References:

[1] Timoshenko - Vibration Problems in Engineering pag 342

[2]

[3]

$\nu := 0.298$ $\qquad E := 198000\ \mathrm{MPa}$ $\qquad \rho := 8000\ \dfrac{\mathrm{kg}}{\mathrm{m}^3}$

$$G := \frac{E}{2\cdot(1+\nu)}$$

$$\kappa\_hollow := 2\cdot\frac{(1+\nu)}{4+3\cdot\nu} = 0.5304$$

Hollow cylinder cross section

$L := 0.32\ \mathrm{m}$ $\qquad mass := 0.98\ \dfrac{\mathrm{kg}}{\mathrm{m}}$ $\qquad Isect := 2.331\cdot10^{-8}\ \mathrm{m}^4$ $\qquad Asect := 1.225\cdot10^{-4}\ \mathrm{m}^2$

$$r(Isect, Asect) := \sqrt{\left(\frac{Isect}{Asect}\right)}$$

$$\varepsilon := \kappa\_hollow$$

$$a(Isect, mass) := \sqrt{\left(E\cdot\frac{Isect}{mass}\right)}$$

$$\beta(\omega, Isect, mass) := \sqrt[4]{\left(\frac{\omega^2}{a(Isect, mass)^2}\right)}$$

$$\alpha(L, Isect, Asect) := \frac{1}{\varepsilon}\cdot\left(\frac{r(Isect, Asect)}{L}\right)^2\cdot\frac{E}{G}$$

$$\gamma(L, Isect, Asect) := \left(\frac{r(Isect, Asect)}{L}\right)^2$$

$$A(\omega, Isect, Asect, mass, L) := (\beta(\omega, Isect, mass)\cdot L)^4\cdot(\alpha(L, Isect, Asect)+\gamma(L, Isect, Asect))$$

$$B(\omega, Isect, Asect, mass, L) := \sqrt{\left((\beta(\omega, Isect, mass)\cdot L)^8\cdot((\alpha(L, Isect, Asect))-(\gamma(L, Isect, Asect)))^2\right.}$$

$$\eta(\omega, Isect, Asect, mass, L) := \frac{\sqrt{2}}{2\cdot L}\cdot\sqrt{(-A(\omega, Isect, Asect, mass, L)+B(\omega, Isect, Asect, mass, L))}$$

$$\theta(\omega, Isect, Asect, mass, L) := \frac{\sqrt{2}}{2\cdot L}\cdot\sqrt{(A(\omega, Isect, Asect, mass, L)+B(\omega, Isect, Asect, mass, L))}$$

$$\sigma 1(\omega, Isect, Asect, mass, L) := \frac{1}{L} \cdot \left(\theta(\omega, Isect, Asect, mass, L) \cdot L - \frac{\alpha(L, Isect, Asect)}{\theta(\omega, Isect, Asect, mass, L) \cdot L} \cdot (\beta\right.$$

$$\sigma 2(\omega, Isect, Asect, mass, L) := \frac{1}{L} \cdot \left(\eta(\omega, Isect, Asect, mass, L) \cdot L + \frac{\alpha(L, Isect, Asect)}{\eta(\omega, Isect, Asect, mass, L) \cdot L} \cdot (\beta\right.$$

$$k(Isect) := E \cdot Isect$$

$$\sigma 11(\omega, Isect, Asect, mass, L) := \left(\theta(\omega, Isect, Asect, mass, L)^{3} - \alpha(L, Isect, Asect) \cdot L^{2} \cdot \beta(\omega, Isect, \right.$$

$$\sigma 21(\omega, Isect, Asect, mass, L) := \left(\eta(\omega, Isect, Asect, mass, L)^{3} + \alpha(L, Isect, Asect) \cdot L^{2} \cdot (\beta(\omega, Isect, \right.$$

$$\sigma 12(\omega, Isect, Asect, mass, L) := \left((\theta(\omega, Isect, Asect, mass, L))^{2} - \alpha(L, Isect, Asect) \cdot L^{2} \cdot (\beta(\omega, Isect \right.$$

