SOLERAS OAMB KERNEL

1. Input data

1.1 Beam nodes matrix

all input values in SI units!

enter all node points in matrix N; start & end of beam are no nodes!

N:=

1	0,0402	-0,002
2	0,1882	-0,005
3	0,3762	0
4	0,6398	-0,005
5	0,7878	-0,002
6	0	0
7	0	0
8	0	0
9	0	0
10	0	0
11	0	0
12	0	0
13	0	0
14	0	0
15	0	0
16	0	0
17	0	0
18	0	0
19	0	0
20	0	0

X:= col(N ; 2)

Y:= col(N ; 3)

```
for i ∈ 1 .. rows(N)
  if N_{i 2} = 0
    | dim:= i + 1
    | break
  else
    dim:= rows(N) + 2
```

dim= 7

1.2 Beam data

beam length

$l_b := X_{dim-2} + 0,050 = 0,8378$

beam width (broadness)

$b_b := 0,058$

beam height

$h_b := 0,005$

area moment of inertia

$I_b := \frac{b_b \cdot h_b^{3}}{12} = 6,0417 \cdot 10^{-10}$

Young's modulus

$E_b := 200 \cdot 10^{9}$

magnet stiffness coefficient

$K_m := \frac{152}{120} = 1,2667$

bending stiffness

$EI_b := K_m \cdot E_b \cdot I_b = 153,0556$

beam density

$\rho_b := 7850$

distributed load (beam weight, factor 1,2 due to magnet presence)

$w_b := \frac{\rho_b \cdot b_b \cdot h_b \cdot l_b \, g_e \frac{kg}{N}}{l_b} \cdot 1,2 = 26,7898$

$w_b := 0$

max. admissible bending stress σadm in case of a repetitive duty cycle is determined by:
* fatigue limit σr for bending (from Smith/Goodman diagram for S235, assume fluctuating load σm=0)
* reduction factor Ksu for fatigue limit due to various surface treatments
* reduction factor Ksi for fatigue limit due to part size (the smaller, the better)
* fatigue notch factor Kf
* safety factor Ksa

since the concerned load is not be considered as repetitive, we take the yield strength as limiting value (source: http://www.tribology-abc.com/sub15.htm)

