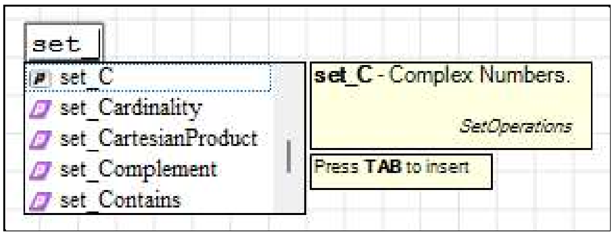
Set Operations plugin

SMath Studio "1.1.8763"

INTRODUCTION

All variables and functions have a **set_** prefix



A dedicated toolbox is available on the right hand side of the canvas

SETS

roster set a list of elements

$A:=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ $B:=\begin{bmatrix}5\\ 2\\ -23\end{bmatrix}$ $C:=\begin{bmatrix}3 & 0 & 1 & -1\end{bmatrix}$ $D:=\begin{bmatrix}3 & 2\end{bmatrix}$ $E:=\begin{bmatrix}3 & 2 & 4 & 1\end{bmatrix}$

NOTE:
- any matrix can be used as a roster set;
- duplicate entries are allowed as input; however they are counted as single items in set operations;
- anything can be an element of a roster set (numbers, strings, variables, matrices, ...)

empty set a set without members $\mathrm{matrix}(0;\ 0)=\mathrm{mat}(0;\ 0)$

set-builder set definition by predicate $\left\{ variables \mid condition\_1 ;\ condition\_2 ;\ \blacksquare ;\ condition\_n \right\}$

the set-builder will evaluate itself if:
- the first argument contains the membership operator and a roster set is given
- the variable given is already defined and is a roster set;
- set_Universe is defined and is a roster set;

$$\left\{ x\in A \mid x>1 ;\ x\le 3\right\}=\begin{bmatrix}2 & 3\end{bmatrix}$$

$$\left\{ \begin{bmatrix}x & y\end{bmatrix}\in\begin{bmatrix}\begin{bmatrix}10 & -5\end{bmatrix}\\ \begin{bmatrix}-10 & 5\end{bmatrix}\\ \begin{bmatrix}4 & 2\end{bmatrix}\\ \begin{bmatrix}-4 & -2\end{bmatrix}\end{bmatrix} \middle| x>-5 ;\ y<0\right\}=\begin{bmatrix}\begin{bmatrix}-4 & -2\end{bmatrix} & \begin{bmatrix}10 & -5\end{bmatrix}\end{bmatrix}$$

$z:=\begin{bmatrix}-5..5\end{bmatrix}$ $P(x):=|x|>4$

$$\left\{ z \mid P(z)\right\}=\begin{bmatrix}-5 & 5\end{bmatrix}$$

otherwise the set-builder won't evalute unless it is used in set membership/operations/subset functions

$$\left\{ x \mid x>1 ;\ x\le 4\right\}=\left\{ x \mid x>1 ;\ x\le 4\right\}$$

$$\pi\in\left\{ x \mid x>1 ;\ x\le 4\right\}=1$$

$$5\in\left\{ x \mid x>1 ;\ x\le 4\right\}=0$$

universe set current universe `set_Universe`

this set is required to be defined to evalute the set_Complement(1) function

$P(x) = |x| > 4$

$Q(x) := x > 3$

QUANTIFIERS

for all	$\forall \blacksquare \blacksquare$	$\forall z\ P(z) = 0$	$\forall x \in A\ Q(x) = 0$
there exists	$\exists \blacksquare \blacksquare$	$\exists z\ P(z) = 1$	$\exists x \in A\ Q(x) = 1$
does not exist	$\nexists \blacksquare \blacksquare$	$\nexists z\ P(z) = 0$	$\nexists x \in A\ Q(x) = 0$
there exists one and only one	$\exists! \blacksquare \blacksquare$	$\exists! z\ P(z) = 0$	$\exists! x \in A\ Q(x) = 1$

NOTE: the enumeration of set elements works in the same way as the set-builder function

MEMBERSHIP

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$B = \begin{bmatrix} 5 \\ 2 \\ -23 \end{bmatrix}$

