MNA Solver

⊞ Numerical Inverse Laplace Transform

⊞ MNA STAMPS

⊟ OpAmp Integrator

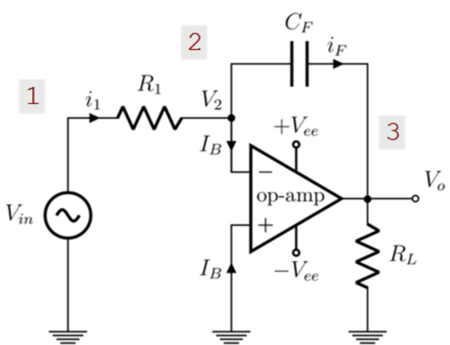
OpAmp integrator

Shorthands:

$$V_{SOL} := MNA\left(\chi, MNA_{AC}\right)_4$$

$$v_o(t) := \Big| ILap\left(V_o(s), t, t_i, t_e, 16\right)$$

$$\Pi := \Big| \begin{cases} v_i(x\ \mathrm{s}) \cdot t_i \le x\ \mathrm{s} \le t_e \\ v_o(x\ \mathrm{s}) \cdot t_i \le x\ \mathrm{s} \le t_e \\ v_{os}(x\ \mathrm{s}) \cdot t_i \le x\ \mathrm{s} \le t_e \end{cases}$$



Ideal integrator

$$\chi := \begin{bmatrix} V_i(s) & [1\ 0] \\ R_1 & [1\ 2] \\ R_L & [0\ 3] \\ C_F & [2\ 3] \\ O & [0\ 3\ 2] \end{bmatrix}$$

Symbolic solution in freq domain

$$V_o(s) := V_{SOL_3} = -\frac{V_i(s)}{s \cdot C_F \cdot R_1}$$

Symbolic solution in time domain

$$v_{os}(t) := -\frac{1}{C_F \cdot R_1} \cdot \int_{t_i}^{t} v_i(\tau\ \mathrm{s})\, d\tau$$

Numerical values

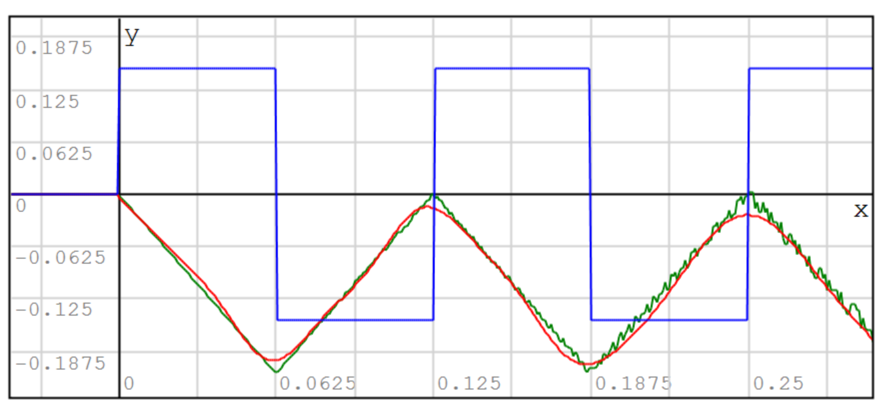
$$\begin{bmatrix} t_i & t_e \end{bmatrix} := \begin{bmatrix} 0\ \mathrm{s} & 300\ \mathrm{ms} \end{bmatrix} \quad f := 8\ \mathrm{Hz} \quad Vo := 150\ \mathrm{mV} \quad \omega := 2 \cdot \pi \cdot f = 50.2655\ \mathrm{Hz}$$

$$R_1 := 220\ \mathrm{k\Omega} \quad R_L := 320\ \Omega \quad C_F := 200\ \mathrm{nF} \quad T := \frac{1}{f} = 0.125\ \mathrm{s}$$

$$\begin{cases} v_i(t) := Vo \cdot \mathrm{sign}(\sin(\omega \cdot t)) \\ V_i(s) := \frac{Vo}{s} \cdot TH\left(\frac{s \cdot T}{4}\right) \end{cases}$$