$\sigma_r := 195 \cdot 10^{6}$

$K_{su} := 1$

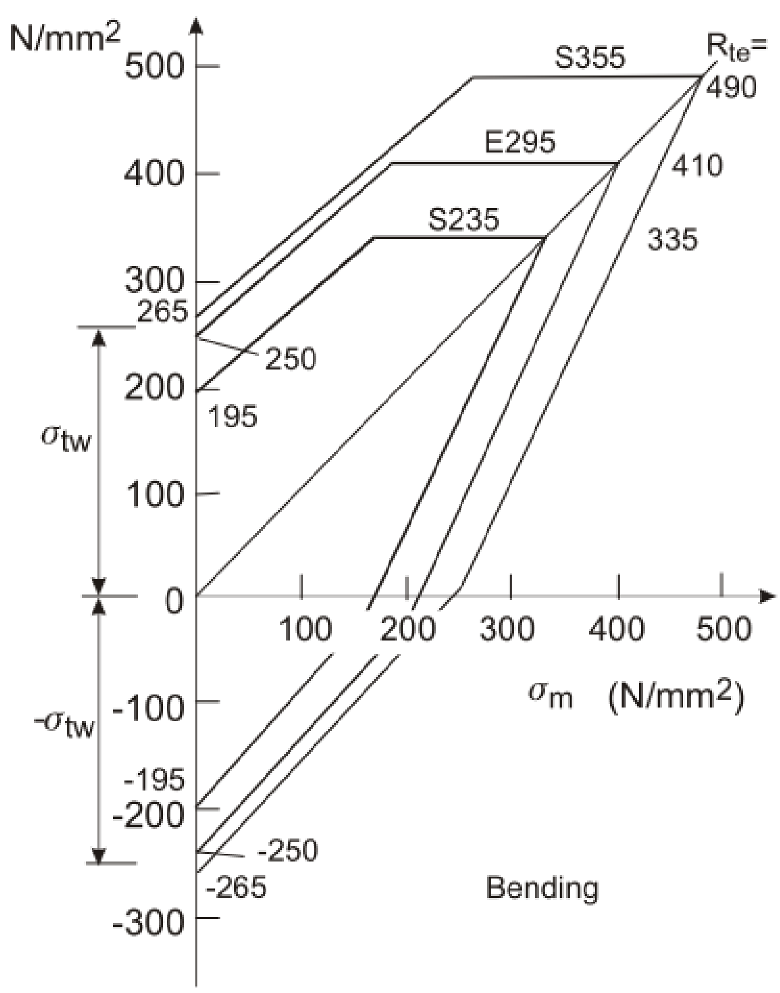
$K_{si} := 1$

$K_f := 1$

$K_{sa} := 1,1$

$\sigma_{adm} := \frac{\sigma_r \cdot K_{su} \cdot K_{si}}{K_f \cdot K_{sa}} = 1,7727 \cdot 10^{8}$

$\sigma_{adm} := 0,8 \cdot 235 \cdot 10^{6} = 1,88 \cdot 10^{8}$



2. Beam equations

singularity function definition
(sources:
* Oscar Campo, Columbia
* "Shigley's Mechanical Engineering Design 9th Ed.", p. 76
* Youtube: Ronald Delahoussaye)

$$f_s(x\,;\,a\,;\,n) := (x-a)^n \cdot (x > a) \cdot (n \geq 0)$$

beam equations for load, shear, moment, slope, deflection (elastic curve) and bending stress (Euler-Bernoulli beam theory)

note that in general, the integration constants C1 and C2 for shear and moment will always be zero if the reaction forces and moments acting on the beam are included in the loading function, because the shear and moment diagrams must close to zero at the end of the beam (source: "Machine Design - 4th Ed." by Robert L. Norton, p. 118)

$$q(x) := -w_b \cdot f_s(x\,;\,0\,;\,0) + w_b \cdot f_s(x\,;\,l_b\,;\,0) + \sum_{i=1}^{dim-2} \left(R_i \cdot f_s(x\,;\,X_i\,;\,-1)\right)$$

$$V(x) := -w_b \cdot f_s(x\,;\,0\,;\,1) + w_b \cdot f_s(x\,;\,l_b\,;\,1) + \sum_{i=1}^{dim-2} \left(R_i \cdot f_s(x\,;\,X_i\,;\,0)\right)$$

$$M(x) := -\frac{w_b}{2} \cdot f_s(x\,;\,0\,;\,2) + \frac{w_b}{2} \cdot f_s(x\,;\,l_b\,;\,2) + \sum_{i=1}^{dim-2} \left(R_i \cdot f_s(x\,;\,X_i\,;\,1)\right)$$

$$\theta(x) := \frac{1}{EI_b} \cdot \left(-\frac{w_b}{6} \cdot f_s(x\,;\,0\,;\,3) + \frac{w_b}{6} \cdot f_s(x\,;\,l_b\,;\,3) + \sum_{i=1}^{dim-2} \left(\frac{R_i}{2} \cdot f_s(x\,;\,X_i\,;\,2)\right) + C_1\right)$$

$$y(x) := \frac{1}{EI_b} \cdot \left(-\frac{w_b}{24} \cdot f_s(x\,;\,0\,;\,4) + \frac{w_b}{24} \cdot f_s(x\,;\,l_b\,;\,4) + \sum_{i=1}^{dim-2} \left(\frac{R_i}{6} \cdot f_s(x\,;\,X_i\,;\,3)\right) + C_1 \cdot x + C_2\right)$$

$$\sigma_{max}(x) := \frac{M(x) \cdot \frac{h_b}{2}}{I_b}$$

applying boundary conditions (X# = lb + 1mm):
* V(x#)=0 (1)
* M(x#)=0 (2)
* y(X1)=Y1 (3)
* y(X2)=Y2 (4)
* y(X3)=Y3 (5)
* y(X4)=Y4 (6)
* y(X5)=Y5 (7)
results in a system of 7 equations and 7 unknown variables (R1, R2, R3, R4, R5, C1, C2)

$$X\# := l_b + 0{,}001$$

$$\sum_{i=1}^{dim-2} \left(R_i \cdot f_s(X\#\,;\,X_i\,;\,0)\right) = w_b \cdot f_s(X\#\,;\,0\,;\,1) - w_b \cdot f_s(X\#\,;\,l_b\,;\,1)$$

