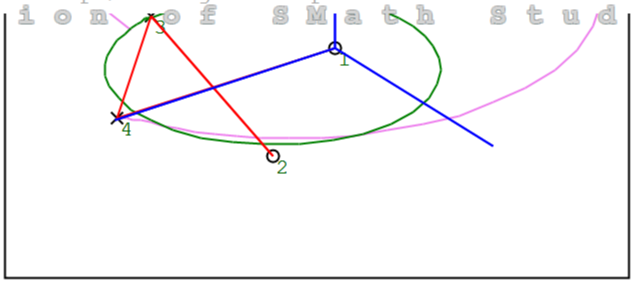
[ 2 3 4 ]



$$E := \begin{bmatrix} 3 & 4 & 1 \\ b & f & a \end{bmatrix} \quad \tau := \frac{t-1}{N}$$

$$BarE(\theta, x, y, z) := \begin{bmatrix} BarR\left(\theta, x_4, y_4\right) \\ z_3 \\ z_4 - zo \cdot \tau \end{bmatrix}$$

$$[\,P \;\; W\,] := BarPW(\theta o, E, Po, Pg)$$



BarPlot(t, Po, P, W)

**Double rocker**

In this example we use Bar4 for get the angle bounds and construct θo for the forward and retrun paths.

$$N := 60 \quad n := [1..(N+1)] \quad \theta o := pR(29°, 88°, 0.5 \cdot N)$$

$$\theta o := eval(stack(\theta o, reverse(\theta o)))$$

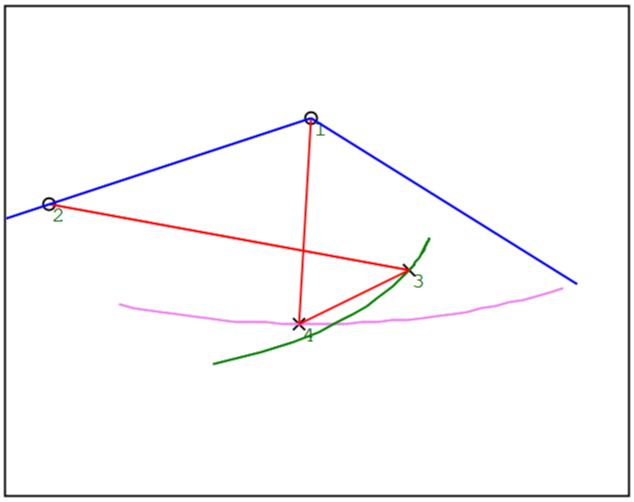
$$a := 1.4 \quad b := 1.2 \quad f := 0.5 \quad g := 1$$

$$Po := \begin{bmatrix} 0 & 0 & 0 \\ g & 0 & 0 \end{bmatrix} \quad Pg := \begin{bmatrix} 0 & b & 0 \\ f & b & 0 \end{bmatrix}$$

$$E := \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 1 \\ b & f & a \end{bmatrix}$$

$$BarE(\theta, x, y, z) := \begin{bmatrix} BarR\left(\theta, x_4, y_4\right) \\ z_3 \\ z_4 \end{bmatrix}$$

$$[\,P \;\; W\,] := BarPW(\theta o, E, Po, Pg)$$



BarPlot(t, Po, P, W)

$Bar4(a, b, f, g) =$

| "Inp link a" | "θmin" | "θmax" | "Out link b" | "ψmin" | "ψmax" |
|---|---|---|---|---|---|
| "rocker" | 28.0981 | 88.5675 | "rocker" | 60.8236 | 132.7786 |

