⊞—Include

⊟—Atwood

Ideal Atwood Machine.

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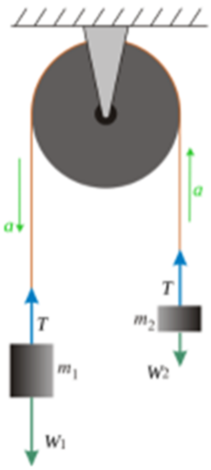

$$eq := \begin{cases} m_1 \cdot g - T = m_1 \cdot a & \text{equation for m.1} \\ T - m_2 \cdot g = m_2 \cdot a & \text{equation for m.2} \end{cases}$$

$$\begin{cases} T := isol\left(eq_1, T\right) \\ a := isol\left(eq_2, a\right) \end{cases}$$

$$T = \frac{2 \cdot m_1 \cdot m_2 \cdot g}{m_1 + m_2} \qquad a = \frac{g \cdot \left(m_1 - m_2\right)}{m_1 + m_2}$$

Numerical example

$$\begin{cases} m_1 := 4\ \text{kg} \\ m_2 := 12\ \text{kg} \\ g := g_e \end{cases} \qquad T = 6\ \text{kgf} \qquad a = -0.5\ g_e$$

Atwood Machine with pulley with inertia and friction.

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$$eq := \begin{cases} m_1 \cdot g - T_1 = m_1 \cdot a \\ T_2 - m_2 \cdot g = m_2 \cdot a \\ \left(T_1 - T_2\right) \cdot r - \tau_f = I \cdot \alpha & \text{equation for pulley} \\ a = \alpha \cdot r & \text{Non-sleep condition} \end{cases}$$

$$\begin{cases} T_1 := isol\left(eq_1, T_1\right) \\ T_2 := isol\left(eq_2, T_2\right) \\ \alpha := isol\left(eq_3, \alpha\right) \end{cases}$$

No solve for a yet

$$T_1 = -m_1 \cdot \left(-g + a\right) \qquad T_2 = m_2 \cdot \left(g + a\right)$$

$$\alpha = -\frac{\tau_f + \left(m_1 \cdot \left(-g + a\right) + m_2 \cdot \left(g + a\right)\right) \cdot r}{I} \qquad a := isol\left(eq_4, a\right) = -\frac{\left(\tau_f + g \cdot \left(-m_1 + m_2\right) \cdot r\right) \cdot r}{I + r^2 \cdot \left(m_1 + m_2\right)}$$

If $\begin{cases} \tau_f := 0 \\ I := 0 \end{cases}$ we recover the ideal case

$$T_1 = \frac{2 \cdot m_1 \cdot m_2 \cdot g}{m_1 + m_2} \qquad T_2 = \frac{2 \cdot m_2 \cdot m_1 \cdot g}{m_1 + m_2} \qquad a = -\frac{g \cdot \left(-m_1 + m_2\right)}{m_1 + m_2}$$

Numerical example

$$\begin{cases} m_1 := 4\ \text{kg} \\ m_2 := 12\ \text{kg} \\ g := g_e \end{cases} \qquad \begin{cases} r := 12\ \text{cm} & \text{Pulley radious} \\ M := 0.5\ \text{kg} & \text{mass} \\ I := 0.5 \cdot M \cdot r^2 & \text{Inertia} \\ \tau_f := 0.1\ \text{N m} & \text{frictional torque} \end{cases}$$

$$T_1 = 5.9901\ \text{kgf} \qquad T_2 = 6.0296\ \text{kgf} \qquad a = -0.4975\ g_e$$

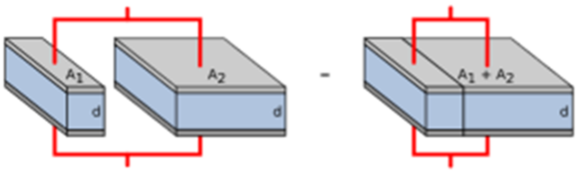
Almost the same values for the ideal case, for τ.f small.

⊟—Capacitor

Parallel Capacitors

Cle

$$eq := \begin{cases} Q = q_1 + q_2 \\ q_1 = C_1 \cdot V \\ q_2 = C_2 \cdot V \\ Q = V \cdot C_{eq} \end{cases} \qquad \begin{cases} q_1 := isol\left(eq_1, q_1\right) \\ q_2 := isol\left(eq_2, q_2\right) \\ V := isol\left(eq_3, V\right) \\ C_{eq} := isol\left(eq_4, C_{eq}\right) \end{cases}$$



$$q_1 = \frac{C_1 \cdot Q}{C_1 + C_2} \qquad q_2 = \frac{Q \cdot C_2}{C_1 + C_2} \qquad V = \frac{Q}{C_1 + C_2} \qquad \boxed{C_{eq} = C_2 + C_1}$$

Sereies Capacitors

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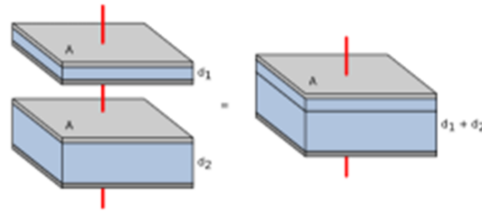
$$eq := \begin{cases} Q = V_1 \cdot C_1 \\ Q = V_2 \cdot C_2 \\ V = V_1 + V_2 \\ Q = V \cdot C_{eq} \end{cases} \qquad \begin{cases} Q := isol\left(eq_1, Q\right) \\ V_1 := isol\left(eq_2, V_1\right) \\ V_2 := isol\left(eq_3, V_2\right) \\ C_{eq} := isol\left(eq_4, C_{eq}\right) \end{cases}$$



$$V_1 = \frac{V \cdot C_2}{C_1 + C_2} \qquad V_2 = \frac{V \cdot C_1}{C_1 + C_2} \qquad Q = \frac{V \cdot C_1 \cdot C_2}{C_1 + C_2} \qquad \boxed{C_{eq} = \frac{C_1 \cdot C_2}{C_1 + C_2}}$$

Alvaro