$$\sum_{i=1}^{dim-2} \left(R_i \cdot f_s(X\#\,;\,X_i\,;\,1)\right) = \frac{w_b}{2} \cdot f_s(X\#\,;\,0\,;\,2) - \frac{w_b}{2} \cdot f_s(X\#\,;\,l_b\,;\,2)$$

$$\sum_{i=1}^{dim-2} \left(\frac{R_i}{6} \cdot f_s(X_1\,;\,X_i\,;\,3)\right) + C_1 \cdot X_1 + C_2 = Y_1 \cdot EI_b + \frac{w_b}{24} \cdot f_s(X_1\,;\,0\,;\,4) - \frac{w_b}{24} \cdot f_s(X_1\,;\,l_b\,;\,4)$$

$$\sum_{i=1}^{dim-2} \left(\frac{R_i}{6} \cdot f_s(X_2\,;\,X_i\,;\,3)\right) + C_1 \cdot X_2 + C_2 = Y_2 \cdot EI_b + \frac{w_b}{24} \cdot f_s(X_2\,;\,0\,;\,4) - \frac{w_b}{24} \cdot f_s(X_2\,;\,l_b\,;\,4)$$

$$\sum_{i=1}^{dim-2} \left(\frac{R_i}{6} \cdot f_s(X_3\,;\,X_i\,;\,3)\right) + C_1 \cdot X_3 + C_2 = Y_3 \cdot EI_b + \frac{w_b}{24} \cdot f_s(X_3\,;\,0\,;\,4) - \frac{w_b}{24} \cdot f_s(X_3\,;\,l_b\,;\,4)$$

$$\sum_{i=1}^{dim-2} \left(\frac{R_i}{6} \cdot f_s(X_4\,;\,X_i\,;\,3)\right) + C_1 \cdot X_4 + C_2 = Y_4 \cdot EI_b + \frac{w_b}{24} \cdot f_s(X_4\,;\,0\,;\,4) - \frac{w_b}{24} \cdot f_s(X_4\,;\,l_b\,;\,4)$$

$$\sum_{i=1}^{dim-2} \left(\frac{R_i}{6} \cdot f_s(X_5\,;\,X_i\,;\,3)\right) + C_1 \cdot X_5 + C_2 = Y_5 \cdot EI_b + \frac{w_b}{24} \cdot f_s(X_5\,;\,0\,;\,4) - \frac{w_b}{24} \cdot f_s(X_5\,;\,l_b\,;\,4)$$

$COEF \cdot REACTIONS = FORCES$

$$COEF\_EXPLICIT = \begin{pmatrix} f_s(X\#; X_1; 0) & f_s(X\#; X_2; 0) & f_s(X\#; X_3; 0) & f_s(X\#; X_4; 0) & f_s(X\#; X_5; 0) & 0 & 0 \\ f_s(X\#; X_1; 1) & f_s(X\#; X_2; 1) & f_s(X\#; X_3; 1) & f_s(X\#; X_4; 1) & f_s(X\#; X_5; 1) & 0 & 0 \\ \frac{1}{6} \cdot f_s(X_1; X_1; 3) & \frac{1}{6} \cdot f_s(X_1; X_2; 3) & \frac{1}{6} \cdot f_s(X_1; X_3; 3) & \frac{1}{6} \cdot f_s(X_1; X_4; 3) & \frac{1}{6} \cdot f_s(X_1; X_5; 3) & X_1 & 1 \\ \frac{1}{6} \cdot f_s(X_2; X_1; 3) & \frac{1}{6} \cdot f_s(X_2; X_2; 3) & \frac{1}{6} \cdot f_s(X_2; X_3; 3) & \frac{1}{6} \cdot f_s(X_2; X_4; 3) & \frac{1}{6} \cdot f_s(X_2; X_5; 3) & X_2 & 1 \\ \frac{1}{6} \cdot f_s(X_3; X_1; 3) & \frac{1}{6} \cdot f_s(X_3; X_2; 3) & \frac{1}{6} \cdot f_s(X_3; X_3; 3) & \frac{1}{6} \cdot f_s(X_3; X_4; 3) & \frac{1}{6} \cdot f_s(X_3; X_5; 3) & X_3 & 1 \\ \frac{1}{6} \cdot f_s(X_4; X_1; 3) & \frac{1}{6} \cdot f_s(X_4; X_2; 3) & \frac{1}{6} \cdot f_s(X_4; X_3; 3) & \frac{1}{6} \cdot f_s(X_4; X_4; 3) & \frac{1}{6} \cdot f_s(X_4; X_5; 3) & X_4 & 1 \\ \frac{1}{6} \cdot f_s(X_5; X_1; 3) & \frac{1}{6} \cdot f_s(X_5; X_2; 3) & \frac{1}{6} \cdot f_s(X_5; X_3; 3) & \frac{1}{6} \cdot f_s(X_5; X_4; 3) & \frac{1}{6} \cdot f_s(X_5; X_5; 3) & X_5 & 1 \end{pmatrix}$$

```
COEF:= | COEF:= matrix(dim ; dim)
       | for i ∈ 1 ..dim
       |   for j ∈ 1 ..dim-2
       |     if i<3
       |       COEF_{i j} := f_s(X# ; X_j ; i-1)
       |     else
       |       COEF_{i j} := 1/6 · f_s(X_{i-2} ; X_j ; 3)
       | for i ∈ 3 ..dim
       |   | COEF_{i dim-1} := X_{i-2}
       |   | COEF_{i dim} := 1
       | COEF
```