$D = \begin{bmatrix} 3 & 2 \end{bmatrix}$

$E = \begin{bmatrix} 3 & 2 & 4 & 1 \end{bmatrix}$

element of set	$\blacksquare \in \blacksquare$	$3 \in A = 1$	$3 \in B = 0$
set contains an element	$\blacksquare \ni \blacksquare$	$A \ni 3 = 1$	$B \ni 3 = 0$
not an element of set	$\blacksquare \notin \blacksquare$	$3 \notin A = 0$	$3 \notin B = 1$
set doesn't contains an element	$\blacksquare \not\ni \blacksquare$	$A \not\ni 3 = 0$	$B \not\ni 3 = 1$

SUBSETS

subset	$\blacksquare \subseteq \blacksquare$	$D \subseteq A = 1$	$A \subseteq E = 1$	$A \subseteq B = 0$
superset	$\blacksquare \supseteq \blacksquare$	$D \supseteq A = 0$	$A \supseteq E = 1$	$A \supseteq B = 0$
proper subset	$\blacksquare \subsetneq \blacksquare$	$D \subsetneq A = 1$	$A \subsetneq E = 0$	$A \subsetneq B = 0$
proper superset	$\blacksquare \supsetneq \blacksquare$	$D \supsetneq A = 0$	$A \supsetneq E = 0$	$A \supsetneq B = 0$
not subset	$\blacksquare \nsubseteq \blacksquare$	$D \nsubseteq A = 0$	$A \nsubseteq E = 0$	$A \nsubseteq B = 1$
not superset	$\blacksquare \nsupseteq \blacksquare$	$D \nsupseteq A = 1$	$A \nsupseteq E = 0$	$A \nsupseteq B = 1$

OPERATIONS

union	$\blacksquare \cup \blacksquare$	$\blacksquare \cup \blacksquare \cup \blacksquare$	$A \cup B = \begin{bmatrix} -23 & 1 & 2 & 3 & 4 & 5 \end{bmatrix}$
intersection	$\blacksquare \cap \blacksquare$	$\blacksquare \cap \blacksquare \cap \blacksquare$	$A \cap B = \begin{bmatrix} 2 \end{bmatrix}$
difference	$\blacksquare \setminus \blacksquare$	$\blacksquare \setminus \blacksquare \setminus \blacksquare$	$A \setminus B = \begin{bmatrix} 1 & 3 & 4 \end{bmatrix}$
symmetric difference	$\blacksquare \Delta \blacksquare$	$\blacksquare \Delta \blacksquare \Delta \blacksquare$	$A \Delta B = \begin{bmatrix} -23 & 1 & 3 & 4 & 5 \end{bmatrix}$
cartesian product	$\blacksquare \times \blacksquare$	$\blacksquare \times \blacksquare \times \blacksquare$	$B \times D = \begin{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} & \begin{bmatrix} 5 \\ 2 \end{bmatrix} & \begin{bmatrix} 2 \\ 3 \end{bmatrix} & \begin{bmatrix} 2 \\ 2 \end{bmatrix} & \begin{bmatrix} -23 \\ 3 \end{bmatrix} & \begin{bmatrix} -23 \\ 2 \end{bmatrix} \end{bmatrix}$
power set	$\mathcal{P}(\blacksquare)$		$\mathcal{P}(D) = \begin{bmatrix} \mathrm{mat}(0;0) & \begin{bmatrix} 3 \end{bmatrix} & \begin{bmatrix} 2 \end{bmatrix} & \begin{bmatrix} 3 & 2 \end{bmatrix} \end{bmatrix}$

set cardinality $|\blacksquare|$ $|A|=4$ $\left|\left[\begin{matrix}2 & a\\ \sqrt{2} & \sqrt{4}\end{matrix}\right]\right|=3$

complement $\blacksquare^{C}$ $A^{C}=\blacksquare$

lastError = "Set 'set_Universe' is not defined."