**Crank - rocker**

We add an small zo, which not change the mechanism type, and the angles ψ

$$N := 40 \quad n := [1..(N+1)]$$

$$\theta o := pR\left(0, 2 \cdot \pi, N, e^{3 \cdot x} - 1\right)$$

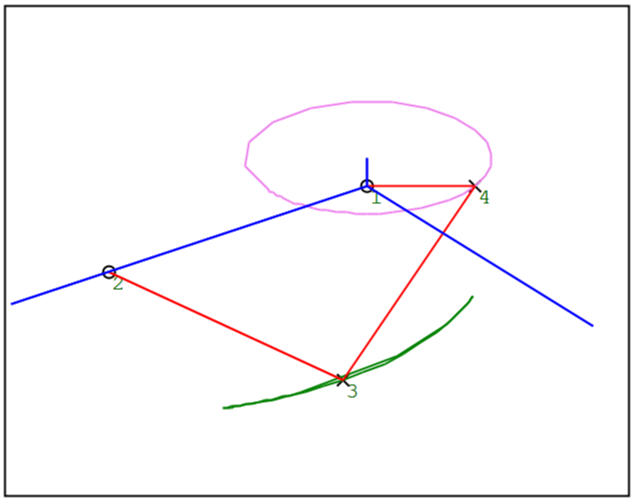
$$a := 0.4 \quad b := 1.2 \quad f := 1.4 \quad g := 1 \quad zo := 0.1$$

$$Po := \begin{bmatrix} 0 & 0 & 0 \\ g & 0 & 0 \end{bmatrix} \quad Pg := \begin{bmatrix} g & b & zo \\ a & 0 & zo \end{bmatrix}$$

$$E := \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 1 \\ b & f & a \end{bmatrix}$$

$$BarE(\theta, x, y, z) := \begin{bmatrix} BarR\left(\theta, x_4, y_4\right) \\ z_3 - zo \\ z_4 - zo \end{bmatrix}$$

$$[\,P \;\; W\,] := BarPW(\theta o, E, Po, Pg)$$



BarPlot(t, Po, P, W)

$Bar4(a, b, f, g) =$

| "Inp link a" | "θmin" | "θmax" | "Out link b" | "ψmin" | "ψmax" |
|---|---|---|---|---|---|
| "crank" | "∄" | "∄" | "rocker" | 70.5288 | 126.8699 |

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⊟— Chebyshev Linkages

**Chebyshev Linkage**

It's a four bar linkage with the interesting point at the center of bar f

$N := 60 \quad n := [1..(N+1)] \quad \theta o := pRR(37°, 103°, N)$

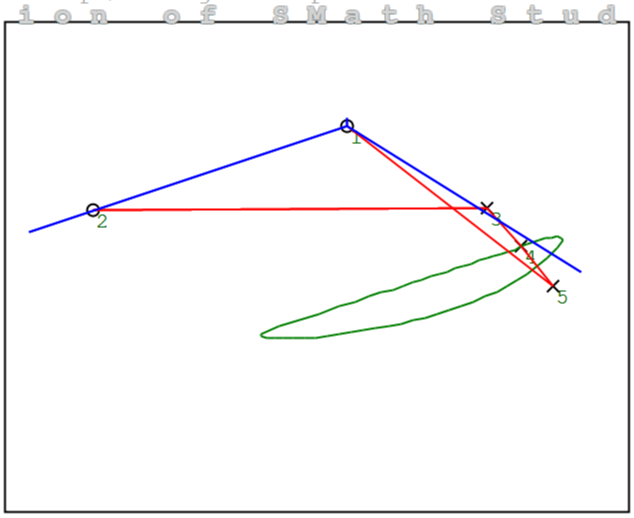
$f := 1 \quad a := 5 \cdot f \quad b := 5 \cdot f \quad g := 4 \cdot f \quad zo := 0.1$

$Po := \begin{bmatrix} 0 & 0 & 0 \\ g & 0 & 0 \end{bmatrix} \quad Pg := \begin{bmatrix} g & b & zo \\ f & b & 0 \\ 0 & b & -zo \end{bmatrix}$

$E := \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 1 \\ b & f & f & a \end{bmatrix}$

$BarE(\theta, x, y, z) := \begin{bmatrix} BarR(\theta, x_5, y_5) \\ BarA(180°, x, y, 3, 5, 4) \\ z_3 - zo \\ z_4 \\ z_5 + zo \end{bmatrix}$

$[P \; W] := BarPW(\theta o, E, Po, Pg)$




BarPlot(t, Po, P, W, 4)

$Bar4(a, b, 2 \cdot f, g) = \begin{bmatrix} \text{"Inp link a"} & \text{"θmin"} & \text{"θmax"} & \text{"Out link b"} & \text{"ψmin"} & \text{"ψmax"} \\ \text{"rocker"} & 36.9 & 101.5 & \text{"rocker"} & 78.5 & 143.1 \end{bmatrix}$

## Chebyshev Lambda Linkage

It's a 5 bar based in a previous example of 4 bar.
We introduce also some noise in the node 2 and a small height.

