Pipe Network Analysis by HARDY-CROSS Method

Following three methods of solution used.

1. Successive Steps of Corrections (3 steps : long procedure).
2. With assumed number of Iterations (1 step : better and quick).
3. While Loop (1 step : better)

appVersion(4) = "0.99.7691.4821"

$t_{start} := \text{time}(0)$

Example 2:

Calculate the head losses and the corrected flows in the various pipes of a distributed network shown in fig. 20.6. The diameters and the lengths of the pipes used are given against each pipe. Make se of Hardy-cross method with Hazen-William's formula. Compute the corrected flows after two corrections.

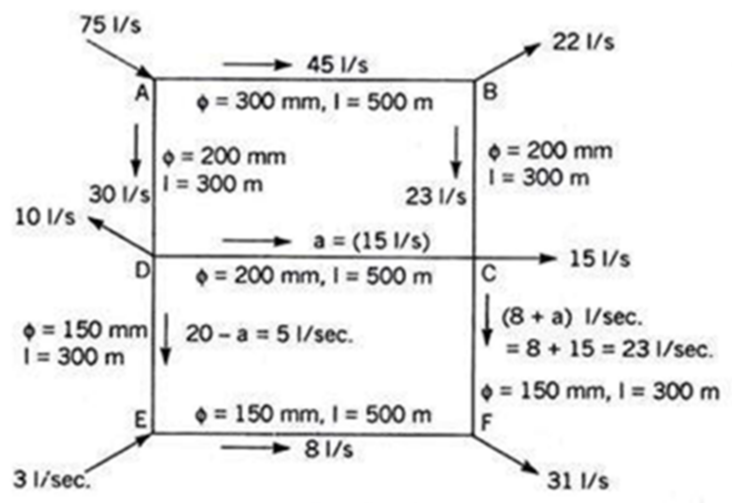


Fig. 20.7

Solution:

First of all the magnitudes as well as the directions of the possible flows in each pipe are assumed keeping in consideration the law of continuity at each junction (i.e., input equals the output at each junction). These assumed flows aare shown in fig. 20.7. The twon closed loops ie. ABCDA and DCFED are then analysed by Hardy-Cross method.

The analysis of the pipe loops will require computation of head loss (H2) in each pipe, which is to be computed by Hazen's William's formula ~~as below:~~

Flow into Node:	+ve
Flow out of Node:	-ve

Flows at Nodes

$$QJ := \begin{bmatrix} 75 \\ -22 \\ -15 \\ -10 \\ -31 \\ 3 \end{bmatrix} \frac{L}{s}$$

Lengths LOOP 1

$$L1 := \begin{bmatrix} 500 \\ 300 \\ 500 \\ 300 \end{bmatrix} m$$

Dia. LOOP 1

$$D1 := \begin{bmatrix} 300 \\ 200 \\ 200 \\ 200 \end{bmatrix} mm$$

Lengths LOOP 2

$$L2 := \begin{bmatrix} 500 \\ 300 \\ 500 \\ 300 \end{bmatrix} m$$

Dia. LOOP 2

$$D2 := \begin{bmatrix} 200 \\ 150 \\ 150 \\ 150 \end{bmatrix} mm$$

Continuity of the whole network.
Sum of Nodal flows should be zero

$$\sum QJ = 0$$

Flows in LOOPS should be balanced first to start with. Otherwise, will yield wrong answers

For Darcy-Weisbach: n = 2

Assumed Flows for Continuity in LOOP 1

$$Q1 := \begin{bmatrix} 45 \\ 23 \\ -15 \\ -30 \end{bmatrix} \frac{L}{s}$$

Assumed Flows for Continuity in LOOP 2

$$Q2 := \begin{bmatrix} 15 \\ 23 \\ -8 \\ -5 \end{bmatrix} \frac{L}{s}$$

Friction Factor

$f := 0.02$

Hazen William 'n'

$n := 1.85$

Flows in LOOPS Sign Convention
1. Clockwise: +ve
2. Anticlockwise: -ve

Following 3 Methods will be used in this excercise, using Hardy Cross Method

1. **Successive Steps – Pipe flows assumed to balance at each node. 3 steps in this example.**
2. **One step for a defined number of Iterations. More Iterations, better accuracy. 5 Iterations in this example.**
3. **One step using a "While Loop" with a pre-defined degree of accuracy. Flow accuracy 0.001 L/s.**

