

Differentials

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dSymbol := "d"
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Enter an expression, and say that  
have some constants

$$z := x \cdot y^2 + \frac{y}{a \cdot x}$$

$$da := 0$$

Total differential

$$dz := d(z) = \frac{-y \cdot (1 - x^2 \cdot y \cdot a) \cdot dx + (1 + 2 \cdot x^2 \cdot y \cdot a) \cdot dy \cdot x}{a \cdot x^2}$$

Partial derivatives

$$d(dz, dx) = -\frac{y \cdot (1 - x^2 \cdot y \cdot a)}{a \cdot x^2} \qquad d(dz, dy) = \frac{1 + 2 \cdot x^2 \cdot y \cdot a}{x \cdot a}$$

or

$$d(z, x) = -\frac{y \cdot (1 - x^2 \cdot y \cdot a)}{a \cdot x^2} \qquad d(z, y) = \frac{1 + 2 \cdot x^2 \cdot y \cdot a}{a \cdot x}$$

Notice that

$$d(dz, dy^2) = 0 \qquad \text{because} \qquad d(d(dz, dy), dy) = 0$$

but

$$d(dz, y^2) = 2 \cdot dx$$

Some examples

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dSymbol := "∂"
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$$d\left(\begin{bmatrix} r \cdot \cos(\theta) \\ r \cdot \sin(\theta) \end{bmatrix}\right) = \begin{bmatrix} \cos(\theta) \cdot \partial_r - r \cdot \sin(\theta) \cdot \partial_\theta \\ \sin(\theta) \cdot \partial_r + r \cdot \cos(\theta) \cdot \partial_\theta \end{bmatrix}$$

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∂a := 0
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$$d(a \cdot e^{b \cdot x}) = e^{b \cdot x} \cdot a \cdot (x \cdot \partial b + b \cdot \partial x)$$

$$d(a \cdot e^{\sin(x)}) = e^{\sin(x)} \cdot \cos(x) \cdot a \cdot \partial x$$

$$d(x^2 \cdot \ln(x \cdot y)) = \frac{x \cdot ((1 + 2 \cdot \ln(x \cdot y)) \cdot \partial x \cdot y + x \cdot \partial y)}{y}$$

$$\text{Clear}(\partial a) = 1$$

$$d1 := d(a \cdot b) = b \cdot \partial a + a \cdot \partial b$$

$$d2 := d(d1) = \partial a \cdot (\partial b + b \cdot \partial a + \partial b) + a \cdot \partial b^2$$

$$d3 := d(d2) = 2 \cdot ((\partial b + b \cdot \partial a) \cdot \partial a^2 + (\partial a + \partial b \cdot a) \cdot \partial b^2) + \partial b \cdot \partial a \cdot (\partial b + \partial a)$$

$$\partial a := 0$$

$$d1 := d(a \cdot b) = a \cdot \partial b$$

$$d2 := d(d1) = a \cdot \partial b^2$$

$$d3 := d(d2) = 2 \cdot \partial b^3 \cdot a$$

Chain rule

$$c:=d\left(a\cdot f\left(g\left(x\right)\right)\right)=a\cdot \frac{d}{d\,x}f\left(g\left(x\right)\right)\cdot \partial x$$

$$f\left(x\right):=\ln\left(x\right)\qquad g\left(x\right):=\cos\left(x\right)\qquad c=-a\cdot \tan\left(x\right)\cdot \partial x$$

$$d\left(a\cdot \ln\left(\cos\left(x\right)\right)\right)=-\frac{\sin\left(x\right)\cdot a\cdot \partial x}{\cos\left(x\right)}$$

Total differential

$$d\left(f\left(t,\,x,\,x'\right)\right)=\frac{d}{d\,t}f\left(t,\,x,\,x'\right)\cdot \partial t+\frac{d}{d\,x}f\left(t,\,x,\,x'\right)\cdot \partial x+\frac{d}{d\,x'}f\left(t,\,x,\,x'\right)\cdot \partial x'$$

Alvaro

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