$$\sigma 21(\omega, Isect, Asect, mass, L) := \left((\eta(\omega, Isect, Asect, mass, L))^{3} + \alpha(L, Isect, Asect) \cdot L^{2} \cdot (\beta(\omega, Isec \right.$$

$$\sigma 22(\omega, Isect, Asect, mass, L) := \left((\eta(\omega, Isect, Asect, mass, L))^{2} + \alpha(L, Isect, Asect) \cdot L^{2} \cdot (\beta(\omega, Isec \right.$$

$$\varphi 11(\omega, Isect, Asect, mass, L) := \frac{1}{\sigma 12(\omega, Isect, Asect, mass, L) + \sigma 22(\omega, Isect, Asect, mass, L)} \cdot (\sigma 22$$

$$\varphi 12(\omega, Isect, Asect, mass, L) := \frac{1}{\sigma 1(\omega, Isect, Asect, mass, L) \cdot \sigma 21(\omega, Isect, Asect, mass, L) + \sigma 2(\omega,}$$

$$\varphi 13(\omega, Isect, Asect, mass, L) := \frac{1}{(k(Isect)) \cdot (\sigma 1(\omega, Isect, Asect, mass, L) \cdot \sigma 21(\omega, Isect, Asect, mass}$$

$$\varphi 14(\omega, Isect, Asect, mass, L) := \frac{1}{(k(Isect)) \cdot (\sigma 12(\omega, Isect, Asect, mass, L) + \sigma 22(\omega, Isect, Asect, ma}$$

$$\varphi 21(\omega, Isect, Asect, mass, L) := \frac{1}{\sigma 12(\omega, Isect, Asect, mass, L) + \sigma 22(\omega, Isect, Asect, mass, L)} \cdot (\sigma 1(\omega$$

$$\varphi 22(\omega, Isect, Asect, mass, L) := \frac{1}{\sigma 1(\omega, Isect, Asect, mass, L) \cdot \sigma 21(\omega, Isect, Asect, mass, L) + \sigma 2(\omega,}$$

$$\varphi 23(\omega, Isect, Asect, mass, L) := \frac{\sigma 1(\omega, Isect, Asect, mass, L) \cdot}{(k(Isect)) \cdot (\sigma 1(\omega, Isect, Asect, mass, L) \cdot \sigma 21(\omega, Isect, Asect, mass}$$

$$\varphi 24(\omega, Isect, Asect, mass, L) := \frac{1}{(k(Isect)) \cdot (\sigma 12(\omega, Isect, Asect, mass, L) + \sigma 22(\omega, Isect, Asect, ma}$$

$$\varphi 31(\omega, Isect, Asect, mass, L) := \frac{(k(Isect))}{(\sigma 12(\omega, Isect, Asect, mass, L) + \sigma 22(\omega, Isect, Asect, mass, L))} \cdot (\sigma 1$$

$$\varphi 32(\omega, Isect, Asect, mass, L) := \frac{(-k(Isect) \cdot \sigma 11(\omega, Isect, Asect, mass, L) \cdot}{(\sigma 1(\omega, Isect, Asect, mass, L) \cdot \sigma 21(\omega, Isect, Asect, mass, L) + \sigma 2(\omega}$$

$$\varphi 33(\omega, Isect, Asect, mass, L) := \frac{1}{(\sigma 1(\omega, Isect, Asect, mass, L) \cdot \sigma 21(\omega, Isect, Asect, mass, L) + \sigma 2(\omega}$$

$$\varphi 34(\omega, Isect, Asect, mass, L) := \frac{1}{\sigma 12(\omega, Isect, Asect, mass, L) + \sigma 22(\omega, Isect, Asect, mass, L)} \cdot ((-\sigma$$

$$\varphi 41(\omega, Isect, Asect, mass, L) := \frac{-(k(Isect)) \cdot \sigma 12(\omega, Isect, Asect, mass, L) \cdot \sigma 22(\omega, Isect, Asect, ma}{(\sigma 12(\omega, Isect, Asect, mass, L) + \sigma 22(\omega, Isect, Asect, mass, L)}$$