htt

$N := 40 \quad n := [1..(N+1)] \quad \theta o := pR(0, 2 \cdot \pi, N)$

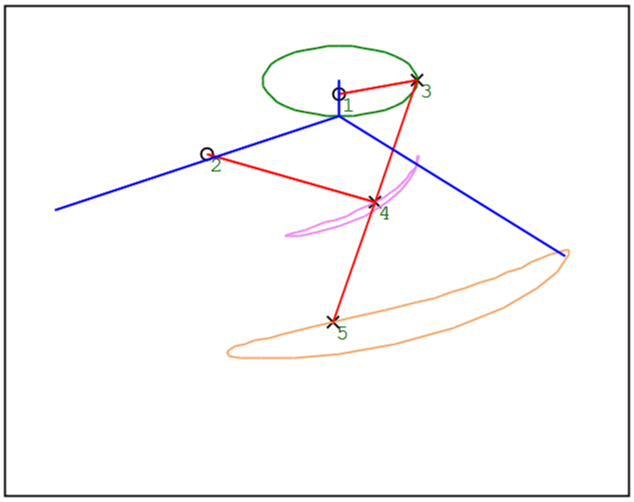
$a := 1 \quad zo := 0.3 \quad \tau := \frac{t-1}{N}$

$Po := \begin{bmatrix} 0 & 0 & zo \\ 2 \cdot a & 0.2 \cdot \sin(2 \cdot \pi \cdot \tau) & 0 \end{bmatrix} \quad Pg := \begin{bmatrix} -a & 0 & 0 \\ 0.5 \cdot a & 2 \cdot a & 0 \\ 2 \cdot a & 4 \cdot a & 0 \end{bmatrix}$

$E := \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 4 & 5 \\ a & 2.5 \cdot a & 2.5 \cdot a & 2.5 \cdot a \end{bmatrix}$

$BarE(\theta, x, y, z) := \begin{bmatrix} BarR(\theta, x_3, y_3) \\ BarA(180°, x, y, 3, 4, 5) \\ z_3 - 0.5 \\ z_4 \\ z_5 + 0.5 \end{bmatrix}$

$[P \; W] := BarPW(\theta o, E, Po, Pg)$




BarPlot(t, Po, P, W)

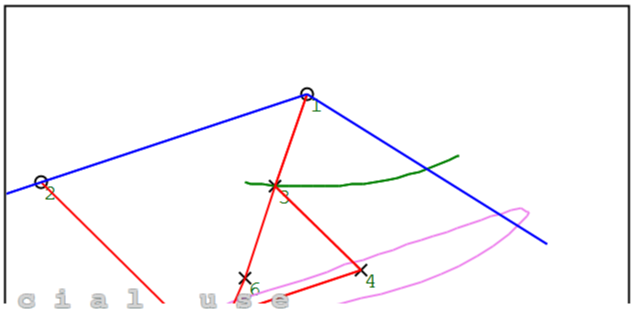
$Bar4(a, 2.5 \cdot a, 2.5 \cdot a, 2 \cdot a) = \begin{bmatrix} \text{"Inp link a"} & \text{"θmin"} & \text{"θmax"} & \text{"Out link b"} & \text{"ψmin"} & \text{"ψmax"} \\ \text{"crank"} & \text{"∄"} & \text{"∄"} & \text{"rocker"} & 78.5 & 143.1 \end{bmatrix}$

## Chebyshev Table Linkage

$N := 40 \quad n := [1..(N+1)] \quad \theta o := pRR(105°, 35°, N)$

$a := 1 \quad b := 5 \cdot a \quad g := 4 \cdot a$

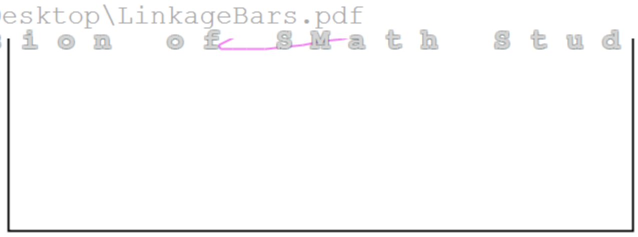
$Po := \begin{bmatrix} 0 & 0 & 0 \\ 4 \cdot a & 0 & 0 \end{bmatrix} \quad Pg := \begin{bmatrix} 2 \cdot a & a & 0 \\ 2 \cdot a & 4 \cdot a & 0 \\ 4 \cdot a & 4 \cdot a & 0 \\ 4 \cdot a & 3 \cdot a & 0 \\ 4 \cdot a & 5 \cdot a & 0 \end{bmatrix}$