$$FORCES\_EXPLICIT = \begin{pmatrix} w_b \cdot f_s(X\#; 0; 1) - w_b \cdot f_s(X\#; l_b; 1) \\ \frac{w_b}{2} \cdot f_s(X\#; 0; 2) - \frac{w_b}{2} \cdot f_s(X\#; l_b; 2) \\ Y_1 \cdot EI_b + \frac{w_b}{24} \cdot f_s(X_1; 0; 4) - \frac{w_b}{24} \cdot f_s(X_1; l_b; 4) \\ Y_2 \cdot EI_b + \frac{w_b}{24} \cdot f_s(X_2; 0; 4) - \frac{w_b}{24} \cdot f_s(X_2; l_b; 4) \\ Y_3 \cdot EI_b + \frac{w_b}{24} \cdot f_s(X_3; 0; 4) - \frac{w_b}{24} \cdot f_s(X_3; l_b; 4) \\ Y_4 \cdot EI_b + \frac{w_b}{24} \cdot f_s(X_4; 0; 4) - \frac{w_b}{24} \cdot f_s(X_4; l_b; 4) \\ Y_5 \cdot EI_b + \frac{w_b}{24} \cdot f_s(X_5; 0; 4) - \frac{w_b}{24} \cdot f_s(X_5; l_b; 4) \end{pmatrix}$$

```
FORCES:= | for i ∈ 1 ..dim
         |   if i<3
         |     FORCES_i := w_b/i · f_s(X# ; 0 ; i) - w_b/i · f_s(X# ; l_b ; i)
         |   else
         |     FORCES_i := Y_{i-2} · EI_b + w_b/24 · f_s(X_{i-2} ; 0 ; 4) - w_b/24 · f_s(X_{i-2} ; l_b ; 4)
         | FORCES
```

$$R := COEF^{-1} \cdot FORCES = \begin{pmatrix} 594,1429 \\ -1516,0498 \\ 1515,6093 \\ -1074,2062 \\ 480,5039 \\ -5,2715 \\ -0,0942 \end{pmatrix}$$

$C_1 := R_{dim-1} = -5,2715$ $\qquad$ $C_2 := R_{dim} = -0,0942$

$$MARKERS\_ALL := \begin{pmatrix} X_1 & 0 & "1" & 12 & "red" \\ X_2 & 0 & "2" & 12 & "red" \\ X_3 & 0 & "3" & 12 & "red" \\ X_4 & 0 & "4" & 12 & "red" \\ X_5 & 0 & "5" & 12 & "red" \\ X_6 & 0 & "6" & 12 & "red" \\ X_7 & 0 & "7" & 12 & "red" \\ X_8 & 0 & "8" & 12 & "red" \\ X_9 & 0 & "9" & 12 & "red" \\ X_{10} & 0 & "10" & 12 & "red" \\ X_{11} & 0 & "11" & 12 & "red" \\ X_{12} & 0 & "12" & 12 & "red" \\ X_{13} & 0 & "13" & 12 & "red" \\ X_{14} & 0 & "14" & 12 & "red" \\ X_{15} & 0 & "15" & 12 & "red" \\ X_{16} & 0 & "16" & 12 & "red" \\ X_{17} & 0 & "17" & 12 & "red" \\ X_{18} & 0 & "18" & 12 & "red" \\ X_{19} & 0 & "19" & 12 & "red" \\ X_{20} & 0 & "20" & 12 & "red" \end{pmatrix}$$

