

Drawing lines

`pLine(A)`

If $\text{cols}(A) \equiv 3$, draws the line $ax+by+c \equiv 0$, where a, b, c are the cols of A
 If $\text{cols}(A) \equiv 2$, draws the line between the points defined by the A rows
 If $\text{cols}(A) \equiv 1$, assume that A have submatrices with two or three cols.

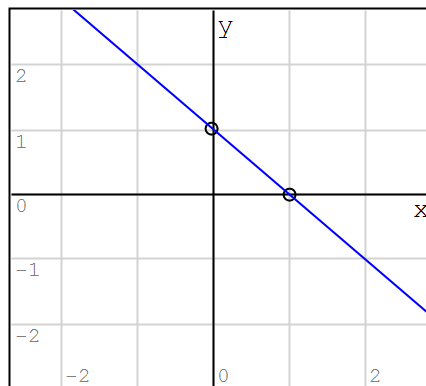
Examples

$$P := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

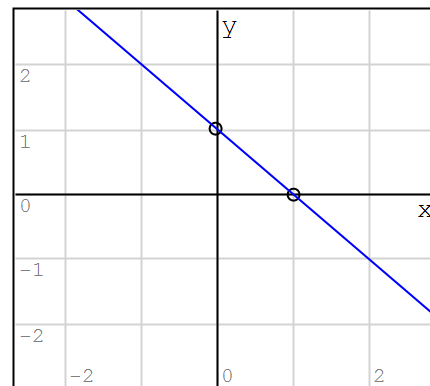
$$M := \begin{bmatrix} 3 & 3 & -3 \end{bmatrix}$$

$$\Pi(MP) := \begin{cases} pLine(MP) \\ \text{augment}(P, "o") \end{cases}$$

The same lines defined by its coefficients or two points



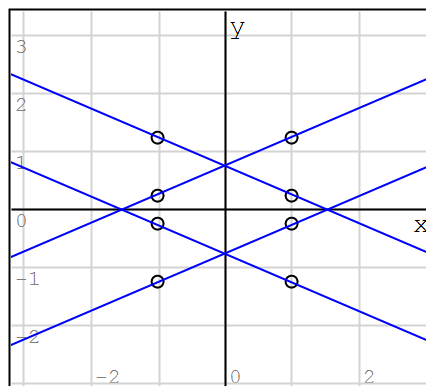
$\Pi(M)$



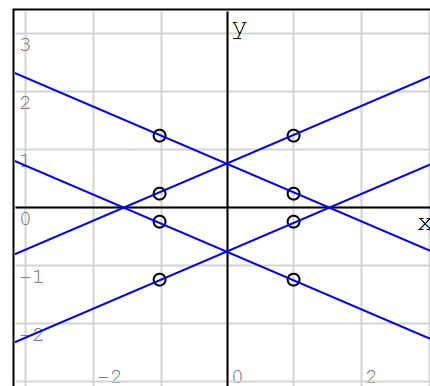
$\Pi(P)$

$$M := \begin{bmatrix} 2 & 4 & -3 \\ 2 & -4 & -3 \\ -2 & 4 & -3 \\ -2 & -4 & -3 \end{bmatrix}$$

$$P := \begin{bmatrix} \begin{bmatrix} 1 & 0.25 \\ -1 & 1.25 \end{bmatrix} \\ \begin{bmatrix} 1 & -0.25 \\ -1 & -1.25 \end{bmatrix} \\ \begin{bmatrix} 1 & 1.25 \\ -1 & 0.25 \end{bmatrix} \\ \begin{bmatrix} 1 & -1.25 \\ -1 & -0.25 \end{bmatrix} \end{bmatrix}$$

$$\Pi(MP) := \begin{cases} pLine(MP) \\ \text{mat2sys}_1(\text{augment}(P, "o")) \end{cases}$$