METHOD 1: Hardy Cross Method using Successive Steps of Corrections

PROGRAM 1 starts with assumed flow for both LOOPS

Program 1: Using Vectorize Function.
Find 'Corrected Flow' for both LOOPS

$$Calc\_\Delta Q\#(Q\#, L\#, D\#) := \left| \begin{array}{l} k\# := \overrightarrow{\dfrac{8 \cdot f \cdot L\#}{g_e \cdot (D\#)^5 \cdot \pi^2}} \\ HL\# := \overrightarrow{k\# \cdot Q\# \cdot (|Q\#|)^{n-1}} \\ \delta HL\# := \left| \overrightarrow{\dfrac{HL\#}{Q\#}} \right| \\ \Delta Q\# := -\dfrac{\sum HL\#}{n \cdot \left(\sum \delta HL\#\right)} \end{array} \right.$$

<---------- K values for all pipes

<---------- Head Loss for all pipes

<---------- Net Head Loss

<---------- Corrections for both LOOPS .
To be minimized with successive steps

1st correction (Starts with assumed flows in both LOOPS)

Apply PROGRAM 1 to get 1st correction

$$\Delta Q_{cor1} := \overrightarrow{Calc\_\Delta Q\#\left(\begin{bmatrix} Q1 \\ Q2 \end{bmatrix}, \begin{bmatrix} L1 \\ L2 \end{bmatrix}, \begin{bmatrix} D1 \\ D2 \end{bmatrix}\right)} = \begin{bmatrix} 2.06 \\ -4.93 \end{bmatrix} \frac{L}{s}$$

$$\Delta Q_{cor1_1} = 2.06 \frac{L}{s} \qquad \Delta Q_{cor1_2} = -4.93 \frac{L}{s}$$

1st Corrected flows in LOOP 1

$$Q1_{cor1} := Q1 + \Delta Q_{cor1_1}$$

1st Corrected flows in LOOP 2

$$Q2_{cor1} := Q2 + \Delta Q_{cor1_2} = \begin{bmatrix} 10.07 \\ 18.07 \\ -12.93 \\ -9.93 \end{bmatrix} \frac{L}{s}$$

Now Apply 1st Correction for Pipe CD common to both loops

Adjusted flow for Pipe CD in LOOP 1

$$\Delta CD1 := Q1_{cor1_3} - \Delta Q_{cor1_2} = -8 \frac{L}{s}$$

$$Q1_{cor1_3} := \Delta CD1$$

$$Q1_{cor1_3} = -0.008 \frac{m^3}{s} \qquad Q1_{cor1} = \begin{bmatrix} 47.06 \\ 25.06 \\ -8 \\ -27.94 \end{bmatrix} \frac{L}{s}$$

Adjusted flow for Pipe DC in LOOP 2

$$Q2_{cor1_1} := -\Delta CD1$$

$$Q2_{cor1} = \begin{bmatrix} 8 \\ 18.1 \\ -12.9 \\ -9.9 \end{bmatrix} \frac{L}{s}$$

2nd correction (Starts with flows in correction 1)

Apply PROGRAM 1 to get 2nd correction

$$\Delta Q_{cor2} := \overrightarrow{Calc\_\Delta Q\#\left(\begin{bmatrix} Q1_{cor1} \\ Q2_{cor1} \end{bmatrix}, \begin{bmatrix} L1 \\ L2 \end{bmatrix}, \begin{bmatrix} D1 \\ D2 \end{bmatrix}\right)} = \begin{bmatrix} -1.22 \\ 0.45 \end{bmatrix} \frac{L}{s}$$

$$\Delta Q_{cor2_1} = -1.22 \frac{L}{s} \qquad \Delta Q_{cor2_2} = 0.45 \frac{L}{s}$$

2nd Corrected flows in LOOP 1

$$Q1_{cor2} := Q1_{cor1} + \Delta Q_{cor2_1} = \begin{bmatrix} 45.8 \\ 23.8 \\ -9.2 \\ -29.2 \end{bmatrix} \frac{L}{s}$$

2nd Corrected flows in LOOP 2

$$Q2_{cor2} := Q2_{cor1} + \Delta Q_{cor2_2} = \begin{bmatrix} 8.5 \\ 18.5 \\ -12.5 \\ -9.5 \end{bmatrix} \frac{L}{s}$$