$$\varphi 42(\omega, Isect, Asect, mass, L) := \frac{-(k(Isect))}{(\sigma 1(\omega, Isect, Asect, mass, L) \cdot \sigma 21(\omega, Isect, Asect, mass, L) + \sigma 2(\omega}$$

$$\varphi 43(\omega, Isect, Asect, mass, L) := \frac{1}{(\sigma 1(\omega, Isect, Asect, mass, L) \cdot \sigma 21(\omega, Isect, Asect, mass, L) + \sigma 2(\omega}$$

$$\varphi 44(\omega, Isect, Asect, mass, L) := \frac{1}{(\sigma 12(\omega, Isect, Asect, mass, L) + \sigma 22(\omega, Isect, Asect, mass, L))} \cdot (\sigma 1$$

added eval here

$$\Phi(\omega) := \mathrm{eval}\left(\left[\begin{matrix} \varphi 11(\omega, Isect, Asect, mass, L) & \varphi 12(\omega, Isect, Asect, mass, L) & \varphi 13(\omega, Isect, Asect, mass \\ \varphi 21(\omega, Isect, Asect, mass, L) & \varphi 22(\omega, Isect, Asect, mass, L) & \varphi 23(\omega, Isect, Asect, mass \\ \varphi 31(\omega, Isect, Asect, mass, L) & \varphi 32(\omega, Isect, Asect, mass, L) & \varphi 33(\omega, Isect, Asect, mass \\ \varphi 41(\omega, Isect, Asect, mass, L) & \varphi 42(\omega, Isect, Asect, mass, L) & \varphi 43(\omega, Isect, Asect, mass \end{matrix}\right.\right.$$

$$detZ := \varphi 31(\omega, Isect, Asect, mass, L) \cdot \varphi 42(\omega, Isect, Asect, mass, L) - \varphi 32(\omega, Isect, Asect, mass, L) \cdot$$

$\mathrm{strlen}(\mathrm{num2str}(detZ)) = 25257$ very very long

$detZ := (ratsimp(detZ))$

$\mathrm{strlen}(\mathrm{num2str}(detZ)) = 8158$ very long

$detZ(\omega) := detZ$

$detZ' := (ratsimp(\mathrm{Diff}(detZ(xx), xx)))$ can't use ω here, looks like a bug in maxima for the greek letter.

$detZ(\omega) := \mathrm{eval}(detZ)$ $detZ'(xx) := \mathrm{eval}(detZ')$

```
NR(F(1), x) := | [MI  h  εx] := [25  10^-5·UnitsOf(x)  10^-5·UnitsOf(x)]
               | for iter ∈ [1..MI]
               |   | Fx := F(x)
               |   | x' := eval(x − h·Fx / (F(x + h) − Fx))
               |   | if |x − x'| > εx
               |   |    x := x'
               |   | else
               |   |    break
               | x
```

for $n \in [1..8]$
$\Omega_n := NR(detZ(x), 20 \cdot n\ \mathrm{kHz})$

for $n \in [1..7]$
$\Omega'_n := NR(detZ'(x), (10 + 20 \cdot (n-1))\ \mathrm{kHz})$

$\mathrm{normi}\left(\Delta Z := \overrightarrow{detZ(\Omega)\ \mathrm{MN}^{-2}}\right) = 3.4 \cdot 10^{-6}$

$\mathrm{normi}\left(\Delta Z' := \overrightarrow{detZ'(\Omega')\ \mathrm{MN}^{-2}\ \mathrm{kHz}}\right) = 1.6 \cdot 10^{-8}$

$$\Omega = \begin{bmatrix} 13.7131 \\ 32.8356 \\ 55.3997 \\ 78.9188 \\ 102.4893 \\ 125.3542 \\ 146.9698 \\ 162.0383 \end{bmatrix} \text{kHz} \qquad \Omega' = \begin{bmatrix} 10.9222 \\ 27.4924 \\ 47.5194 \\ 68.9282 \\ 90.8209 \\ 112.5643 \\ 133.5257 \end{bmatrix} \text{kHz}$$