$$E := \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 5 & 3 \\ 3 & 7 & 4 & 5 & 6 & 7 & 6 \\ 2.5 \cdot a & 5 \cdot a & 2.5 \cdot a & 2 \cdot a & a & a & 2.5 \cdot a \end{bmatrix}$$

$$BarE(\theta, x, y, z) := \begin{bmatrix} BarR\left(\theta, x_3, y_3\right) \\ BarA(180°, x, y, 1, 3, 6) \\ BarA(180°, x, y, 6, 5, 7) \end{bmatrix}$$

$$[P\ W] := BarPW(\theta o, E, Po, Pg, \text{"J"})$$



BarPlot(t, Po, P, W, [3 5])

Hoecken linkage - sliding joint

## Hoecken linkage

This example shows how to implement an sliding joint, using 0 as distance between nodes in the E matrix.

$$[N := 60 \quad n := [1..(N+1)] \quad \theta o := pR(0°, 360°, N)]$$

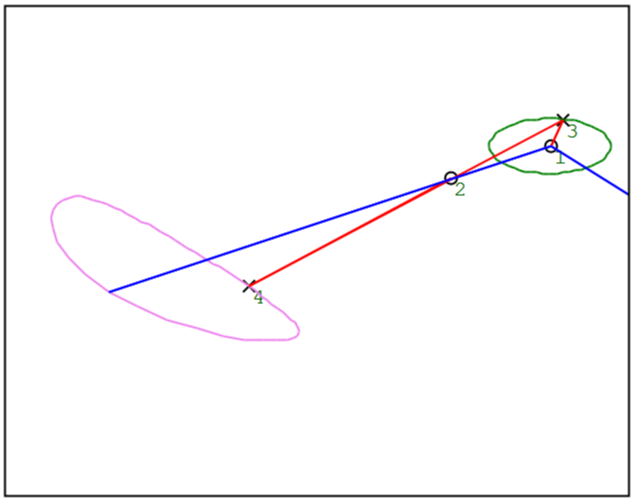
$$[a := 1 \quad v := 6]$$

$$Po := \begin{bmatrix} 0 & 0 & 0 \\ 2 \cdot a & 0 & 0 \end{bmatrix} \quad Pg := \begin{bmatrix} a & 0 & 0 \\ v \cdot a & 0 & 0 \end{bmatrix}$$

$$E := \begin{bmatrix} 1 & 3 & 2 \\ 3 & 4 & 4 \\ a & 2 \cdot a + v \cdot a & 0 \end{bmatrix}$$

$$BarE(\theta, x, y, z) := \begin{bmatrix} BarR\left(\theta, x_3, y_3\right) \\ BarA(180°, x, y, 3, 2, 4) \end{bmatrix}$$

$$[P\ W] := BarPW(\theta o, E, Po, Pg)$$




BarPlot(t, Po, P, W)

Roberts linkage

## Roberts linkage

It is a four-bar linkage which converts a rotational motion to approximate straight-line motion

$$[N := 60 \quad n := [1..(N+1)] \quad \theta o := pRR(30°, 97°, N)]$$

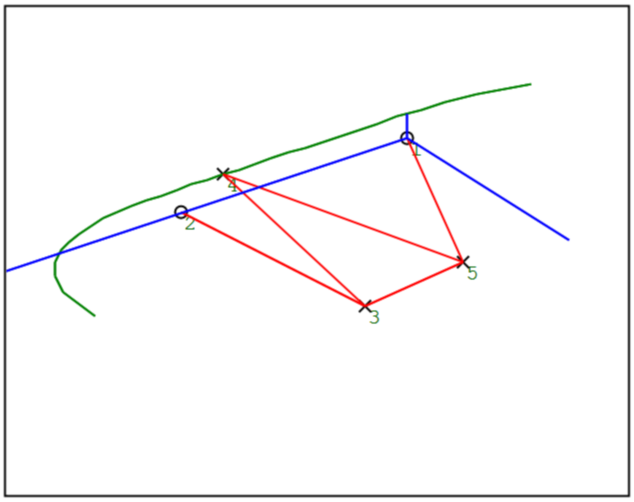
$$[a := 2 \quad b := 1 \quad zo := 0.2]$$