Now Apply 2nd Correction for Pipe CD common to both loops

Adjusted flow for Pipe CD in LOOP 1

$\Delta CD2 := Q1_{cor2_3} - \Delta Q_{cor2_2} = -9.68\ \frac{L}{s}$

$Q1_{cor2_3} := \Delta CD2$

$Q1_{cor2_3} = -9.7\ \frac{L}{s}$

Adjusted flow for Pipe DC in LOOP 2

$Q2_{cor2_1} := -\Delta CD2$

$Q2_{cor2_1} = 9.7\ \frac{L}{s}$

Corrected flows in LOOP 1

$$Q1_{cor2} = \begin{bmatrix} 45.8 \\ 23.8 \\ -9.7 \\ -29.2 \end{bmatrix} \frac{L}{s}$$

Corrected flows in LOOP 2

$$Q2_{cor2} = \begin{bmatrix} 9.7 \\ 18.5 \\ -12.5 \\ -9.5 \end{bmatrix} \frac{L}{s}$$

3rd correction (Starts with flows in correction 2)

Apply PROGRAM 2 to get 3rd correction

$$\Delta Q_{cor3} := \overrightarrow{Calc\_\Delta Q\#\left(\begin{bmatrix} Q1_{cor2} \\ Q2_{cor2} \end{bmatrix}, \begin{bmatrix} L1 \\ L2 \end{bmatrix}, \begin{bmatrix} D1 \\ D2 \end{bmatrix}\right)} = \begin{bmatrix} 0.12 \\ -0.09 \end{bmatrix} \frac{L}{s}$$

$\Delta Q_{cor3_1} = 0.12\ \frac{L}{s}$

$\Delta Q_{cor3_2} = -0.09\ \frac{L}{s}$

3rd Corrected flows in LOOP 1

$$Q1_{cor3} := Q1_{cor2} + \Delta Q_{cor3_1} = \begin{bmatrix} 46 \\ 24 \\ -9.6 \\ -29 \end{bmatrix} \frac{L}{s}$$

$Q1_{cor3_3} = -9.56\ \frac{L}{s}$

3rd Corrected flows in LOOP 2

$$Q2_{cor3} := Q2_{cor2} + \Delta Q_{cor3_2} = \begin{bmatrix} 9.6 \\ 18.4 \\ -12.6 \\ -9.6 \end{bmatrix} \frac{L}{s}$$

$Q2_{cor3_1} = 9.59\ \frac{L}{s}$

Now Apply 3rd Correction for Pipe CD common to both loops

Adjusted flow for Pipe CD in LOOP 1

$\Delta CD3 := Q1_{cor3_3} - \Delta Q_{cor3_2} = -9.47\ \frac{L}{s}$

$Q1_{cor3_3} := \Delta CD3$

$Q1_{cor3_3} = -9.47\ \frac{L}{s}$

Adjusted flow for Pipe DC in LOOP 2

$Q2_{cor3_1} := -\Delta CD3$

$Q2_{cor3_1} = 9.47\ \frac{L}{s}$

OK

Corrected flows in LOOP 1

$$Q1_{cor3} = \begin{bmatrix} 45.96 \\ 23.96 \\ -9.47 \\ -29.04 \end{bmatrix} \frac{L}{s}$$

Diff of flow in CD/DC in LOOP1 and LOOP 2

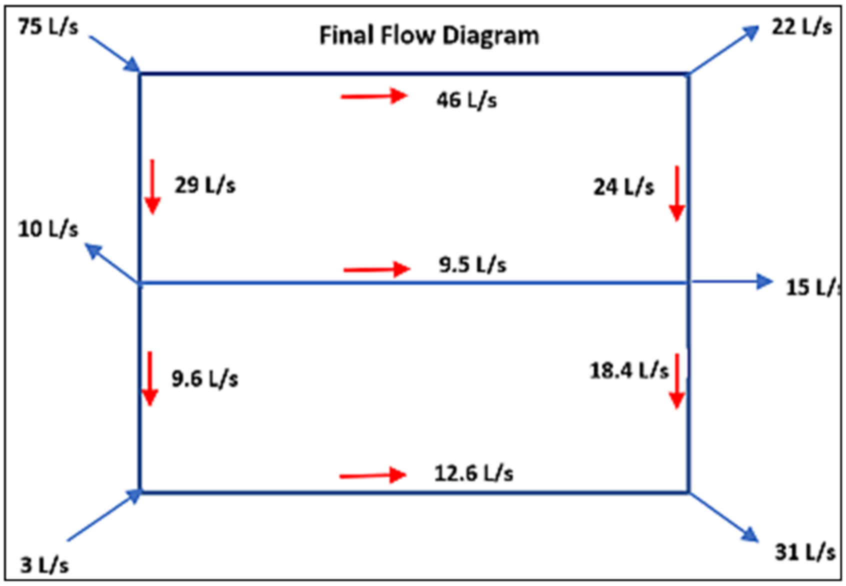
$$\left| Q1_{cor3_3} + Q2_{cor3_1} \right| = 0\ \frac{L}{s}$$

