⊞—svd

⊟—svd Examples

$TOL := 10^{-9}$

Ex 1

$A := \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$ $\quad [U\ S\ V] := MSVD(A)$ $\quad \text{norme}\left(A - U \cdot S \cdot V^{T}\right) = 2.4 \cdot 10^{-9}$

$$[U\ S\ V] = \left[\begin{bmatrix} 0.7071 & -0.7071 \\ 0.7071 & 0.7071 \end{bmatrix} \begin{bmatrix} 3.4641 & 0 & 0 \\ 0 & 3.1623 & 0 \end{bmatrix} \begin{bmatrix} 0.4082 & -0.8944 & -0.1826 \\ 0.8165 & 0.4472 & -0.3651 \\ 0.4082 & 0 & 0.9129 \end{bmatrix}\right]$$

Ex 1'

$A := A^{T}$ $\quad [U\ S\ V] := MSVD(A)$ $\quad \text{norme}\left(A - U \cdot S \cdot V^{T}\right) = 2.1 \cdot 10^{-9}$

$$[U\ S\ V] = \left[\begin{bmatrix} 0.4082 & -0.8944 & -0.1826 \\ 0.8165 & 0.4472 & -0.3651 \\ 0.4082 & 0 & 0.9129 \end{bmatrix} \begin{bmatrix} 3.4641 & 0 \\ 0 & 3.1623 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0.7071 & -0.7071 \\ 0.7071 & 0.7071 \end{bmatrix}\right]$$

Ex 2

$A := \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$ $\quad [U\ S\ V] := MSVD(A)$ $\quad \text{norme}\left(A - U \cdot S \cdot V^{T}\right) = 1.1 \cdot 10^{-9}$

$$[U\ S\ V] = \left[\begin{bmatrix} -0.7071 & 0.7071 \\ -0.7071 & -0.7071 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix} \begin{bmatrix} -0.7071 & 0.2357 & -0.6667 \\ -0.7071 & -0.2357 & 0.6667 \\ 0 & 0.9428 & 0.3333 \end{bmatrix}\right]$$

Ex 2'

$A := A^{T}$ $\quad [U\ S\ V] := MSVD(A)$ $\quad \text{norme}\left(A - U \cdot S \cdot V^{T}\right) = 5.8 \cdot 10^{-10}$

$$[U\ S\ V] = \left[\begin{bmatrix} 0.7071 & -0.2357 & -0.6667 \\ 0.7071 & 0.2357 & 0.6667 \\ 0 & -0.9428 & 0.3333 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0.7071 & -0.7071 \\ 0.7071 & 0.7071 \end{bmatrix}\right]$$

Ex 3

$$A := \begin{bmatrix} 92 & 99 & 1 & 8 & 15 \\ 98 & 80 & 7 & 14 & 16 \\ 4 & 81 & 88 & 20 & 22 \\ 85 & 87 & 19 & 21 & 3 \\ 86 & 93 & 25 & 2 & 9 \\ 17 & 24 & 76 & 83 & 90 \\ 23 & 5 & 82 & 89 & 91 \\ 79 & 6 & 13 & 95 & 97 \\ 10 & 12 & 94 & 96 & 78 \\ 11 & 18 & 100 & 77 & 84 \end{bmatrix} \quad [U\ S\ V] := MSVD(A) \quad \text{norme}\left(A - U \cdot S \cdot V^{T}\right) = 1.4 \cdot 10^{-9}$$

$$S = \begin{bmatrix} 363.4811 & 0 & 0 & 0 & 0 \\ 0 & 231.5016 & 0 & 0 & 0 \\ 0 & 0 & 117.8766 & 0 & 0 \\ 0 & 0 & 0 & 22.4612 & 0 \\ 0 & 0 & 0 & 2.8156 \cdot 10^{-8} & 17.7241 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.2226 & 0.4705 & -0.0536 & 0.4124 & -0.2407 & -0.2459 & 0.3826 & -0.2205 & 0.2094 & 0.4459 \\ 0.2282 & 0.4178 & -0.1317 & -0.1262 & 0.2822 & -0.0235 & -0.3761 & -0.6485 & -0.2416 & -0.199 \\ 0.2565 & 0.0734 & 0.6669 & 0.2445 & -0.2193 & -0.3739 & -0.1709 & 0.1731 & -0.2849 & -0.3099 \\ 0.2291 & 0.3978 & 0.0278 & -0.5274 & -0.3461 & 0.3566 & 0.1289 & 0.2819 & -0.3719 & 0.1704 \\ 0.2258 & 0.4274 & 0.1097 & -0.0248 & 0.4538 & 0.1425 & 0.0068 & 0.4047 & 0.5421 & -0.2642 \\ 0.3809 & -0.2089 & 0.02 & 0.4446 & -0.2286 & 0.7122 & -0.0954 & -0.1468 & 0.0721 & -0.1338 \\ 0.3865 & -0.2617 & -0.0581 & -0.094 & 0.2943 & -0.0857 & 0.723 & -0.1132 & -0.2371 & -0.2903 \\ 0.3703 & -0.0538 & -0.6888 & 0.1533 & -0.1458 & -0.3055 & -0.2595 & 0.3868 & -0.1133 & -0.1327 \\ 0.3874 & -0.2816 & 0.1014 & -0.4953 & -0.334 & -0.2093 & -0.1142 & -0.2422 & 0.5366 & 0.0046 \\ 0.3841 & -0.2521 & 0.1833 & 0.0073 & 0.4659 & -0.0047 & -0.2325 & 0.1223 & -0.1494 & 0.6685 \end{bmatrix}$$

$$V = \begin{bmatrix} 0.3727 & 0.5861 & -0.473 & -0.3459 & 0.4174 \\ 0.349 & 0.6295 & 0.4358 & 0.3409 & -0.4192 \\ 0.4805 & -0.2661 & 0.6503 & -0.3274 & 0.4102 \\ 0.5043 & -0.3105 & -0.2819 & -0.3825 & -0.6507 \\ 0.504 & -0.3048 & -0.2898 & 0.7146 & 0.2416 \end{bmatrix}$$

Ex 4 Check the time and accuracy and compare with other softwre. As expected, isn't very fast.

$A := \text{matrix}(15, 20)$ $\quad r := [1..\text{length}(A)]$ $\quad A_r := \text{eval}\left(200 \cdot 10^{-9} \cdot \text{random}\left(10^{9}\right) - 100\right)$

$TOL := 10^{-5}$ $\quad tol := 10^{-4}$

$t_o := \text{time}(0)$ $[U\ S\ V] := MSVD(A, TOL, tol)$ $\text{time}(0) - t_o = 145.79\ s$

$SUV := \text{dn\_LinAlgSVD}(A)$ $[U_{dn}\ S_{dn}\ V_{dn}] := [SUV_2\ SUV_1\ SUV_3^T]$

$\text{norme}(A - U \cdot S \cdot V^T) = 0$

$\text{norme}(A - U_{dn} \cdot S_{dn} \cdot V_{dn}^T) = 3 \cdot 10^{-12}$

$\text{norme}(U - U_{dn}) = 5.7$

$\text{norme}(S - S_{dn}) = 0$

$\text{norme}(V - V_{dn}) = 6.1$

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