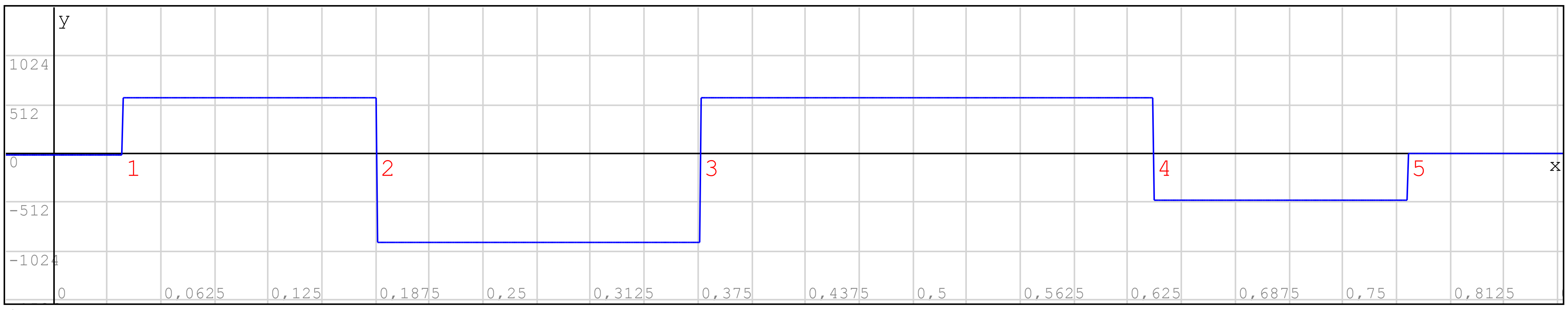
```
for i ∈ 1 .. dim - 2
  for j ∈ 1 .. 5
    MARKERS_{i j} := MARKERS_ALL_{i j}
```

$$MARKERS = \begin{pmatrix} 0,0402 & 0 & "1" & 12 & "red" \\ 0,1882 & 0 & "2" & 12 & "red" \\ 0,3762 & 0 & "3" & 12 & "red" \\ 0,6398 & 0 & "4" & 12 & "red" \\ 0,7878 & 0 & "5" & 12 & "red" \end{pmatrix}$$

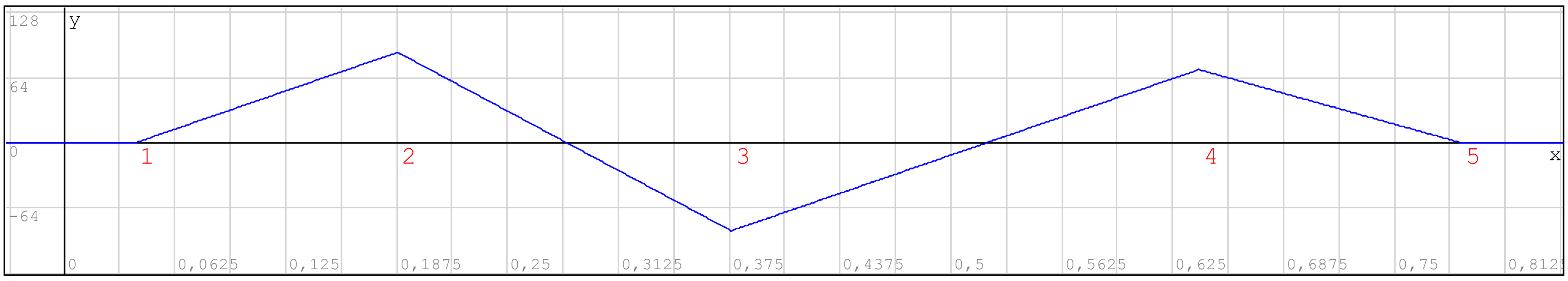
```
REACTIONS := | for i ∈ 1 .. dim - 2
             |   | REACTIONS_{i 1} := i
             |   | REACTIONS_{i 2} := R_i
             | REACTIONS
```

$$\text{REACTIONS}=\begin{pmatrix}1 & 594,1429\\ 2 & -1516,0498\\ 3 & 1515,6093\\ 4 & -1074,2062\\ 5 & 480,5039\end{pmatrix}$$