$\Pi(M)$



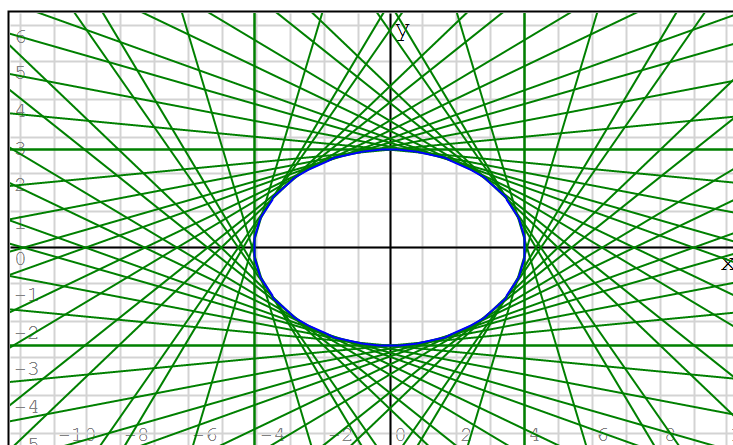
$\Pi(P)$

Tangents to an implicit plot

$$f(x, y) := \frac{x^2}{4} + \frac{y^2}{\sqrt{3}} - 4$$

$$\left[B := \begin{bmatrix} -4 & 4 \\ -4 & 4 \end{bmatrix} \quad N := \begin{bmatrix} 30 \\ 30 \end{bmatrix} \right]$$

$$C := pContour("f", B, N)$$

$$\Pi := \begin{cases} pCycleC(C) \\ "" \\ pLine(C) \end{cases}$$


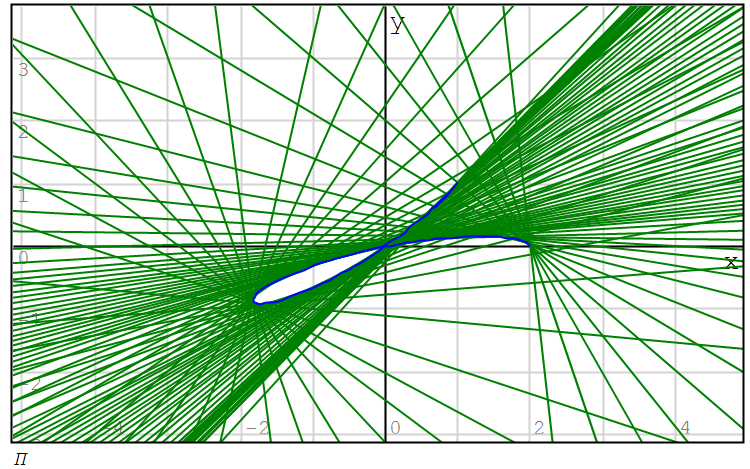
Π

Tangent to an adaptive plot

```

ρ(θ) := eval(2 · cos(7 · θ) · ei · θ)
C := ReIm(pAdapt(ρ, 0, 0.25 · π))
Π := {
  C
  " "
  pLine(C)
}

```



Π

□ — pEnvelope

Enevelopes

`pEnvelope(A)` Returns the intersections of two consecutive lindes defined in A as the coefficients of a rect line given by its general formula $ax+by+c=0$