$$Po := \begin{bmatrix} 0 & 0 & 0 \\ 2 \cdot b & 0 & 0 \end{bmatrix} \quad Pg := \begin{bmatrix} 3 \cdot b & a & 0 \\ b & 0 & zo \\ 0.5 \cdot b & a & 0 \end{bmatrix}$$

$$E := \begin{bmatrix} 2 & 3 & 4 & 5 & 3 \\ 3 & 4 & 5 & 1 & 5 \\ a & a & a & a & b \end{bmatrix}$$

$$BarE(\theta, x, y, z) := \begin{bmatrix} BarR\left(\theta, x_5, y_5\right) \\ z_3 \\ z_4 - zo \\ z_5 \end{bmatrix}$$

$$[P\ W] := BarPW(\theta o, E, Po, Pg)$$




BarPlot(t, Po, P, W, 4)

$$Bar4(a, a, b, 2 \cdot b) = \begin{bmatrix} \text{"Inp link a"} & \text{"θmin"} & \text{"θmax"} & \text{"Out link b"} & \text{"ψmin"} & \text{"ψmax"} \\ \text{"rocker"} & 29.0 & 97.2 & \text{"rocker"} & 82.8 & 151.0 \end{bmatrix}$$

Parallel motion linkage

Parallel motion linkage

**Parallel motion linkage**

It is a six-bar mechanical linkage described by James Watt

$N := 60 \quad n := [1..(N+1)] \quad \theta o := pRR(-5°, 30°, N)$

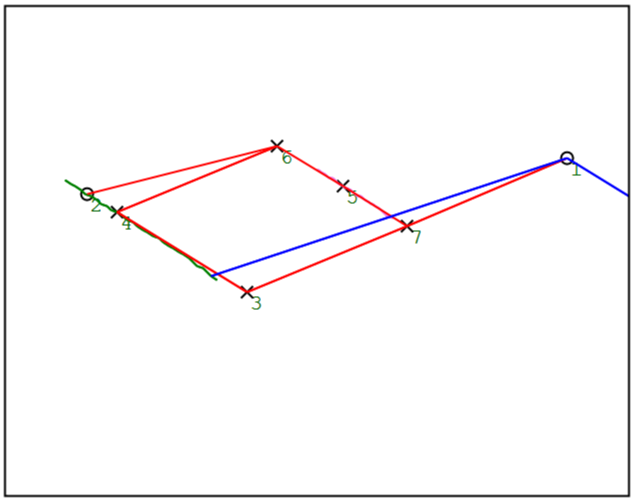
$a := 2 \quad b := 1 \quad zo := 0.0$

$$Po := \begin{bmatrix} 0 & 0 & 0 \\ 2 \cdot a & -2 \cdot b & 0 \end{bmatrix} \quad Pg := \begin{bmatrix} 2 \cdot a & 0 & 0 \\ 2 \cdot a & -b & 0 \\ a & -b & 0 \\ a & -2 \cdot b & 0 \\ a & 0 & 0 \end{bmatrix}$$

$$E := \begin{bmatrix} 1 & 7 & 3 & 7 & 5 & 4 & 6 \\ 7 & 3 & 4 & 5 & 6 & 6 & 2 \\ a & a & 2 \cdot b & b & b & a & a \end{bmatrix}$$

$$BarE(\theta, x, y, z) := \begin{bmatrix} BarR(\theta, x_7, y_7) \\ BarA(180°, x, y, 1, 7, 3) \\ BarA(180°, x, y, 7, 5, 6) \end{bmatrix}$$

$[P \; W] := BarPW(\theta o, E, Po, Pg)$



BarPlot(t, Po, P, W, [4 5])

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Peaucellier-Lipkin linkage

**Peaucellier–Lipkin linkage**

It is the first true planar straight line mechanism.