$set\_Universe:=\left[\begin{matrix}5 & 3 & 2\\ 4 & 1 & 0\end{matrix}\right]$ $A^{C}=\left[\begin{matrix}0 & 5\end{matrix}\right]$

NUMBER SETS

Natural numbers set of natural numbers **set_N**

$2\in$ **set_N** $=1$	$-2\in$ **set_N** $=0$	$\pi\in$ **set_N** $=0$	$\frac{5}{3}\in$ **set_N** $=0$
$2+3\cdot i\in$ **set_N** $=0$	$-5\cdot i\in$ **set_N** $=0$	$\infty\in$ **set_N** $=0$	$0\in$ **set_N** $=0$

Integrers set of integers **set_Z**

$2\in$ **set_Z** $=1$	$-2\in$ **set_Z** $=1$	$\pi\in$ **set_Z** $=0$	$\frac{5}{3}\in$ **set_Z** $=0$
$2+3\cdot i\in$ **set_Z** $=0$	$-5\cdot i\in$ **set_Z** $=0$	$\infty\in$ **set_Z** $=0$	$0\in$ **set_Z** $=1$

Rational numbers set of rational numbers **set_Q**

$2\in$ **set_Q** $=1$	$-2\in$ **set_Q** $=1$	$\pi\in$ **set_Q** $=0$	$\frac{5}{3}\in$ **set_Q** $=1$
$2+3\cdot i\in$ **set_Q** $=0$	$-5\cdot i\in$ **set_Q** $=0$	$\infty\in$ **set_Q** $=0$	$0\in$ **set_Q** $=1$

Real numbers set of real numbers **set_R**

$2\in$ **set_R** $=1$	$-2\in$ **set_R** $=1$	$\pi\in$ **set_R** $=1$	$\frac{5}{3}\in$ **set_R** $=1$
$2+3\cdot i\in$ **set_R** $=0$	$-5\cdot i\in$ **set_R** $=0$	$\infty\in$ **set_R** $=0$	$0\in$ **set_R** $=1$

Complex numbers set of complex numbers **set_C**

$2\in$ **set_C** $=1$	$-2\in$ **set_C** $=1$	$\pi\in$ **set_C** $=1$	$\frac{5}{3}\in$ **set_C** $=1$
$2+3\cdot i\in$ **set_C** $=1$	$-5\cdot i\in$ **set_C** $=1$	$\infty\in$ **set_C** $=0$	$0\in$ **set_C** $=1$

Imaginary numbers set of imaginary numbers **set_I**

$2\in$ **set_I** $=0$	$-2\in$ **set_I** $=0$	$\pi\in$ **set_I** $=0$	$\frac{5}{3}\in$ **set_I** $=0$
$2+3\cdot i\in$ **set_I** $=0$	$-5\cdot i\in$ **set_I** $=1$	$\infty\in$ **set_I** $=0$	$0\in$ **set_I** $=1$

Whole numbers set of whole numbers **set_W**

$2\in$ **set_W** $=1$	$-2\in$ **set_W** $=0$	$\pi\in$ **set_W** $=0$	$\frac{5}{3}\in$ **set_W** $=0$
$2+3\cdot i\in$ **set_W** $=0$	$-5\cdot i\in$ **set_W** $=0$	$\infty\in$ **set_W** $=0$	$0\in$ **set_W** $=1$

Prime numbers set of prime numbers **set_P**

$2 \in \mathbf{set\_P} = 1$ $-2 \in \mathbf{set\_P} = 0$ $\pi \in \mathbf{set\_P} = 0$ $\frac{5}{3} \in \mathbf{set\_P} = 0$

$2 + 3 \cdot i \in \mathbf{set\_P} = 0$ $-5 \cdot i \in \mathbf{set\_P} = 0$ $\infty \in \mathbf{set\_P} = 0$ $0 \in \mathbf{set\_P} = 0$

$393919 \in \mathbf{set\_P} = 1$ $999999000001 \in \mathbf{set\_P} = 1$ $999999000003 \in \mathbf{set\_P} = 0$

Irrational numbers set of irrational numbers **set_J**

$2 \in \mathbf{set\_J} = 0$ $-2 \in \mathbf{set\_J} = 0$ $\pi \in \mathbf{set\_J} = 1$ $\frac{5}{3} \in \mathbf{set\_J} = 0$

$2 + 3 \cdot i \in \mathbf{set\_J} = 0$ $-5 \cdot i \in \mathbf{set\_J} = 0$ $\infty \in \mathbf{set\_J} = 0$ $0 \in \mathbf{set\_J} = 0$

COMBINATORICS

number of expected results: $C(n; k) := \frac{n!}{(k!) \cdot ((n-k)!)}$

Choose

set_Choose([a b c]) = [mat(0; 0) [a] [b] [c] [a b] [a c] [b c] [a b c]]

set_Choose([a b c]; 0) = [mat(0; 0)] $C(3; 0) = 1$

set_Choose([a b c]; 1) = [[a] [b] [c]] $C(3; 1) = 3$

set_Choose([a b c]; 2) = [[a b] [a c] [b c]] $C(3; 2) = 3$

set_Choose([a b c]; 3) = [[a b c]] $C(3; 3) = 1$

number of expected results: n!