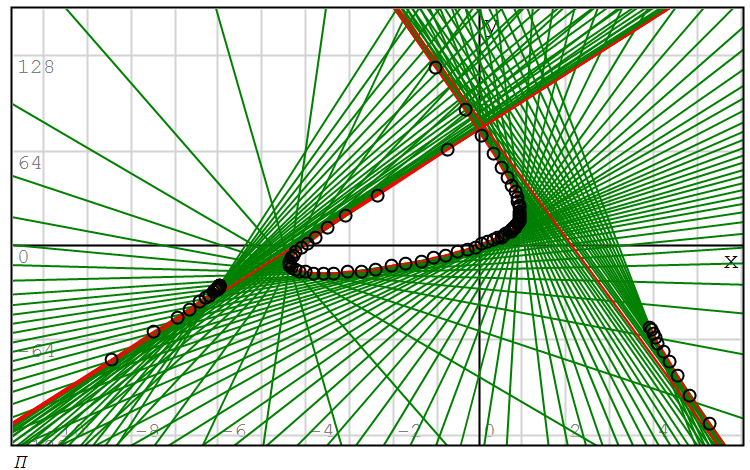
Examles

This procedure draws the envelope of the uniparametric family of lines given by λ

```

λ(t) := [ t - 1 0.1 · cos(t) t2 ]
T := pR(-4, 4, 100)
λ := pRGrid("λ", T)
E := pEnvelope(λ)
Π := {
  augment(E, "o")
  E
  pLine(λ)
}

```



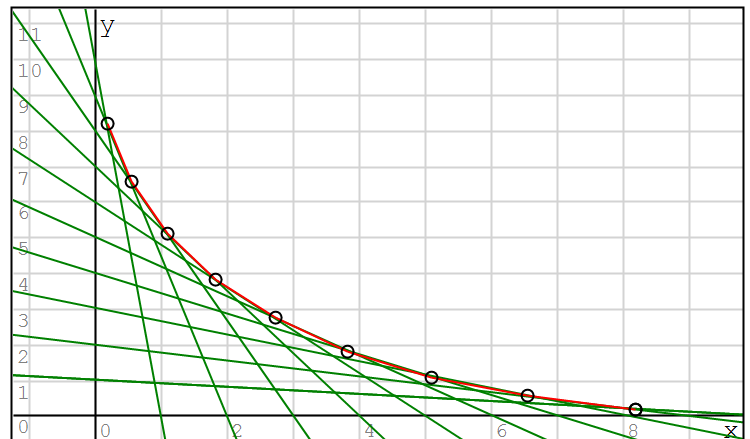
Π

This example compare the numerical procedure with the symbolic one.

```

λ(t) := [ 1/t 1/(11-t) -1 ]
T := pR(1, 11, 10)
λ := pRGrid("λ", T)
E := pEnvelope(λ)
Π := {
  augment(E, "o")
  E
  pLine(λ)
}

```

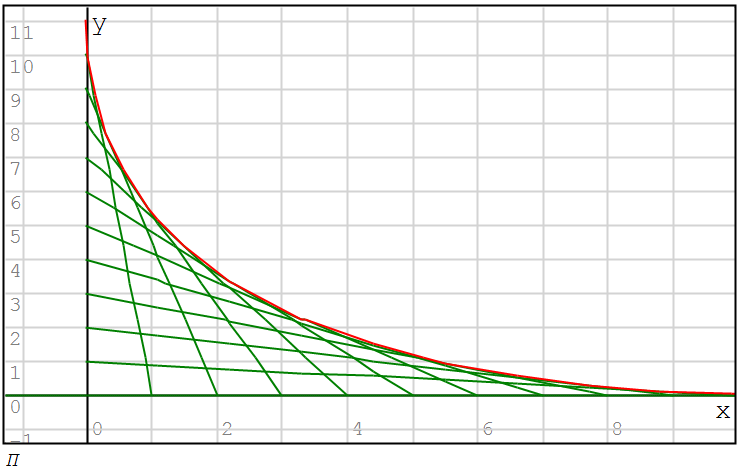


The symbolic solution isn't immediate:

```

E(x,y):=(x-y)2-22.(x+y)+121
C(x,y,t):=[x y 1]T.λ(t)T
[B:= [0 11] N:= [20]
 [0 11] [20]]
for k ∈ [1..length(T)]
    C(x,y):=C(x,y,Tk)
    Cok:=pCycleC(pContour("C",B,N))
Π:= {
    ""
    pCycleC(pContour("E",B,N))
    pCycleC(Co)
}

```



Alvaro