

$$f(x, y, a, c) := (x^2 + y^2)^2 - 2 \cdot c^2 \cdot (x^2 - y^2) - (a^4 - c^4)$$

$$\begin{cases} f_1(x, y) := f(x, y, \sqrt{2}, 1) \\ f_2(x, y) := f(x - 0.5, y - 0.5, 2, 2) \end{cases}$$

Utilities $z := \begin{bmatrix} x \\ y \end{bmatrix}$ $F(z) := \begin{bmatrix} f_1(z_1, z_2) \\ f_2(z_1, z_2) \end{bmatrix}$ $J(z) := \text{Jacob} \left(F(z), \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \right)$ $I := \text{identity}(2)$

Guess values

$$\text{Plot} := \begin{cases} f_1(x, y) \\ f_2(x, y) \\ NR_a(-3, -2) \\ NR_a(5, -1) \\ NR_a(-1, 3) \\ NR_a(4.5, 2.5) \end{cases}$$

Algorithm Iteration for some NR.f formula

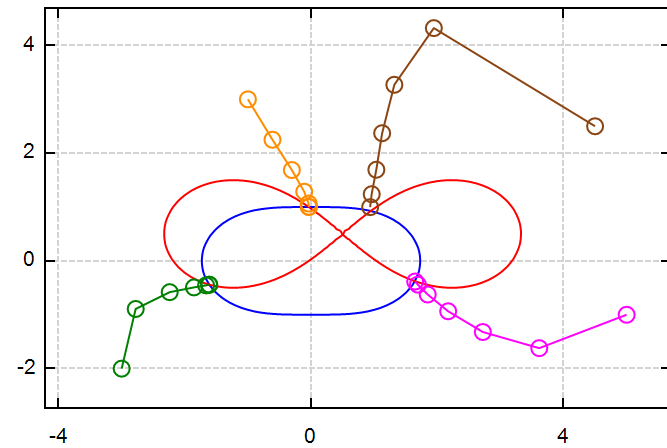
$$NR_a(x, y) := \begin{cases} xy := \text{stack}(x, y) \\ \text{for } j \in [1..n] \\ \quad xy := \text{augment} \left(xy, \begin{bmatrix} x \\ y \end{bmatrix} := NR_f(x, y) \right) \\ xy^T \end{cases}$$

Formula: Newton Raphson

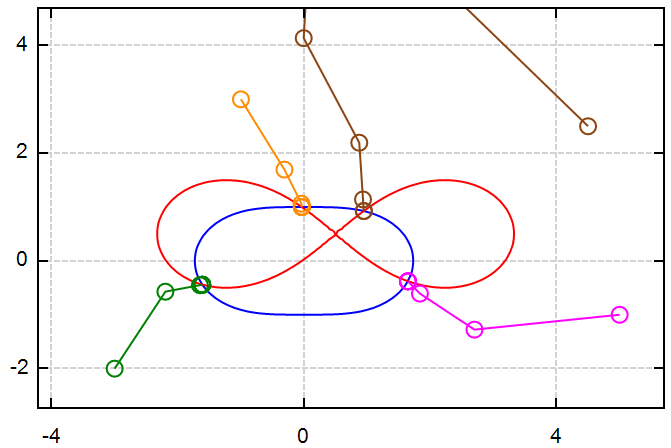
$$\begin{cases} NR_f(x, y) := \left| z - J(z)^{-1} \cdot F(z) \right. \\ Z1 := \text{Plot} \end{cases}$$

Formula: Modified King's method

$$\begin{cases} NR_f(x, y) := \begin{cases} z^* := z - J(z)^{-1} \cdot F(z) \\ t := z^* - \left(2 \cdot I - J(z)^{-1} \cdot J(z^*) \right) \cdot J(z)^{-1} \cdot F(z^*) \\ t - \left(2 \cdot I - J(z)^{-1} \cdot J(z^*) \right) \cdot J(z)^{-1} \cdot F(t) \end{cases} \\ Z2 := \text{Plot} \end{cases}$$



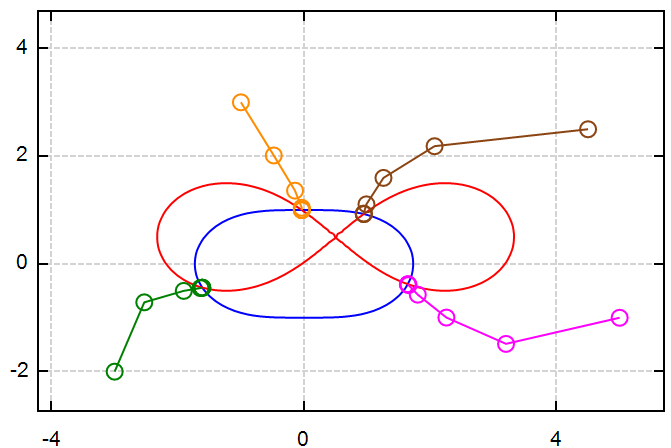
Z1



Z2

Formula: Modified Newton Raphson

$$\begin{cases} NR_f(x, y) := \begin{cases} z^* := z - J(z)^{-1} \cdot F(z) \\ z - J(z)^{-1} \cdot (F(z) + F(z^*)) \end{cases} \\ Z3 := \text{Plot} \end{cases}$$



Z3

Comparing

$$Z := \text{augment} \left(\begin{matrix} Z1 \\ Z2 \\ Z3 \end{matrix} \begin{bmatrix} 3 \dots 6 \end{bmatrix}' \right)$$

normi(F(z))	P1(-3,-2)	P2(5,-1)	P3(-1,3)	P4(4.5,2.5)
Method 1	2.1E-004	7.9E-002	1.5E-004	7.4E-001
Method 2	1.6E-013	8E-014	2.1E-014	4.4E-014
Method 3	1.7E-013	2E-013	0E+000	2.7E-014

Alvaro

appVersion(4) = "1.0.8348.30405"