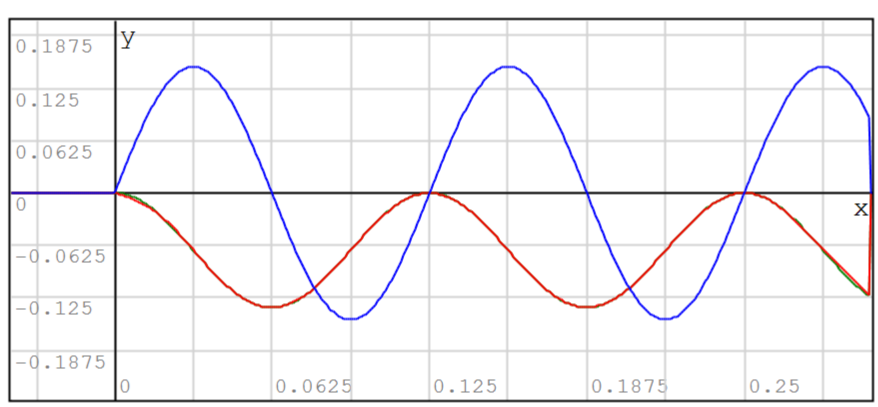
Pulse train input

- blue: input signal
- red: ILap solution
- green: integrate solution

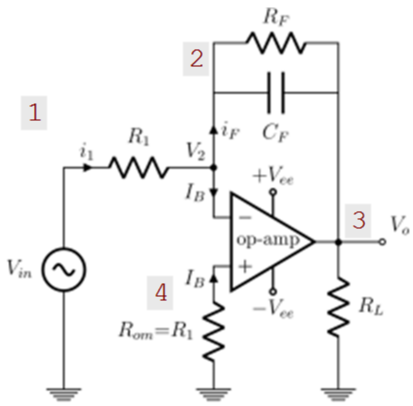


$$\begin{cases} v_i(t) := Vo \cdot \sin(\omega \cdot t) \\ V_i(s) := Vo \cdot \frac{\omega}{s^2 + \omega^2} \end{cases}$$

Sinusoidal input



$$\mathrm{Clear}\left(V_i(s), Vo, T, \omega, R_1, R_L, C_F\right) = 1$$



Real integrator

$$\chi := \begin{bmatrix} V_i(s) & [1\ 0] \\ R_1 & [1\ 2] \\ R_L & [0\ 3] \\ C_F & [2\ 3] \\ O & [4\ 3\ 2] \\ R_F & [2\ 3] \\ R_1 & [0\ 4] \end{bmatrix}$$

Symbolic solution in freq domain

$$V_o(s) := {V_{SOL}}_3 = -\frac{V_i(s) \cdot R_F}{(1 + s \cdot C_F \cdot R_F) \cdot R_1}$$

Symbolic solution in time domain

Can't find a general explicit expression.

$$\mathrm{maple}\left(invlaplace\left(V_o(s), s, t\right)\right) = -\frac{R_F \cdot invlaplace\left(\frac{V_I(s)}{1 + s \cdot C_F \cdot R_F}, s, t\right)}{R_1}$$

Search a particular symbolic solution

Pulse train

$$V_i(s) := \frac{Vo}{s} \cdot \tanh\left(\frac{s \cdot T}{4}\right)$$

$$v_{pulse} := \mathrm{maple}\left(invlaplace\left(V_o(s), s, t\right)\right)$$

$$v_{pulse} = -\frac{Vo \cdot R_F \cdot \left((-1)^{-1 + ceil\left(\frac{2 \cdot t}{T}\right)} - C_F \cdot R_F \cdot invlaplace\left(\frac{\tanh\left(\frac{s \cdot T}{4}\right)}{1 + s \cdot C_F \cdot R_F}, s, t\right)\right)}{R_1}$$

