appVersion(4) = "1.0.8348.30405"

appVersion(−4) = "1.0.8348.30405"

Define

$$S_{BEAM} := \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

Program 1 : Pairing non-diagonal indices of stiffness matrix

```
Pair_STF(B#) := | nr# := rows(B#) − 1
                | for k ∈ [1..(nr#)]
                |   | Z#_k := stack(B#_k, B#_(k+1))
                | Z#
```

Program 2 : To find the Stiffness Natrix : Calls Program 1
fdn#=1 if foundatio column base is FIXED ", fdn#=0.75 if foundation column base is PINNED

Find_K(L#, I#_UC, I#_LC, I#_beam, H#_UC, H#_LC, fdn#) :=

nr := rows(L#) + 1

for $j \in [1..(nr)]$

$$M\#_j := 4 \cdot E \cdot \begin{cases} \dfrac{I\#_{UC_j}}{H\#_{UC_j}} + \dfrac{I\#_{LC_j} \cdot fdn\#}{H\#_{LC_j}} + \dfrac{I\#_{beam_j}}{L\#_j} & \text{if } j = 1 \\ \dfrac{I\#_{UC_j}}{H\#_{UC_j}} + \dfrac{I\#_{LC_j} \cdot fdn\#}{H\#_{LC_j}} + \dfrac{I\#_{beam_j}}{L\#_{j-1}} + \dfrac{I\#_{beam_j}}{L\#_j} & \text{if } 1 < j < nr \\ \dfrac{I\#_{UC_j}}{H\#_{UC_j}} + \dfrac{I\#_{LC_j} \cdot fdn\#}{H\#_{LC_j}} + \dfrac{I\#_{beam_{j-1}}}{L\#_{j-1}} & \text{otherwise} \end{cases}$$

(STF := Diag(M#)) = "Create diagonal matrix"

C# := [1..nr]

(D# := Pair_STF(C#)) = "Indices of non-diagonal elements"

Clear(j)

for $j \in [1..rows(L\#)]$

$\left(M2\#_j := 2 \cdot E \cdot \dfrac{I\#_{beam_j}}{L\#_j}\right)$ = "Non-diagonal elements"

$\left(r_j := D\#_{j_1}\right)$ = "Row index of non-diagonal elements"

$\left(c_j := D\#_{j_2}\right)$ = "Column index of non-diagonal elements"

$\left(STF_{r_j, c_j} := M2\#_j\right)$ = "Assign appropriate M2# to STF"

$\left(STF_{c_j, r_j} := M2\#_j\right)$ = "Assign appropriate M2# to STF"

STF

PROGRAM 3 : To find joint moments & ST

```
Find_ST#(Z, aa(▪), kk) := | j := [1..rows(Z)]
                          | jm_j := aa(Z_j)
                          | st_j := −kk^(−1) · jm_j
                          | st
```

Program 4 : Any function can be passed to this program

```
Arrange(Z#, a(▪)) := | j := [1..rows(Z#)]
                     | D_j := a(Z#_j)
```

Program 5 : Find lower column moments. Valid even if foundation columns have equal or diff heights

```
Find_MLC(st, d) := | for j ∈ [1..rows(st)]
                   |   | a# := st_j
                   |   | A#_j := vectorize(a# · d)
                   | A#
```