Permute

set_Permute([1..1]) = [[1]] $1! = 1$

set_Permute([1..2]) = [[1 2] [2 1]] $2! = 2$

set_Permute([1..3]) = [[1 2 3] [2 1 3] [3 1 2] [1 3 2] [2 3 1] [3 2 1]] $3! = 6$

P := set_Permute([1..4]) = [[1 2 3 4] [2 1 3 4] [3 1 2 4] [1 3 2 4] ...] $4! = 24$

length(P) = 24

number of expected results: $\frac{n!}{(n-k)!}$

set_Permute([1..3]; 2) = [[1 2] [2 1] [1 3] [3 1] [2 3] [3 2]] $\frac{3!}{(3-2)!} = 6$

P := set_Permute([1..5]; 3) = [[1 2 3] [2 1 3] [3 1 2] ...] $\frac{5!}{(5-3)!} = 60$

length(P) = 60

TOOLS

Remove duplicates $\text{set\_Unique}\left(\left[\begin{matrix}-1 & 2\\ 3 & -2\cdot 0.5\end{matrix}\right]\right)=\left[\begin{matrix}-1 & 2 & 3\end{matrix}\right]$

NOTE: a single level comparision is performed; this means that matrices like $\left[\begin{matrix}1\\ 2\end{matrix}\right]$ and $\left[\begin{matrix}2\\ 1\end{matrix}\right]$ are considered different items

Sort elements $\text{set\_Sort}\left(\left[\begin{matrix}-1 & 2\\ 3 & -7\end{matrix}\right]\right)=\left[\begin{matrix}-7 & -1 & 2 & 3\end{matrix}\right]$

Shuffle elements $\text{set\_Shuffle}\left(\left[\begin{matrix}-1 & 2\\ 3 & -7\end{matrix}\right]\right)=\left[\begin{matrix}-1 & -7 & 2 & 3\end{matrix}\right]$ *press F9 to shuffle*

$\text{set\_Shuffle}\left(\left[\begin{matrix}1 & 2 & 3 & 4 & 5\end{matrix}\right]; 42\right)=\left[\begin{matrix}3 & 2 & 5 & 1 & 4\end{matrix}\right]$ *deterministic shuffle*

$\text{set\_Shuffle}\left(\left[\begin{matrix}j & k & l & m & n\end{matrix}\right]; 42\right)=\left[\begin{matrix}l & k & n & j & m\end{matrix}\right]$

$A:=\left[-3..3\right]=\left[\begin{matrix}-3\\ -2\\ -1\\ 0\\ 1\\ 2\\ 3\end{matrix}\right]$

$B:=\text{set\_Shuffle}(A)=\left[\begin{matrix}0 & 1 & 2 & -2 & 3 & -1 & -3\end{matrix}\right]$

$C:=\text{set\_Sort}(B)=\left[\begin{matrix}-3 & -2 & -1 & 0 & 1 & 2 & 3\end{matrix}\right]$

SETTINGS

$set\_Settings\_Orientation:=\text{"horizontal"}$ alternatively: "h", "r", "row"

$A\cup B=\left[\begin{matrix}-3 & -2 & -1 & 0 & 1 & 2 & 3\end{matrix}\right]$

$set\_Settings\_Orientation:=\text{"vertical"}$ alternatively: "v" , "c", "column"

$A\cup B=\left[\begin{matrix}-3\\ -2\\ -1\\ 0\\ 1\\ 2\\ 3\end{matrix}\right]$

$\text{Clear}(\textbf{set\_Settings\_Orientation})=1$ $A\cup B=\left[\begin{matrix}-3 & -2 & -1 & 0 & 1 & 2 & 3\end{matrix}\right]$