

$$X := \begin{bmatrix} 30.4 \\ 0 \\ 63.9 \\ 387.1 \\ 330 \\ 187.3 \\ 495.9 \\ 18 \\ 163.3 \\ 128.8 \end{bmatrix}$$

278.7
13.22

$$\text{appVersion}(3) = "0.99.7251"$$

$$X_{mean} := \text{Mean}(X) = 180.47$$

$$LN X(x\#) := \begin{cases} 0 & \text{if } x\# = 0 \\ \ln(x\#) & \text{otherwise} \end{cases}$$

$$\text{Mean}(\overrightarrow{\ln(X)}) = \blacksquare$$

$$LN X := \overrightarrow{LN X(X)}$$

$$LN X_{mean} := \text{Mean}(LN X) = 4.3613$$

$$A := \ln(X_{mean}) - LN X_{mean} = 0.8343$$

$$\varepsilon := 10^{-12}$$

$$\alpha := \text{eval}\left(\frac{1}{4 \cdot A} \cdot \left(1 + \sqrt{1 + \frac{4 \cdot A}{3}}\right)\right) = 0.7352$$

$$\alpha = 0.7352$$

$$\beta := \text{eval}\left(\frac{X_{mean}}{\alpha}\right) = 245.4756$$

Works correct only if
numerically evaluated.
Otherwise gives different
results in evaluating F & G
when using the **vectorize**
operator (only if $\alpha \geq 1$)

If $0 < \alpha < 1$ in **Eqn 1** throwss an error . This behaviour seems to be a bug, either
in **SS 7251** OR evaluation of **Eqn 1**. But it evaluates correctly in EXCEL

$$F(z) := \frac{1}{\beta^\alpha \cdot \text{Gamma}(\alpha)} \cdot \text{eval} \left(\int_{\varepsilon}^z x^{\alpha-1} \cdot \exp\left(-\frac{x}{\beta}\right) dx \right)$$

Eqn 1

$$Q(u) := \begin{bmatrix} \text{for } j \in [1..rows(u)] \\ \quad \text{eval}(B_j := F(u_j)) \\ B \end{bmatrix}$$

$$G := Q(X) = \blacksquare$$

$$\overrightarrow{F(X)} = \blacksquare$$

$$F\left(X_1\right) = 1.29$$

lastError = "x - not defined."

As per Jean's method

$$\varepsilon = 1 \cdot 10^{-12}$$



This solves the problem

$$CDF(z, \alpha, \beta) := \frac{1}{\Gamma(\alpha) \cdot \beta^\alpha} \cdot \text{eval} \left(\int_{\varepsilon}^z x^{\alpha-1} \cdot e^{-\frac{x}{\beta}} dx \right)$$

eval(,) drops timing ~ 0.75
acc accuracy 50/100 plenty.

$$CDF2(z, \alpha, \beta) := \begin{cases} 0 & \text{if } z = 0 \\ \frac{1}{\Gamma(\alpha) \cdot \beta^\alpha} \cdot \text{eval} \left(\int_{\varepsilon}^z x^{\alpha-1} \cdot e^{-\frac{x}{\beta}} dx \right) & \text{otherwise} \end{cases}$$

Mathcad result**Excel Result**

$$\overrightarrow{CDF(X, \alpha, \beta)} = \blacksquare$$

$$G(X_j) =$$

0.154
0.0702
0.2964
0.8811
0.8372
0.6431
0.9346
0.0943
0.5927
0.5076

X	LN(X)	X_Mean	Gamma CDF
30.4	3.414443	181.792	0.1540
13.22	2.581731	LN(X)_Mean	0.0702
63.9	4.157319	4.6195	0.2964
387.1	5.958683	A	0.8811
330	5.799093	0.58341	0.8372
187.3	5.232712	Alpha	0.6431
495.9	6.206374	0.9999	0.9346
18	2.890372	Beta	0.0943
163.3	5.095589	181.8117	0.5927
128.8	4.858261		0.5076

$$X = \begin{bmatrix} 30.4 \\ 0 \\ 63.9 \\ 387.1 \\ 330 \\ 187.3 \\ 495.9 \\ 18 \\ 163.3 \\ 128.8 \end{bmatrix} \xrightarrow{CDF2(X, \alpha, \beta)} \begin{bmatrix} 1.29 \\ 0 \\ 2.6078 \\ 14.4668 \\ 12.421 \\ 7.2425 \\ 18.3415 \\ 0.7866 \\ 6.3574 \\ 5.0736 \end{bmatrix}$$