$N := 60 \quad n := [1..(N+1)] \quad \theta o := pRR(30°, -30°, N)$

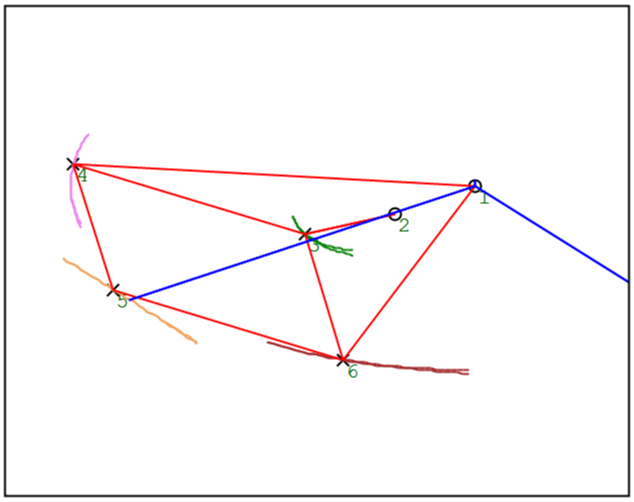
$a := 0.8 \quad b := 2.2 \quad c := 3.2 \quad zo := 0.0$

$$Po := \begin{bmatrix} 0 & 0 & 0 \\ a & 0 & 0 \end{bmatrix} \quad Pg := \begin{bmatrix} 2 \cdot a & a & 0 \\ c & 0 & 0 \\ 2 \cdot c & b & 0 \\ c+b & 2 \cdot b & 0 \end{bmatrix}$$

$$E := \begin{bmatrix} 1 & 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 3 & 4 & 5 & 6 & 3 \\ c & c & a & b & b & b & b \end{bmatrix}$$

$$BarE(\theta, x, y, z) := \begin{bmatrix} BarR(\theta, x_3 - a, y_3) \end{bmatrix}$$

$[P \; W] := BarPW(\theta o, E, Po, Pg)$



BarPlot(t, Po, P, W)

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Jansen's linkage

**Jansen's linkage**

It is a planar leg mechanism. Here we use the option "J" for the solver.
Values are from https://github.com/cvigoe/Jansen/blob/master/jansen.pdf

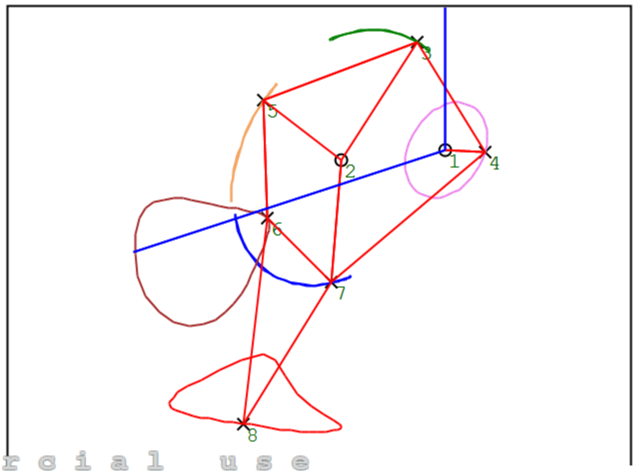
$N := 40 \quad n := [1..(N+1)] \quad \theta o := pR(0°, 360°, N)$

$k_{32} := 4.15 \quad k_{27} := 3.93 \quad k_{25} := 4.01 \quad k_{35} := 5.58$

$k_{56} := 3.94 \quad k_{67} := 3.67 \quad k_{68} := 6.57 \quad k_{78} := 4.90$

$k_{34} := 5.00 \quad k_{47} := 6.19 \quad k_{14} := 1.50 \quad \lambda := 0.78 \quad a := 3.80$

$$Po := \begin{bmatrix} 0 & 0 & 0 \\ a & 0 & \lambda \end{bmatrix} \quad Pg := \begin{bmatrix} k_{14} & 0 & k_{34} \\ -k_{34} & 0 & 0 \\ a+k_{25} & 0 & \lambda \\ a+k_{25} & 0 & -\lambda \\ a & 0 & -k_{27} \end{bmatrix}$$



$$\left[ a + k_{25} \quad 0 \quad -\left(k_{27} + k_{78}\right) \right]$$

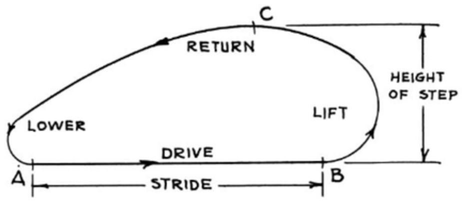
$$E := \begin{bmatrix} 2 & 4 & 4 & 3 & 3 & 2 & 5 & 6 & 6 & 7 & 1 \\ 5 & 3 & 7 & 2 & 5 & 7 & 6 & 7 & 8 & 8 & 4 \\ k_{25} & k_{34} & k_{47} & k_{32} & k_{35} & k_{27} & k_{56} & k_{67} & k_{68} & k_{78} & k_{14} \end{bmatrix}$$