Fails, take $v_{pulse} := 0$

Sinusoidal

$$V_i(s) := Vo \cdot \frac{\omega}{s^2 + \omega^2}$$

$$v_{sin} := \mathrm{maple}\left(invlaplace\left(V_o(s), s, t\right)\right)$$

$$v_{sin} = -\frac{Vo \cdot R_F \cdot \left(-C_F \cdot R_F \cdot \cos(\omega \cdot t) \cdot \omega + \sin(\omega \cdot t) + C_F \cdot R_F \cdot \exp\left(-\frac{t}{C_F \cdot R_F}\right) \cdot \omega\right)}{R_1 \cdot \left(1 + {C_F}^2 \cdot {R_F}^2 \cdot \omega^2\right)}$$

It's OK

Numerical values

$[t_i\ \ t_e] := [0\ \mathrm{s}\ \ 300\ \mathrm{ms}]$ $\quad f := 8\ \mathrm{Hz}$ $\quad Vo := 150\ \mathrm{mV}$ $\quad R_F := 250\ \mathrm{k\Omega}$

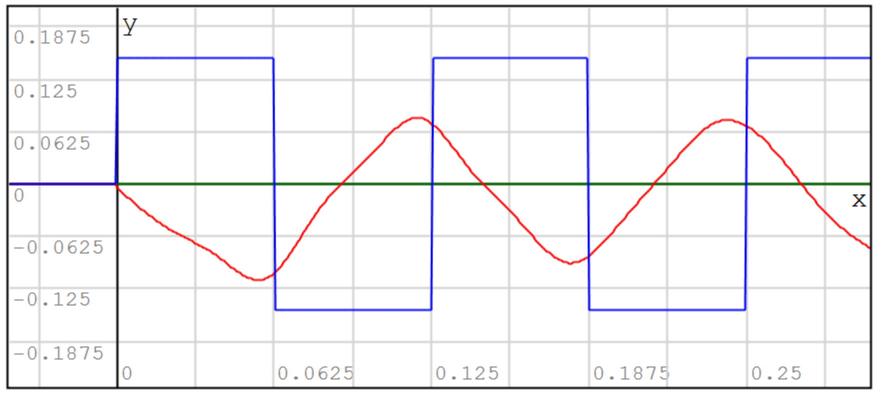
$R_1 := 220\ \mathrm{k\Omega}$ $\quad R_L := 320\ \Omega$ $\quad C_F := 200\ \mathrm{nF}$

$$R_{om} := \frac{1}{\frac{1}{R_1} + \frac{1}{R_F} + \frac{1}{R_L}} = 0.3191\ \mathrm{k\Omega}$$

$\omega := 2 \cdot \pi \cdot f = 50.2655\ \mathrm{Hz}$ $\quad T := \frac{1}{f} = 0.125\ \mathrm{s}$

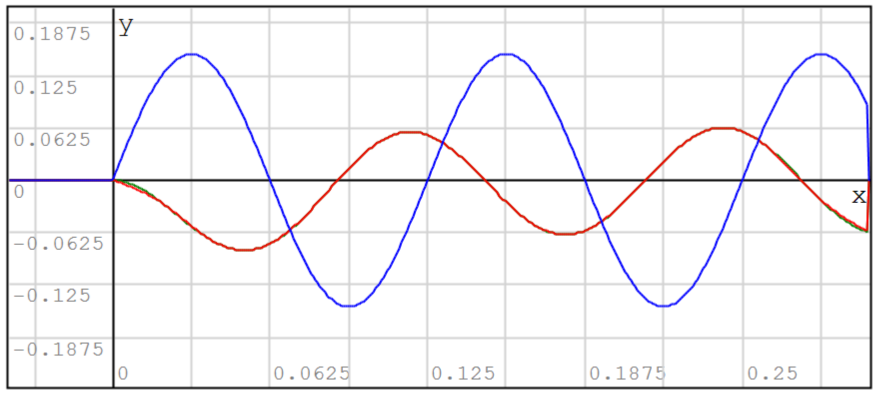
$$\begin{cases} v_i(t) := Vo \cdot \mathrm{sign}(\sin(\omega \cdot t)) \\ V_i(s) := \dfrac{Vo}{s} \cdot TH\left(\dfrac{s \cdot T}{4}\right) \\ v_{os}(t) := 0 \end{cases}$$