Corrected flows in LOOP 2

$$Q2_{cor3} = \begin{bmatrix} 9.47 \\ 18.43 \\ -12.57 \\ -9.57 \end{bmatrix} \frac{L}{s}$$

Calculated Flows - *After 3 Corrections*

From Book Example - *After 2 Corrections*



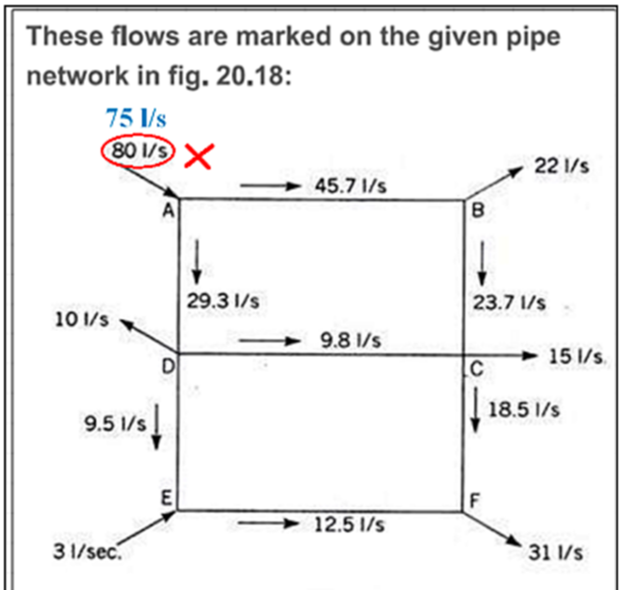


Fig. 20.8

METHOD 2: Hardy Cross Method: For Any Given Number of Iterations

Index of common pipe in LOOP 1

$n1 := 3$

Index of common pipe in LOOP 2

$n2 := 1$

PROGRAM 2: Calculates 'Corrected Flows' & 'Corrections' CALLS PROGRAM 1

$$Calc\_All\_\Delta(q1, q2) := \left| \begin{array}{l} \Delta q1 := \overrightarrow{Calc\_\Delta Q\#\left(\begin{bmatrix} q1 \\ q2 \end{bmatrix}, \begin{bmatrix} L1 \\ L2 \end{bmatrix}, \begin{bmatrix} D1 \\ D2 \end{bmatrix}\right)} \\ q1_{cor} := q1 + \Delta q1_1 \\ q2_{cor} := q2 + \Delta q1_2 \\ \Delta cd1 := q1_{cor_{n1}} - \Delta q1_2 \\ q1_{cor_{n1}} := \Delta cd1 \\ q2_{cor_{n2}} := -\Delta cd1 \\ \left[\begin{bmatrix} q1_{cor} \\ q2_{cor} \end{bmatrix} \begin{bmatrix} \Delta q1_1 \\ \Delta q1_2 \end{bmatrix}\right] \end{array} \right.$$

1st Corrections example

$C1 := Calc\_All\_\Delta(Q1, Q2)$

$$C1 = \left[\begin{bmatrix} \begin{bmatrix} 47.06 \\ 25.06 \\ -8 \\ -27.94 \end{bmatrix} \\ \begin{bmatrix} 8 \\ 18.07 \\ -12.93 \\ -9.93 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 2.06 \\ -4.93 \end{bmatrix} \right] \frac{L}{s}$$

