Shooting method

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Solve this Boundary Values Problem using the Shooting method

$$ode := \frac{d^2}{dx^2} y(x) + \frac{1}{2} \cdot \frac{d}{dx} y(x) + y(x) - 5$$

$$\begin{bmatrix} x_0 & x_1 \end{bmatrix} := \begin{bmatrix} -1 & 6 \end{bmatrix} \qquad \begin{bmatrix} y_0 & y_1 \end{bmatrix} := \begin{bmatrix} 7 & 3 \end{bmatrix}$$

Get the symbolic solution for check

$$y_s(x) := \text{maple}\left(evalf\left(subs\left(dsolve\left(\begin{cases} ode \\ y(x_0) = y_0 \\ y(x_1) = y_1 \end{cases}, y(x) \right) \right) \right) \right)$$

The goal is get the slope at x.0 for use with a numerical ode solvers as the initial condition

$$\frac{d}{dx_0} y_s(x_0) = -27.5704$$

Convert to a system

$$D(x, y) := \begin{bmatrix} y_2 \\ -\frac{y_2}{2} - y_1 + 5 \end{bmatrix}$$

Use an ode solver

$$N := 100 \qquad sol(y'_0) := \text{al\_rkckadapt}\left(\begin{bmatrix} y_0 \\ y'_0 \end{bmatrix}, x_0, x_1, N - 1, D \right)$$

Equation to solve

$$Eq(y'_0) := sol(y'_0)_{N\,2} - y_1$$

Guess for a solution

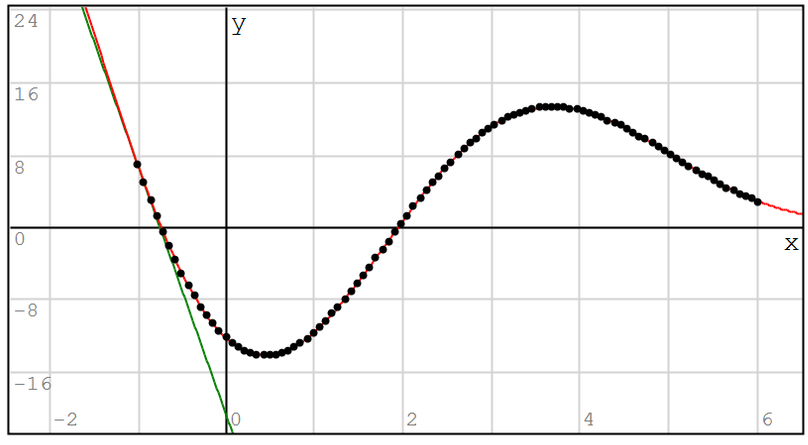
$$y'_0 := -10$$

Solve, this is: shoot and see if holds

$$y'_0 := \text{al\_nleqsolve}(y'_0, Eq)_1 = -27.5704$$

Plot and check

$$\begin{cases} XY := sol(y'_0) \\ \begin{bmatrix} X & Y \end{bmatrix} := \begin{bmatrix} \text{col}(XY, 1) & \text{col}(XY, 2) \end{bmatrix} \end{cases}$$



$$\text{normi}\left(Y - \overrightarrow{y_s(X)} \right) = 7.39 \cdot 10^{-8}$$

$$\left| \frac{d}{dx_0} y_s(x_0) - y'_0 \right| = 1.1534 \cdot 10^{-7}$$

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