Pulse train input



$$\begin{cases} v_i(t) := Vo \cdot \sin(\omega \cdot t) \\ V_i(s) := Vo \cdot \dfrac{\omega}{s^2 + \omega^2} \\ v_{os}(t) := v_{sin} \end{cases}$$

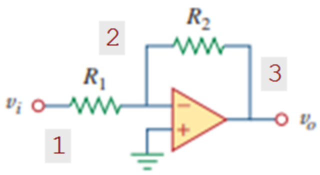
Sinusoidal input



Summary of basic op amp circuits

Clear$(V_i, R_1, \chi) = 1$

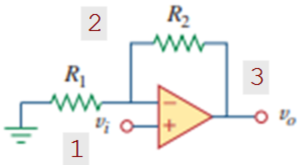
$V_{SOL} := MNA(\chi, MNA_{AC})_4$



Inverter amplifier

$$\chi := \begin{bmatrix} V_i & [1\ 0] \\ R_1 & [1\ 2] \\ R_2 & [2\ 3] \\ O & [0\ 3\ 2] \end{bmatrix}$$

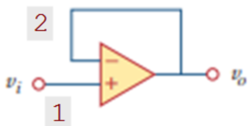
$V_o := V_{SOL_3}$ $\quad V_o = -\frac{V_i \cdot R_2}{R_1}$



Noninverter amplifier

$$\chi := \begin{bmatrix} V_i & [1\ 0] \\ R_1 & [0\ 2] \\ R_2 & [2\ 3] \\ O & [1\ 3\ 2] \end{bmatrix}$$

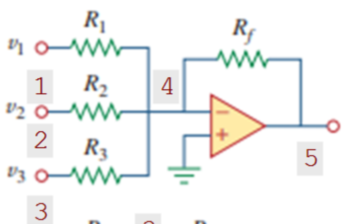
$V_o := V_{SOL_3}$ $\quad V_o = \frac{(R_1 + R_2) \cdot V_i}{R_1}$



Voltage Follower

$$\chi := \begin{bmatrix} V_i & [1\ 0] \\ O & [1\ 2\ 2] \end{bmatrix}$$

$V_o := V_{SOL_2}$ $\quad V_o = V_i$

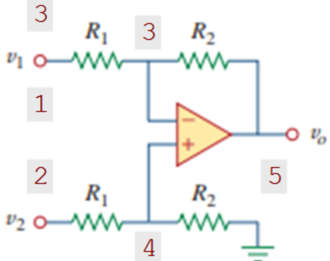


Summer

$$\chi := \begin{bmatrix} V_1 & [1\ 0] \\ V_2 & [2\ 0] \\ V_3 & [3\ 0] \\ R_1 & [1\ 4] \\ R_2 & [2\ 4] \\ R_3 & [3\ 4] \\ R_f & [4\ 5] \\ O & [0\ 5\ 4] \end{bmatrix}$$

$V_o := V_{SOL_5}$

$$V_o = -\frac{R_f \cdot ((V_1 \cdot R_2 + V_2 \cdot R_1) \cdot R_3 + V_3 \cdot R_2 \cdot R_1)}{R_3 \cdot R_2 \cdot R_1}$$



Difference Amplifier

$$\chi := \begin{bmatrix} V_1 & [1\ 0] \\ V_2 & [2\ 0] \\ R_1 & [3\ 1] \\ R_1 & [4\ 2] \\ R_2 & [0\ 4] \\ R_2 & [3\ 5] \\ O & [4\ 5\ 3] \end{bmatrix}$$

$V_o := V_{SOL_5}$ $\quad V_o = \frac{R_2 \cdot (-V_1 + V_2)}{R_1}$

Alvaro