

This QuickSheet illustrates evaluating complex contour integrals.

Input a path in the x-y plane:

$$x(t) := 2 \cdot \sin(5t)$$

$$y(t) := 3 \cdot \cos(t)$$

$$t_{\text{initial}} := 0$$

$$t_{\text{final}} := \pi$$

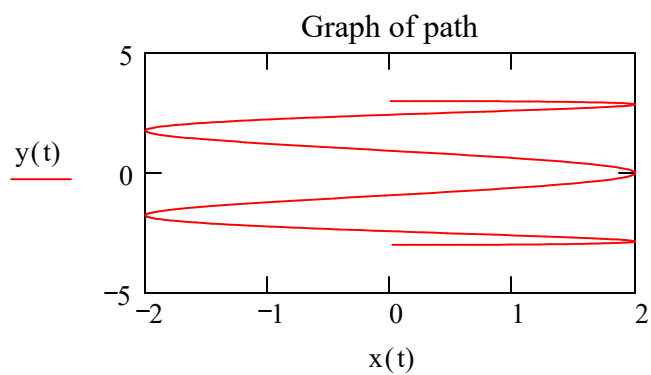
Input a function of complex variable z to be integrated:

$$f(z) := \frac{1}{z^2}$$

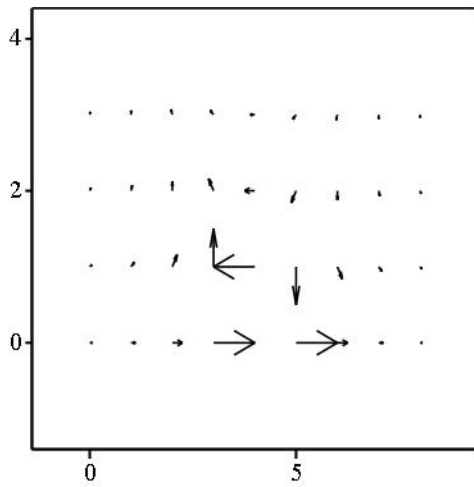
Path in complex z plane:

Type li for imaginary unit:

$$z(t) := x(t) + i \cdot y(t)$$



The formulas underlying the plots are past the right margin.



$\longrightarrow$
 $f(Z)$

Contour integral:

$$\int_{t_{\text{initial}}}^{t_{\text{final}}} f(z(t)) \cdot \frac{d}{dt} z(t) dt = -0.667i$$

$$t := t_{\text{initial}}, t_{\text{initial}} + 0.01 \ldots t_{\text{final}} \qquad n := 0 \ldots 3$$

$$m := -4 \ldots 4$$

$$Z_{m+4,n} := \frac{m}{5} + i \cdot \frac{n}{5} \qquad f(z) := \text{if} \left(|z| > 0, \frac{1}{z^2}, 0 \right)$$

$$Z = \begin{pmatrix} -0.8 & -0.8 + 0.2i & -0.8 + 0.4i & -0.8 + 0.6i \\ -0.6 & -0.6 + 0.2i & -0.6 + 0.4i & -0.6 + 0.6i \\ -0.4 & -0.4 + 0.2i & -0.4 + 0.4i & -0.4 + 0.6i \\ -0.2 & -0.2 + 0.2i & -0.2 + 0.4i & -0.2 + 0.6i \\ 0 & 0.2i & 0.4i & 0.6i \\ 0.2 & 0.2 + 0.2i & 0.2 + 0.4i & 0.2 + 0.6i \\ 0.4 & 0.4 + 0.2i & 0.4 + 0.4i & 0.4 + 0.6i \\ 0.6 & 0.6 + 0.2i & 0.6 + 0.4i & 0.6 + 0.6i \\ 0.8 & 0.8 + 0.2i & 0.8 + 0.4i & 0.8 + 0.6i \end{pmatrix}$$

$$\begin{array}{c} \longrightarrow \\ f(Z) = \end{array} \begin{pmatrix} 1.562 & 1.298 + 0.692i & 0.75 + i & 0.28 + 0.96i \\ 2.778 & 2 + 1.5i & 0.74 + 1.775i & 1.389i \\ 6.25 & 3 + 4i & 3.125i & -0.74 + 1.775i \\ 25 & 12.5i & -3 + 4i & -2 + 1.5i \\ 0 & -25 & -6.25 & -2.778 \\ 25 & -12.5i & -3 - 4i & -2 - 1.5i \\ 6.25 & 3 - 4i & -3.125i & -0.74 - 1.775i \\ 2.778 & 2 - 1.5i & 0.74 - 1.775i & -1.389i \\ 1.562 & 1.298 - 0.692i & 0.75 - i & 0.28 - 0.96i \end{pmatrix}$$