To solve an ODE directly without creating a solve block, use one of the *ODE solvers*, which solve systems of ODEs of the form

$$\frac{d}{dx}y = \begin{bmatrix} f_1(x,y) \\ f_2(x,y) \\ \dots \\ f_n(x,y) \end{bmatrix}$$

where y is vector of unknown functions of the independent variable x. To solve a higher-order ODE, rewrite it as a system of first order ODEs.

Rewriting a Higher-Order ODE as a System of First-Order ODEs

To rewrite the n^{th}-order ODE

$$\frac{d^n}{dx^n}y(x) + a_{n-1}(x)\cdot\frac{d^{n-1}}{dx^{n-1}}y(x) + \dots + a_1(x)\left(\frac{d}{dx}y\right) + a_0(x)\cdot y(x) = f(x)$$

as a system of first-order ODEs, define variables y_0, y_1, ... y_n as follows:

$$y_0 = y$$

$$y_1 = \frac{d}{dx}y_0$$

$$y_2 = \frac{d}{dx}y_1$$

$$\dots$$

$$y_n = \frac{d}{dx}y_{n-1}$$

In terms of the new variables, the original equation becomes

$$y_n + a_{n-1}(x)\cdot y_{n-1} + \dots + a_1(x)\,y_1 + a_0(x)\,y_0 = f(x)$$

This is equivalent to the following system of first-order ODEs:

$$\frac{d}{dx}y = \begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_n \end{pmatrix} \quad \text{where} \quad y = \begin{pmatrix} y_0 \\ y_1 \\ \dots \\ y_{n-1} \end{pmatrix}$$

If you solve the original equation for y_n and substitute the result in the vector, you get a vector function that you can apply the ODE solvers to:

$$\frac{d}{dx}y = D(x,y) = \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_{n-1} \\ -a_{n-1}(x)\cdot y_{n-1} - \dots - a_1(x)\cdot y_1 - a_0(x)\cdot y_0 + f(x) \end{bmatrix}$$

Example:

For example, you can rewrite the second-order ODE

$$\frac{d^2}{dx^2}y(x) + 3x\cdot\left(\frac{d}{dx}y(x)\right) - 7\,y(x) = 4x$$

using vector subscripts, as

$$D(x,y) := \begin{pmatrix} y_1 \\ 4\cdot x + 7\cdot y_0 - 3\cdot x\cdot y_1 \end{pmatrix}$$ with implied left-hand-side $$\begin{pmatrix} \frac{d}{dx}y_0 \\ \frac{d}{dx}y_1 \end{pmatrix}$$