$$BarE(\theta, x, y, z) := \left[ BarR\left(\theta, x_4, z_4\right) \right]$$

$$[\,P \;\; W\,] := BarPW(\theta o, E, Po, Pg, \text{"J"})$$

BarPlot(t, Po, P, W)

Shigley's Phases of the Foot-Path




Trammel of Archimedes

## Trammel of Archimedes

This example shows how to move a 'fixed' node, using the special variable t, which is the variable in the SMath plots for animations (the range is given by n).

Another way to do that is

$$\tau := \frac{t-1}{N}$$

$$Po := [\, 0 \quad 2 \cdot b \cdot \sin(2 \cdot \pi \cdot \tau) \quad 0 \,]$$

Usually, the trammel have only one node, but here we add 3.
The last equation in BarE is for force x2 to change the sign.

$$[\, N := 40 \quad n := [1..(N+1)] \quad \theta o := pR(0, 2 \cdot \pi, N) \,]$$

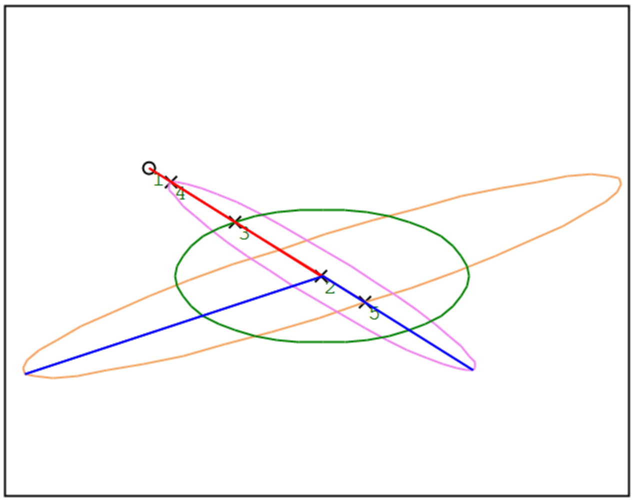
$$a := 2$$

$$Po := \left[\, 0 \quad 2 \cdot a \cdot \sin\left(\theta o_t\right) \quad 0 \,\right] \quad Pg := \begin{bmatrix} 2 \cdot a & 0 & 0 \\ a & 0 & 0 \\ 0.75 \cdot a & 0 & 0 \\ 1.5 \cdot a & 0 & 0 \end{bmatrix}$$

$$E := \text{eval}\left(\begin{bmatrix} 1 & 2 & 3 & 3 \\ 3 & 3 & 4 & 5 \\ a & a & 0.75 \cdot a & 1.5 \cdot a \end{bmatrix}\right)$$

$$BarE(\theta, x, y, z) := \begin{bmatrix} BarA(0°, x, y, 2, 1, 3) \\ BarA(0°, x, y, 2, 4, 3) \\ BarA(0°, x, y, 2, 5, 3) \\ x_2 - 2 \cdot a \cdot \cos(\theta) \end{bmatrix}$$

$$[\,P \;\; W\,] := BarPW(\theta o, E, Po, Pg)$$




BarPlot(t, Po, P, W, [ 3 4 5])