Brute force imag part detection

$\omega i := \Omega_8 \qquad \Delta\omega i := 1\ \text{kHz}$

```
for n ∈ [1..6]
 | while try
 |         Im(detZ(ωi) MN^-2) = 0
 |       on error
 |         0
 |     ωi := ωi + Δωi
 | [ ωi := ωi − Δωi  Δωi := 0.1·Δωi ]
```

$\omega i = 163.0244\ \text{kHz}$

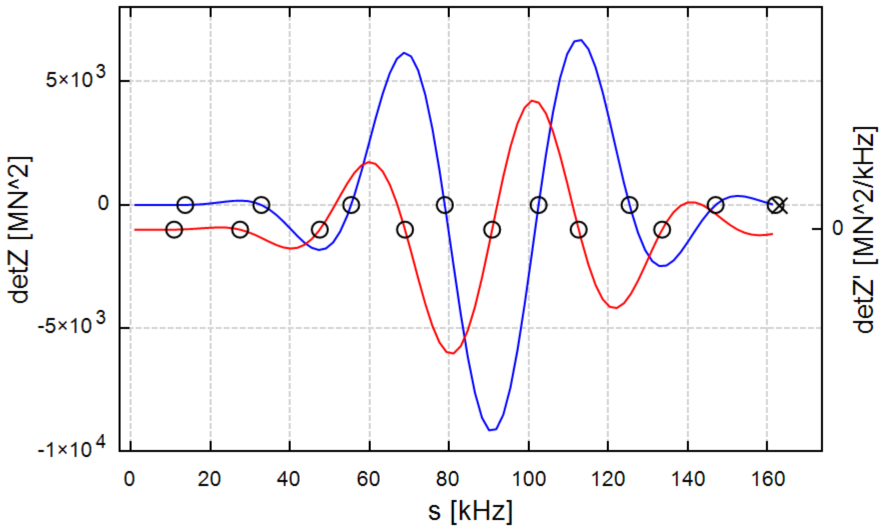
$\omega i - \Omega_8 = 986.04\ \text{Hz}$

'last' real value:

$detZ(\omega i) = -29.9022\ \text{MN}^2$

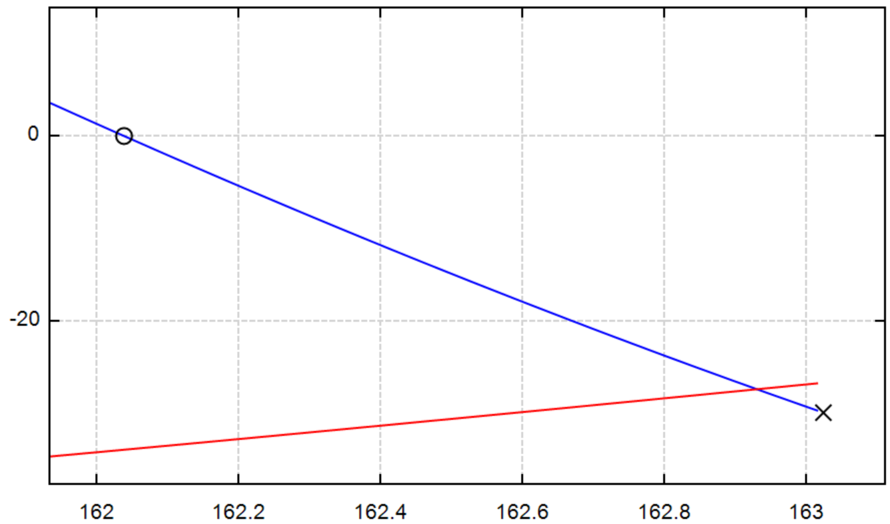
The units for detZ are Mega Newtons.

```
P := { if 0 < ω < ωi/kHz
         eval(detZ(ω kHz) MN^-2)
       else
         ""
       if 0 < ω' < ωi/kHz
         eval(detZ'(ω' kHz) MN^-2 kHz)
       else
         ""
       augment(Ω/kHz, ΔZ, "o")
       augment(Ω'/kHz, ΔZ', "o")
       augment(ωi/kHz, detZ(ωi) MN^-2, "x")
```



P

The issue in the plot is related with the problem that there are not a true NaN in SMath. For example, try to change the value for else in P with zero instead "".



P