Program 6 : Calls Program 4

```
Pair(B#) := | nrr := vectorize(rows(B#))_1 − 1
            | for k ∈ [1..(nrr)]
            |   | Z#_k := [ YY_k ; YY_(k+1) ]
            | FF(YY) := Z#
            | Arrange(B#, FF(YY))
```

```
Program 7 : Main program to find support moments, upper & lower column moments
Calls Programs 3, 4, 5 & 6
```

```
NEW_BMM(ff#, K#, h#_UC, h#_LC, I_b, I_C, I_L, fdn#, L#) :=
    for j ∈ [1..cols(ff#)]
        fem#_j := 1/12 · (ff#_1j · L#^2)→
    nrr := rows(fem#)→_1 + 1
    for j ∈ [1..nrr]
        M#_j := { −Y_j              if  j = 1
                { Y_(j−1) − Y_j     if  1 < j < nrr
                { Y_(nrr−1)         otherwise
    F(Y) := M#
    st := Find_ST#(fem#, F(Y), K#)
    stt := Pair(st)
    for j ∈ [1..cols(ff#)]
        for k ∈ [1..rows(L)]
            M_sup_jk := ( (stt_j_k · E · I_b_k) / L_k )^T · S_BEAM + [ −fem#_j_k ; fem#_j_k ]^T
    Col_bott# := (I_L / h#_LC)→ · 4 · E · fdn#
    M_LCC := Find_MLC(st, Col_bott#)
    Col_upp# := (I_C / h#_UC)→ · 4 · E
    M_UCC := Find_MLC(st, Col_upp#)
    augment(M_sup, M_UCC, M_LCC)
```

Define

$kNm := kN\ m$

Assume

$E := 1\ MPa$

Assume Young's Modulus for RC: This is not really required, but used only for consistent units in intermediate calculations. We can use any numerical value. **Ex E = 1 MPa**

Example 13.1 Simplified analysis of concrete framed building—vertical load

(a) Specification

The cross-section of a reinforced concrete building is shown in Fig. 13.5(a). The frames are at 4.5 m centres, the length of the building is 36 m and the column bases are fixed. Preliminary sections for the beams and columns are shown in Fig. 13.5(b). The floor and roof slabs are designed to span one way between the frames. Longitudinal beams are provided between external columns of the roof and floor levels only.:

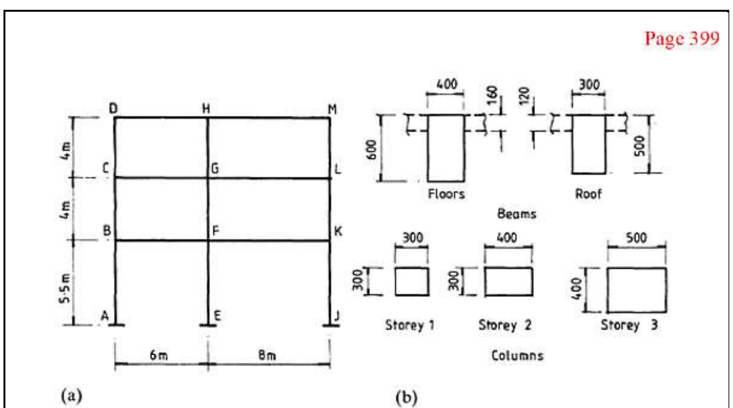

Fig. 13.5 (a) Cross-section; (b) assumed member sections.

Member Stiffness (ST)

cond = 0.75 when Pinned at base.

cond = 1 when fixed at both ends.

This program can handle both cases as shown below.

$$cond := \begin{bmatrix} \text{"Pinned"} & 0.75 \\ \text{"Fixed"} & 1 \end{bmatrix}$$

○ Pinned ◉ Fixed

FDN

DATA

3 upper columns, 3 lower columns and 2 beams

$FDN = 1$

Column

$b_{UC} := \begin{bmatrix} 300 \\ 300 \\ 300 \end{bmatrix} mm$

$h_{UC} := \begin{bmatrix} 400 \\ 400 \\ 400 \end{bmatrix} mm$

$b_{LC} := \begin{bmatrix} 400 \\ 400 \\ 400 \end{bmatrix} mm$

$h_{LC} := \begin{bmatrix} 500 \\ 500 \\ 500 \end{bmatrix} mm$

$H_{UC} := \begin{bmatrix} 4.0 \\ 4.0 \\ 4.0 \end{bmatrix} m$

$H_{LC} := \begin{bmatrix} 5.5 \\ 5.5 \\ 5.5 \end{bmatrix} m$

Beam

$b_{beam} := \begin{bmatrix} 400 \\ 400 \end{bmatrix} mm$

$h_{beam} := \begin{bmatrix} 600 \\ 600 \end{bmatrix} mm$

$L := \begin{bmatrix} 6 \\ 8 \end{bmatrix} m$

Define

$kNm := kN\ m$

Assume

$E := 1\ MPa$

Flanged Beams can be effective only in case of sagging as the concrete is able to take compression. In limit state design we neglect the tensile strength of concrete. As a result, at supports, continuous **'T' beams should be designed as a rectangular beam** ***(with width equal to the web width)***.

