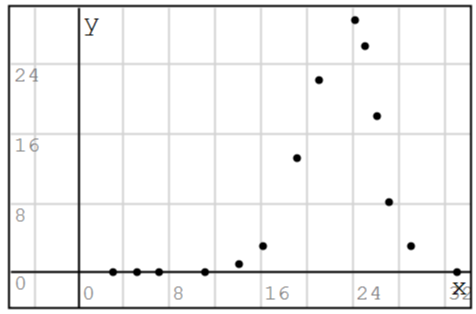
Using splines. CE Tutorial. Numerical methods. Chemical Engineering, October 26, 1987. In regions of sharrply different behavior, splines are often superior to polynomials in generating x - y relationships from data.Example 1

x,time,min	y,dye concentration, g/gal
3	0
5	0
7	0
11	0
14	1
16	3
19	13
21	22
24	29
25	26
26	18
27	8
29	3
33	0
36	0
40	0
44	0

M

$X := M_{[2..17]1}$ $Y := M_{[2..17]2}$



augment(X, Y, ".")

Input data:

$f(x) := \mathrm{cinterp}(X, Y, x)$ integrand

$a := \min(X)$ inferior limit

$b := \max(X)$ superior limit

$maxerr := 10^{-7}$ accuracy

$$simp(\varphi(1), a, b, h, n) := \frac{1}{3} \cdot \left(\frac{h}{2} \cdot (\varphi(a) + \varphi(b)) + h \cdot \sum_{k=1}^{n-1} \varphi(a + k \cdot h) \right) + \frac{2}{3} \cdot h \cdot \sum_{k=1}^{n} \varphi\left(a - \frac{h}{2} + k \cdot h\right)$$

Calculation:

$n := 2$

$h := \frac{b - a}{n}$

$dintp := simp(f(x\#), a, b, h, n)$

$\varepsilon := dintp$

while $|\varepsilon| > maxerr$

$n := 2 \cdot n$

$h := \frac{b - a}{n}$

$dint := simp(f(x\#), a, b, h, n)$

$\varepsilon := \frac{dint - dintp}{}$

$dintp := dint$

$n = 512$

$\varepsilon = 8.8754979 \cdot 10^{-9}$

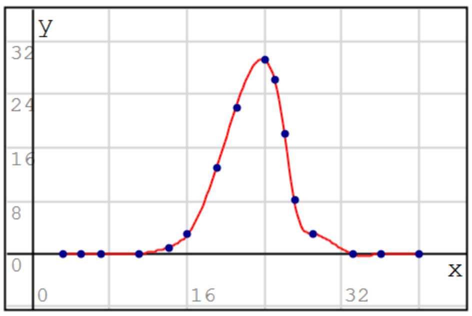
$dint := dint + \varepsilon$

Result:

$dint = 219.8233389$

$AUC := 218.8\ \mathrm{g}\ \dfrac{\mathrm{min}}{\mathrm{gal}} = 3468.0505\ \dfrac{\mathrm{s\ kg}}{\mathrm{m}^3}$

Now we can plot the graph for the experimental data and the interpolated curve with cubic splines.



$$\begin{cases} \mathrm{augment}(X, Y, ".", 10, "Dark\ Blue") \\ \begin{cases} \mathrm{cinterp}(X, Y, x) & \text{if } a \le x \le b \\ "" & \text{otherwise} \end{cases} \end{cases}$$

CSspline

$c := 17$ Tangent point

$MC := CS_V(X, Y)$

$f(x) := CS(MC, x)$

$dc := CS_D(MC, c)$

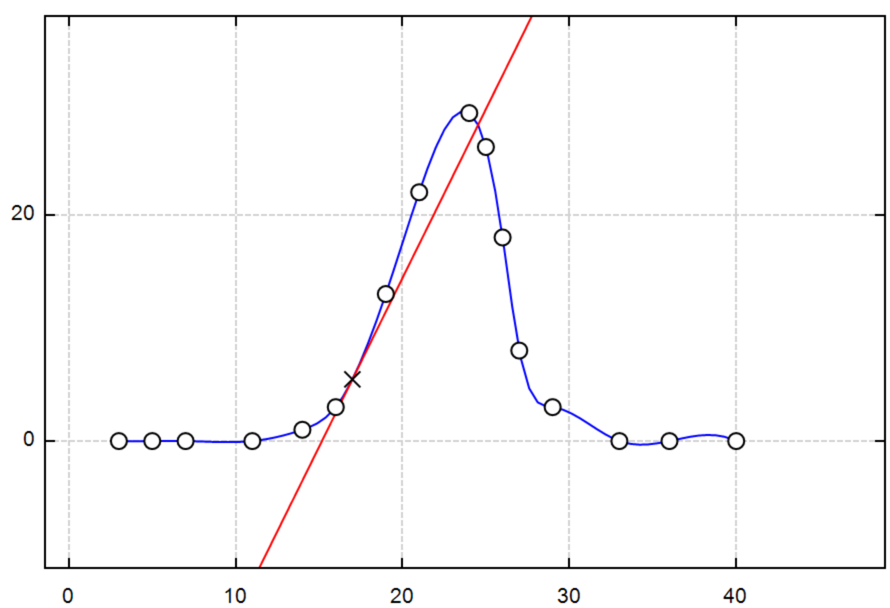
$yc := f(c)$

$$\Pi := \begin{cases} f(x) \\ y - yc - dc \cdot (x - c) \\ \mathrm{augment}(X, Y, ".") \\ \mathrm{augment}(c, yc, "x") \end{cases}$$

$\boxed{CS_I(MC, a, b) = 220.9223}$ $\displaystyle\int_a^b f\, \mathrm{d}x$

$\boxed{CS_D(MC, c) = 2.9863}$ $f'(c)$

$\boxed{CS_{D2}(MC, c) = 0.9739}$ $f''(c)$



Alvaro