## Variation in 3D

$$[\, N := 40 \quad n := [1..(N+1)] \quad \theta o := pR(0°, 360°, N) \,]$$

$$\tau := \frac{t-1}{N}$$

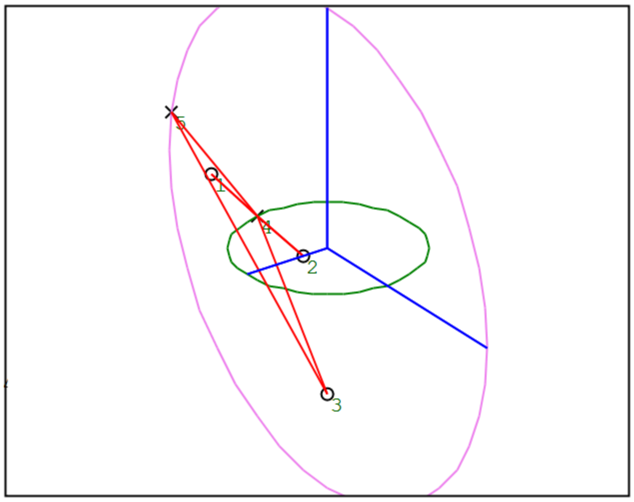
$$\left[\, a := 3 \quad c := a \quad k := 1 \quad \kappa := \left|\frac{1}{\cos(-45°)}\right| \quad b := \frac{2 \cdot a}{\kappa} \,\right]$$

$$b := 4$$

$$Po := \begin{bmatrix} 0 & 2 \cdot a \cdot \sin\left(\theta o_t\right) & 0 \\ 2 \cdot a \cdot \cos\left(\theta o_t\right) & 0 & 0 \\ 0 & 0 & -2 \cdot a \cdot \cos\left(\theta o_t\right) + \end{bmatrix}$$

$$Pg := \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & -2 \cdot b \end{bmatrix}$$

$$E := \begin{bmatrix} 1 & 2 & 3 & 3 & 4 \\ 4 & 4 & 4 & 5 & 5 \\ a & a & 0 & 0 & 0 \end{bmatrix}$$




BarPlot(t, Po, P, W)

$$\left[\; z_4 \;\right]$$

$$BarE(\theta, x, y, z) := \begin{bmatrix} y_5 - 2 \cdot b \cdot \sin(\theta - 0°) \\ z_5 - 2 \cdot b \cdot \cos(\theta - 0°) \\ x_5 \end{bmatrix}$$

$$[P\ W] := BarPW(\theta o, E, Po, Pg, "J")$$

Sewing machine

## Sewing machine

Just trial and error for get some stable lenghts.

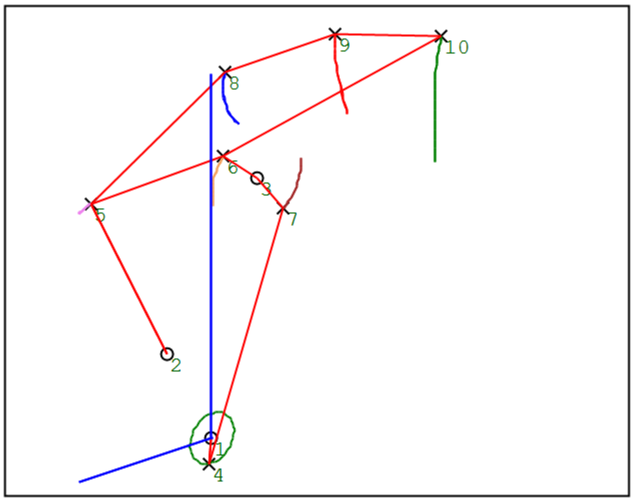
$$[N := 40 \quad n := [1..(N+1)] \quad \theta o := pR(0, 2 \cdot \pi, N)]$$

$$Po := \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ -1 & 0 & 5 \end{bmatrix} \quad Pg := \begin{bmatrix} 0.5 & 0 & 0 \\ 3 & 0 & 5.5 \\ 0 & 0 & 5.5 \\ -2 & 0 & 5.5 \\ 1 & 0 & 7.5 \\ -2 & 0 & 8 \\ -4 & 0 & 10 \end{bmatrix}$$

$$E := \begin{bmatrix} 1 & 2 & 5 & 5 & 6 & 7 & 6 & 8 & 9 & 4 \\ 4 & 5 & 8 & 6 & 3 & 3 & 10 & 9 & 10 & 7 \\ 0.5 & 4 & 3.5 & 3 & 1 & 1 & 5 & 2.5 & 2.5 & 5 \end{bmatrix}$$

$$BarE(\theta, x, y, z) := \begin{bmatrix} BarR(\theta, x_4, z_4) \\ BarA(170°, x, z, 6, 3, 7) \\ BarA(170°, x, z, 10, 6, 5) \\ y_4 \end{bmatrix}$$

$$[P\ W] := BarPW(\theta o, E, Po, Pg)$$



BarPlot(t, Po, P, W)

Alvaro