Defie

$Line_{zero} := \begin{bmatrix} 0 & 0 \\ \sum\left(\frac{L}{m}\right) & 0 \end{bmatrix}$

Define

$LL_{[1..rows(L)]} := 0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Moment of Inertia

Upper Columna

$I_{UC} := \overrightarrow{\frac{1}{12} \cdot b_{UC} \cdot \left(h_{UC}\right)^3} = \begin{bmatrix} 1.6 \cdot 10^9 \\ 1.6 \cdot 10^9 \\ 1.6 \cdot 10^9 \end{bmatrix} mm^4$

Lower Columna

$I_{LC} := \overrightarrow{\frac{1}{12} \cdot b_{LC} \cdot \left(h_{LC}\right)^3} = \begin{bmatrix} 4.1667 \cdot 10^9 \\ 4.1667 \cdot 10^9 \\ 4.1667 \cdot 10^9 \end{bmatrix} mm^4$

Beams

$I_{beam} := \overrightarrow{\frac{1}{12} \cdot b_{beam} \cdot \left(h_{beam}\right)^3} = \begin{bmatrix} 7.2 \cdot 10^9 \\ 7.2 \cdot 10^9 \end{bmatrix} mm^4$

Foundation Columns Fixed at Base

$FDN = 1$

Call Program 2 to find the Stiffness Matrix of the Sustitute Frame

$K1 := Find\_K\left(L, I_{UC}, I_{LC}, I_{beam}, H_{UC}, H_{LC}, FDN\right) = \begin{bmatrix} 9.4303 & 2.4 & 0 \\ 2.4 & 13.0303 & 1.8 \\ 0 & 1.8 & 8.2303 \end{bmatrix} kNm$

Loadind Details

Frames are spaced at 4.5 m

$S_{frame} := 4.5\ m$

$L_{building} := 36\ m$

$\gamma_{cu} := 24\ \frac{kN}{m^3}$

Imposed and Dead loads

$q_{k\_floor} := 3.0\ \frac{kN}{m^2}$

$g_{k\_floor} := 6.2\ \frac{kN}{m^2}$

Design Loads on Beams

Imposed $Q_{k\_floor} := q_{k\_floor} \cdot S_{frame} = 13.5\ \frac{kN}{m}$

Dead $G_{k\_floor} := g_{k\_floor} \cdot S_{frame} = 27.9\ \frac{kN}{m}$

Max Design Load

$F1 := 1.4 \cdot G_{k\_floor} + 1.6 \cdot Q_{k\_floor} = 60.66\ \frac{kN}{m}$

Min Design Load

$F2 := 1.0 \cdot G_{k\_floor} = 27.9\ \frac{kN}{m}$

Define Load Patterns for All 3 Cases

Case I **Case II** **Case III**

$FF := \left[\begin{bmatrix} F1 \\ F1 \end{bmatrix} \begin{bmatrix} F1 \\ F2 \end{bmatrix} \begin{bmatrix} F2 \\ F1 \end{bmatrix}\right] = \left[\begin{bmatrix} 60.66 \\ 60.66 \end{bmatrix} \begin{bmatrix} 60.66 \\ 27.9 \end{bmatrix} \begin{bmatrix} 27.9 \\ 60.66 \end{bmatrix}\right] \frac{kN}{m}$

3 load cases **have been considered in this example**

Call Program 7 to find support moments, upper & lower column moments

$FDN = 1$

$M_{all} := NEW\_BMM\left(FF, K1, H_{UC}, H_{LC}, I_{beam}, I_{UC}, I_{LC}, FDN, L\right)$