X	LN(X)	X_Mean	Gamma CDF
30.4	3.414443	180.47	0.2233
0	0	LN(X)_Mean	0.0000
63.9	4.157319	4.3613	0.3647
387.1	5.958683	A	0.8684
330	5.799093	0.83428	0.8291
187.3	5.232712	Alpha	0.6644
495.9	6.206374	0.7352	0.9194
18	2.890372	Beta	0.1551
163.3	5.095589	245.4756	0.6224
128.8	4.858261		0.5508

$$X_2 = 0 \quad X_1 = 30.4$$

$$CDF(X_1, \alpha, \beta) = 1.29$$

$$CDF2(X_2, \alpha, \beta) = 0$$

$$\alpha = 0.7352$$

$$\beta = 245.4756$$

$$LNX = \begin{bmatrix} 3.414443 \\ 0 \\ 4.157319 \\ 5.958683 \\ 5.799093 \\ 5.232712 \\ 6.206374 \\ 2.890372 \\ 5.095589 \\ 4.858261 \end{bmatrix} \quad \begin{array}{l} X_{mean} = 180.47 \\ LNX_{mean} = 4.3613 \\ A = 0.83428 \\ \alpha = 0.7352 \\ \beta = 245.4756 \end{array}$$

$$\frac{1}{\Gamma(\alpha) \cdot \beta^\alpha} \cdot \text{eval} \left(\int_{\varepsilon}^{X_1} x^{\alpha-1} \cdot \exp\left(-\frac{x}{\beta}\right) dx \right) = 1.29 \quad \varepsilon := 10^{-12}$$

$$\varepsilon = 1 \cdot 10^{-12}$$

$$F^3(u) := \begin{cases} 0 & \text{if } u = 0 \\ \frac{1}{\Gamma(\alpha) \cdot \beta^\alpha} \cdot \text{eval} \left(\int_{\varepsilon}^u x^{\alpha-1} \cdot \exp\left(-\frac{x}{\beta}\right) dx \right) & \text{otherwise} \end{cases}$$

$$F^3\left(\begin{matrix} X \\ 1 \end{matrix}\right) = 1.29$$

$$CDF^3(u, \alpha, \beta) := \begin{cases} j := [1..rows(u)] \\ \text{if } u_j = 0 \\ \quad B_j := 0 \\ \text{else} \\ \quad \text{eval}(B_j := F^3(u_j)) \end{cases}$$

$$\alpha = 0.7352$$

$$\beta = 245.4756$$

Excel Result

X	LN(X)	X Mean	Gamma CDF
30.4	3.414443	180.47	0.2233
0	0	LN _X Mean	0.0000
63.9	4.157319	4.3613	0.3647
387.1	5.958683	A	0.8684
330	5.799093	0.83428	0.8291
187.3	5.232712	Alpha	0.6644
495.9	6.206374	0.7352	0.9194
18	2.890372	Beta	0.1551
163.3	5.095589	245.4756	0.6224
128.8	4.858261		0.5508

Wrong results

Wrong Result

$$CDF^3(X, \alpha, \beta) = \begin{matrix} 1.2925 \\ 0 \\ 2.6131 \\ 14.4981 \\ 12.4477 \\ 7.2577 \\ 18.3811 \\ 0.788 \\ 6.3705 \\ 5.0841 \end{matrix}$$

Data

$$X = \begin{bmatrix} 30.4 \\ 0 \\ 63.9 \\ 387.1 \\ 330 \\ 187.3 \\ 495.9 \\ 18 \\ 163.3 \\ 128.8 \end{bmatrix}$$

Python 3.8.3 Result

3: Mean X

180.47

4: LN_X

[3.4144 0. 4.1573 5.9587 5.7991 5.2327 6.2064 2.8904 5.0956 4.8583]

5: Mean LN_X

4.3613

6: A | alpha | beta

[0.8343, 0.7352, 245.4756]

7: Gamma distrb. CDF

[0.2233 0. 0.3647 0.8684 0.8291 0.6644 0.9194 0.1551 0.6224 0.5508]

Excel Result

X	LN(X)	X _{Mean}	Gamma CDF
30.4	3.414443	180.67	0.2063
2	0.693147	LN _X Mean	0.0253
63.9	4.157319	4.4306	0.3487
387.1	5.958683	A	0.8711
330	5.799093	0.76607	0.8307
187.3	5.232712	Alpha	0.6594
495.9	6.206374	0.7903	0.9229
18	2.890372	Beta	0.1395
163.3	5.095589	228.6037	0.6155
128.8	4.858261		0.5409

Excel Result

X	LN(X)	X _{Mean}	Gamma CDF
30.4	3.414443	180.47	0.2233
0	0	LN _X Mean	0.0000
63.9	4.157319	4.3613	0.3647
387.1	5.958683	A	0.8684
330	5.799093	0.83428	0.8291
187.3	5.232712	Alpha	0.6644
495.9	6.206374	0.7352	0.9194
18	2.890372	Beta	0.1551
163.3	5.095589	245.4756	0.6224
128.8	4.858261		0.5508

3: Mean X

180.67

4: LN_X

[3.4144 0.6931 4.1573 5.9587 5.7991 5.2327 6.2064 2.8904 5.0956 4.8583]

5: Mean LN_X

4.4306

6: A | alpha | beta

[0.7661, 0.7903, 228.6037]

7: Gamma distrb. CDF

[0.2063 0.0253 0.3487 0.8711 0.8307 0.6594 0.9229 0.1395 0.6155 0.5409]

3: Mean X

180.47

4: LN_X

[3.4144 0. 4.1573 5.9587 5.7991 5.2327 6.2064 2.8904 5.0956 4.8583]

5: Mean LN_X

4.3613

6: A | alpha | beta

[0.8343, 0.7352, 245.4756]

7: Gamma distrb. CDF

[0.2233 0. 0.3647 0.8684 0.8291 0.6644 0.9194 0.1551 0.6224 0.5508]