Solution by Repeated Iteration Method using PROGRAM 2

PROGRAM 3: Repeated Iteration Method. CALLS PROGRAM 2

$$Q(q1, q2, iter) := \left| \begin{array}{l} \text{for } j \in [1..iter] \\ \quad \left| \begin{array}{l} \text{if } j = 1 \\ \quad P_j := Calc\_All\_\Delta(q1, q2) \\ \text{else} \\ \quad P_j := Calc\_All\_\Delta\left(P_{j-1_{1_1}}, P_{j-1_{1_2}}\right) \end{array} \right. \\ P_{iter} \end{array} \right.$$

1st Corrections example

$C\_Iter := Q(Q1, Q2, 1)$

$$C\_Iter = \left[\begin{bmatrix} \begin{bmatrix} 47.06 \\ 25.06 \\ -8 \\ -27.94 \end{bmatrix} \\ \begin{bmatrix} 8 \\ 18.07 \\ -12.93 \\ -9.93 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 2.06 \\ -4.93 \end{bmatrix} \right] \frac{L}{s}$$

1st column shows FINAL flows in LOOPS 1 & 2. The 2 nd column shows FINAL Corrections after 3 Iterations

With 3 Iterations:

$$Q(Q1, Q2, 3) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 45.96 \\ 23.96 \\ -9.47 \\ -29.04 \end{bmatrix} \\ \begin{bmatrix} 9.47 \\ 18.43 \\ -12.57 \\ -9.57 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} 0.12 \\ -0.09 \end{bmatrix} \end{bmatrix} \frac{L}{s}$$

Compare

$$Q1_{cor3} = \begin{bmatrix} 45.96 \\ 23.96 \\ -9.47 \\ -29.04 \end{bmatrix} \frac{L}{s} \qquad Q2_{cor3} = \begin{bmatrix} 9.47 \\ 18.43 \\ -12.57 \\ -9.57 \end{bmatrix} \frac{L}{s}$$

METHOD 3: Hardy Cross Method: Using While Loop

PROGRAM 4: Using 'While' loop : CALLS PROHRAM 2

```
Z(q1, q2) := | A := Calc_All_Δ(q1, q2)
             | ΔQ := A_2
             | qq1 := A_1_1
             | qq2 := A_1_2
             | Δ := 0.001 L/s
             | while (|ΔQ_1| ≥ Δ) ∧ (|ΔQ_2| ≥ Δ)
             |     | B := Calc_All_Δ(qq1, qq2)
             |     | ΔQ := B_2
             |     | qq1 := B_1_1
             |     | qq2 := B_1_2
             |     | ΔQ := B_2
             | B
```

With 'While Loop'

$$Z(Q1, Q2) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 45.94 \\ 23.94 \\ -9.5 \\ -29.06 \end{bmatrix} \\ \begin{bmatrix} 9.5 \\ 18.44 \\ -12.56 \\ -9.56 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{bmatrix} \frac{L}{s}$$

Results from Different Methods

'While' Loop Method using PROGRAM 2 & 4 : Both LOOPS

$$Z(Q1, Q2) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 45.94 \\ 23.94 \\ -9.5 \\ -29.06 \end{bmatrix} \\ \begin{bmatrix} 9.5 \\ 18.44 \\ -12.56 \\ -9.56 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{bmatrix} \frac{L}{s}$$

3rd Element of LOOP 1 (**-9.5**) & 1st Element of LOOP 2 (**+9.5**) refer to common Pipe CD/DC. Same value, but opposite sign

Defined Iterations Method using PROGRAM 2 & 3 : Both LOOPS

$$Q(Q1, Q2, 5) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 45.94 \\ 23.94 \\ -9.5 \\ -29.06 \end{bmatrix} \\ \begin{bmatrix} 9.5 \\ 18.44 \\ -12.56 \\ -9.56 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{bmatrix} \frac{L}{s}$$

Successive 3 Steps Method Using PROGRAM 1 for LOOP 1

$$Q1_{cor3} = \begin{bmatrix} 45.96 \\ 23.96 \\ -9.47 \\ -29.04 \end{bmatrix} \frac{L}{s}$$

Since only 3 steps are used, results are slightly different

Successive 3 Step Method Using PROGRAM 1 for LOOP 2

$$Q2_{cor3} = \begin{bmatrix} 9.47 \\ 18.43 \\ -12.57 \\ -9.57 \end{bmatrix} \frac{L}{s}$$

$$\mathrm{time}(0) - t_{start} = 0.3\ s$$