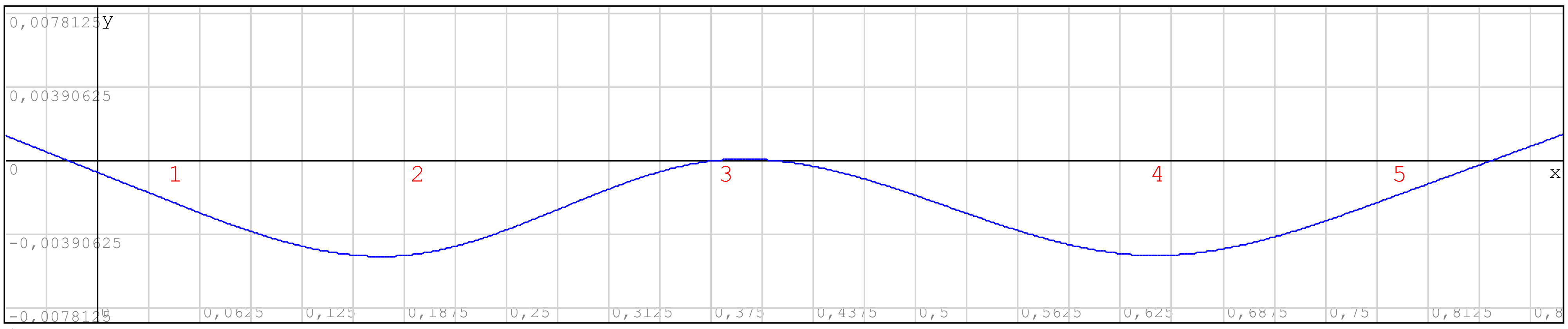
zooming instructions (graphs are not drawn for more than 9 nodes!):
* on X-axis only: "shift" key + scroll wheel
* on Y-axis only: "control" key + scroll wheel



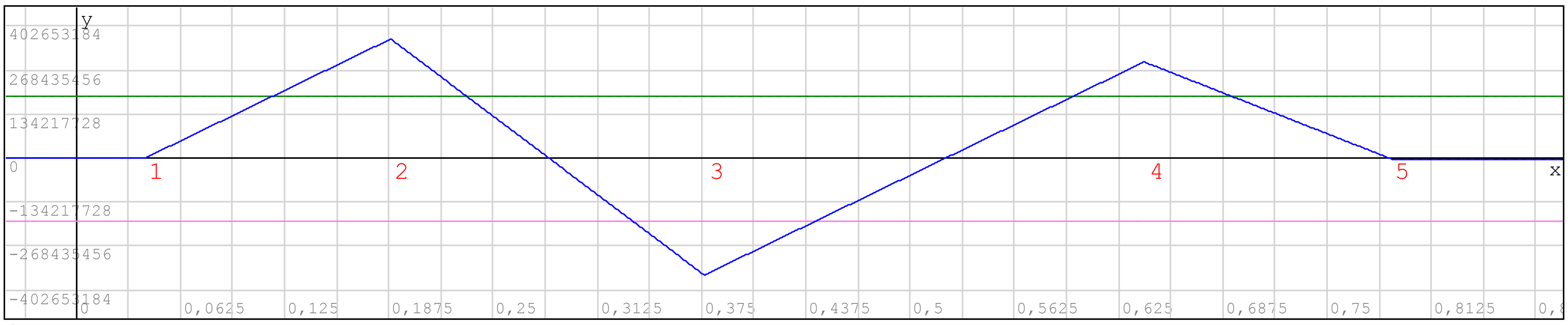
$\begin{cases} V(x) \\ \text{MARKERS} \end{cases}$



$\begin{cases} M(x) \\ \text{MARKERS} \end{cases}$



$\begin{cases} y(x) \\ \text{MARKERS} \end{cases}$



$\begin{cases} \sigma_{max}(x) \\ \text{MARKERS} \\ \sigma_{adm} \\ -\sigma_{adm} \end{cases}$