V. Ochkov rigid catenary example. Reference: http://twt.mpei.ac.ru/MCS/Worksheets/chain.xmcd

Values $\left[x_A \; y_A \right] := \left[0 \; 7 \right] \text{m}$ $\left[x_B \; y_B \right] := \left[30 \; 15 \right] \text{m}$ $L := 37 \text{ m}$ $wo := 0.5 \frac{\text{kgf}}{\text{m}}$

Method 1

Knowing that the shape is a catenary

$$f(x) := a \cdot \cosh\left(\frac{x - xo}{a}\right) + yo$$

$$f'(x) := \frac{\mathrm{d}}{\mathrm{d}x} f(x)$$

$$eq(u) := \left| \begin{array}{l} \left[a \; xo \; yo \right] := u^{\mathrm{T}} \text{ m} \\ \frac{1}{\text{m}} \cdot \begin{bmatrix} f(x_A) - y_A \\ f(x_B) - y_B \\ L - \int_0^{x_B} \sqrt{1 + f'(x)^2} \, \mathrm{d}x \end{bmatrix} \end{array} \right.$$

$$\begin{bmatrix} a \\ xo \\ yo \end{bmatrix} := \text{al\_nleqsolve}\left(\begin{bmatrix} 10 \\ 10 \\ -10 \end{bmatrix}, eq \right) \text{m} = \begin{bmatrix} 13.95 \\ 11.94 \\ -12.37 \end{bmatrix} \text{m}$$

Hor. Force $F_H := a \cdot wo = 6.97 \text{ kgf}$

Angles $\theta_A := \text{atan}\left(f'(x_A)\right) = -43.95 \text{ deg}$ $\theta_B := \text{atan}\left(f'(x_B)\right) = 59.36 \text{ deg}$

Tensions $T_A := \frac{F_H}{\cos(\theta_A)} = 9.6875 \text{ kgf}$ $T_B := \frac{F_H}{\cos(\theta_B)} = 13.6875 \text{ kgf}$

Minimun $x_C := \text{roots}\left(f'(x \text{ m}), x\right) \text{m} = 11.9355 \text{ m}$ $y_C := f(x_C) = 1.5748 \text{ m}$

Method 2 Solving the ODE. All of the parameters are extracting from the numerical solution.

$\text{Clear}(F_H, \theta_A) = 1$

$$\begin{bmatrix} y''(x) = \frac{wo}{F_H} \cdot \sqrt{1 + y'(x)^2} \\ y(x_A) = y_A \qquad y'(x_A) = \tan(\theta_A) \end{bmatrix}$$

$$eq(u) := \left| \begin{array}{l} \left[N := 301 \quad RK := RK\left(u_1, u_2, N - 1\right) \right] \\ \left[X := \text{col}(RK, 1) \quad Y := \text{col}(RK, 2) \right] \\ \begin{bmatrix} Y_N \\ \sum_{k=2}^{N} \text{norme}\left(\begin{bmatrix} X_k - X_{k-1} \\ Y_k - Y_{k-1} \end{bmatrix} \right) \end{bmatrix} - \frac{1}{\text{m}} \cdot \begin{bmatrix} y_B \\ L \end{bmatrix} \end{array} \right.$$

$$\text{RK}(F_H, \theta_A, N) := \text{Rkadapt}\left(y(x), x_B, N\right)$$

$$\begin{bmatrix} F_H \\ \theta_A \end{bmatrix} := \text{diag}\left(\begin{bmatrix} N \\ 1 \end{bmatrix} \right) \cdot \text{al\_nleqsolve}\left(\begin{bmatrix} 100 \\ -45 \text{ deg} \end{bmatrix}, eq \right)$$

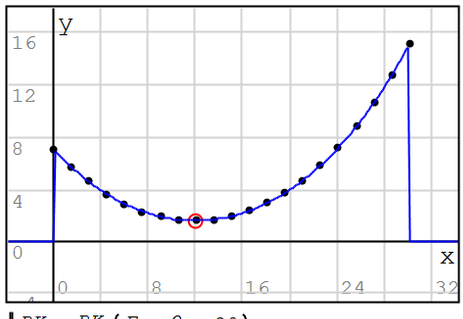
$RK := RK\left(F_H, \theta_A, 601\right)$ $\left[x_C \; y_C \right] := \left(\text{findrows}\left(RK, \min\left(\text{col}(RK, 2)\right), 2\right) \text{m} \right)_{1 \left[1 \; 2 \right]}$

$F_H = 6.9749 \text{ kgf}$

$\theta_A = -43.95 \text{ deg}$ $\theta_B := \text{atan}\left(RK_{\text{rows}(RK)\,3}\right) = 59.36 \text{ deg}$

$T_A := \frac{F_H}{\cos(\theta_A)} = 9.6875 \text{ kgf}$ $T_B := \frac{F_H}{\cos(\theta_B)} = 13.6875 \text{ kgf}$

$x_C = 11.9301 \text{ m}$ $y_C = 1.5748 \text{ m}$



$$\left| \begin{array}{l} RK := RK\left(F_H, \theta_A, 20\right) \\ f(x \text{ m}) \cdot \left(x_A \le x \text{ m} \le x_B\right) \\ \text{augment}\left(\text{col}(RK, 1), \text{col}(RK, 2), "."\right) \\ \left[x_C \; y_C \; "o" \; 12 \; "red" \right] \end{array} \right.$$

Alvaro $\text{appVersion}(4) = "1.2.9018.0"$