appVersion(4) = "0.99.7774.1835"

Numerical Solution of Colenrook-White Equation using SMath Studio 'NewtonRaphson' function

$$\frac{1}{\sqrt{f}} = -2 \cdot \log 10\left(\frac{k}{3.7 \cdot D} + \frac{2.51}{Re \cdot \sqrt{f}}\right)$$

$t_0 := \text{time}(0)$

Numerical Solution

'NewtonRaphson' Method : To find friction factor 'f' of Colebrook-White Eqn

Define $\zeta := 10^{-10}$

Substitute $x = \frac{1}{\sqrt{f}}$ → $\frac{1}{\sqrt{f}} = -2 \cdot \log 10\left(\frac{k}{3.7 \cdot D} + \frac{2.51}{Re \cdot \sqrt{f}}\right)$ → $x + 2 \cdot \log 10\left(\frac{k}{3.7 \cdot d} + \frac{2.51}{Re} \cdot x\right)$

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PROGRAM 1: Use 'NewtonRaphson' function to find 'f' for a pipe
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$NR\_SOLVE(k, d, Re) :=$

$F(x) := x + 2 \cdot \log 10\left(\frac{k}{3.7 \cdot d} + \frac{2.51}{Re} \cdot x\right)$	Linear Eqn for single pipe
$x_{init} := \frac{1}{\sqrt{0.01}}$	Start with **min** **'f'** value in **Moody chart**
$x := \text{NewtonRaphson}(F(x), x_{init}, \zeta)$	Solve for 'x'
$f := \frac{1}{x^2}$	Friction factor

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PROGRAM 2: Find 'f' for ALL PIPES in a LOOP
Calls PROGRAM 1
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$Calc\_LOOP\_f(q, k, d) :=$

$$v := \overrightarrow{\frac{|q|}{\frac{\pi}{4} \cdot d^2}}$$

$$Re := \overrightarrow{\left|\frac{v \cdot d}{\nu}\right|}$$

$$ff := \overrightarrow{NR\_SOLVE(k, d, Re)}$$

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PROGRAM 3 : 'f' values for ALL LOOPS
Calls PROGRAM 2 with Vectorize Function
for Nested Array
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$$Calc\_All\_LOOP\_f(q, k, d) := \overrightarrow{Calc\_LOOP\_f(q, k, d)}$$

1. DATA

Assumed Flows for Continuity in LOOP 1

$$Q1 := \begin{bmatrix} -103.56 \\ 200 \\ 40 \\ -53.56 \end{bmatrix} \frac{L}{s}$$

Assumed Flows for Continuity in LOOP 2

$$Q2 := \begin{bmatrix} -40 \\ 160 \\ 7.22 \\ 7.22 \end{bmatrix} \frac{L}{s}$$

Assumed Flows for Continuity in LOOP 3

$$Q3 := \begin{bmatrix} 53.56 \\ -50 \\ 100.78 \\ -13.8 \end{bmatrix} \frac{L}{s}$$

Kinematic viscosity

$$\nu := 1.005 \cdot 10^{-6} \frac{m^2}{s}$$

Lengths LOOP 1

$$L1 := \begin{bmatrix} 1000 \\ 2000 \\ 1000 \\ 2000 \end{bmatrix} m$$

Lengths LOOP 2

$$L2 := \begin{bmatrix} 1000 \\ 2000 \\ 500 \\ 2200 \end{bmatrix} m$$

Lengths LOOP 3

$$L3 := \begin{bmatrix} 2000 \\ 1000 \\ 750 \\ 2200 \end{bmatrix} m$$

Dia. LOOP 1

$$D1 := \begin{bmatrix} 0.4 \\ 0.45 \\ 0.30 \\ 0.30 \end{bmatrix} m$$

Dia. LOOP 2

$$D2 := \begin{bmatrix} 0.3 \\ 0.3 \\ 0.25 \\ 0.25 \end{bmatrix} m$$

Dia. LOOP 3

$$D3 := \begin{bmatrix} 0.3 \\ 0.4 \\ 0.3 \\ 0.3 \end{bmatrix} m$$

Ks: LOOP 1

$$\begin{bmatrix} 0.00005 \end{bmatrix}$$

Ks: LOOP 2

$$\begin{bmatrix} 0.00005 \end{bmatrix}$$

Ks: LOOP 3

$$\begin{bmatrix} 0.00005 \end{bmatrix}$$

$$Ks1 := \begin{bmatrix} 0.00005 \\ 0.00005 \\ 0.00005 \end{bmatrix} \text{m} \qquad Ks2 := \begin{bmatrix} 0.00003 \\ 0.00003 \\ 0.00003 \end{bmatrix} \text{m} \qquad Ks3 := \begin{bmatrix} 0.00003 \\ 0.00003 \\ 0.00003 \end{bmatrix} \text{m}$$

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2. CALCULATIONS

Define Nested Arrays for All LOOPS

$$QQ := \begin{bmatrix} Q1 \\ Q2 \\ Q3 \end{bmatrix} \qquad DD := \begin{bmatrix} D1 \\ D2 \\ D3 \end{bmatrix} \qquad KK := \begin{bmatrix} Ks1 \\ Ks2 \\ Ks3 \end{bmatrix}$$

'f' values for Each LOOP using Program 2

LOOP 1 $f1 := Calc\_LOOP\_f(Q1, Ks1, D1)$

LOOP 2 $f2 := Calc\_LOOP\_f(Q2, Ks2, D2)$

LOOP 3 $f3 := Calc\_LOOP\_f(Q3, Ks3, D3)$

3. RESULTS: 'f' values for ALL LOOPS

Call PROGRAM 3 : for all LOOPS

$$ff := Calc\_All\_LOOP\_f(QQ, KK, DD) = \begin{bmatrix} \begin{bmatrix} 0.0155 \\ 0.0144 \\ 0.0173 \\ 0.0166 \end{bmatrix} \\ \begin{bmatrix} 0.0173 \\ 0.014 \\ 0.0228 \\ 0.0228 \end{bmatrix} \\ \begin{bmatrix} 0.0166 \\ 0.0169 \\ 0.0147 \\ 0.0206 \end{bmatrix} \end{bmatrix}$$

Compare

$$f1 = \begin{bmatrix} 0.0155 \\ 0.0144 \\ 0.0173 \\ 0.0166 \end{bmatrix}$$

$$f2 = \begin{bmatrix} 0.0173 \\ 0.014 \\ 0.0228 \\ 0.0228 \end{bmatrix}$$

$$f3 = \begin{bmatrix} 0.0166 \\ 0.0169 \\ 0.0147 \\ 0.0206 \end{bmatrix}$$

$\text{time}(0) - t_0 = 0.3 \text{ s}$