Foundation Columns Fixed at Base

Upper Col / Lower Col

$$M_{all} = \begin{bmatrix} \begin{bmatrix} -73.1 & 286.1 \end{bmatrix} & \begin{bmatrix} -350 & 196 \end{bmatrix} & \begin{bmatrix} 25.3 \\ 22.1 \\ -67.7 \end{bmatrix} & \begin{bmatrix} 47.8 \\ 41.8 \\ -128.3 \end{bmatrix} \\ \begin{bmatrix} -94 & 211.9 \end{bmatrix} & \begin{bmatrix} -193.9 & 79.8 \end{bmatrix} & \begin{bmatrix} 32.5 \\ -6.2 \\ -27.6 \end{bmatrix} & \begin{bmatrix} 61.5 \\ -11.8 \\ -52.2 \end{bmatrix} \\ \begin{bmatrix} -12.8 & 205.8 \end{bmatrix} & \begin{bmatrix} -317.1 & 206.4 \end{bmatrix} & \begin{bmatrix} 4.4 \\ 38.5 \\ -71.3 \end{bmatrix} & \begin{bmatrix} 8.3 \\ 72.9 \\ -135.1 \end{bmatrix} \end{bmatrix} kNm$$

Case I
Case II
Case III

Beam1 **Beam 2**

Get Support Moments

L R L R

$$M_{sup} := M_{all\,[1..rows(M_{all})][1..rows(L)]} = \begin{bmatrix} \begin{bmatrix} -73.1 & 286.1 \end{bmatrix} & \begin{bmatrix} -350 & 196 \end{bmatrix} \\ \begin{bmatrix} -94 & 211.9 \end{bmatrix} & \begin{bmatrix} -193.9 & 79.8 \end{bmatrix} \\ \begin{bmatrix} -12.8 & 205.8 \end{bmatrix} & \begin{bmatrix} -317.1 & 206.4 \end{bmatrix} \end{bmatrix} kNm$$

Case I
Case II
Case III

Get Column Moments

$$M_{col} := M_{all\,[1..rows(M_{all})][(cols(M_{all})-1)..cols(M_{all})]} = \begin{bmatrix} \begin{bmatrix} 25.3 \\ 22.1 \\ -67.7 \end{bmatrix} & \begin{bmatrix} 47.8 \\ 41.8 \\ -128.3 \end{bmatrix} \\ \begin{bmatrix} 32.5 \\ -6.2 \\ -27.6 \end{bmatrix} & \begin{bmatrix} 61.5 \\ -11.8 \\ -52.2 \end{bmatrix} \\ \begin{bmatrix} 4.4 \\ 38.5 \\ -71.3 \end{bmatrix} & \begin{bmatrix} 8.3 \\ 72.9 \\ -135.1 \end{bmatrix} \end{bmatrix} kNm$$

Case I
Case II
Case III

Compare with results using Moment Distribution Program - With 7 iterations

METHOD 1 : With Far End Column Moments - With 7 Iterations
Example 13.1 : Page 398
Reinforced Concrete - Design Theory and Examples
Second Edition - T.J. Macginley & B.S. Choo

	AB	BA	BF	BC	CB	EF	FE	FB	FG	FK	GF	JK	KJ	KF	KL	LK
DF		0.3	0.5	0.2			0.2	0.4	0.1	0.3			0.4	0.4	0.2	
FEM			-182					182		-323.5				323.5		
Bal		58.5	92.6	30.9			32.9	52.1	17.4	39.1			-119.1	-141.5	-62.9	
CO	29.2		26.1		15.4	16.5		46.3		-70.8	8.7	-59.6		19.6		-31.4
Bal		-8.4	-13.3	-4.4			5.7	9	3	6.8			-7.2	-8.6	-3.8	
CO	-4.2		4.5		-2.2	2.8		-6.6		-4.3	1.5	-3.6		3.4		-1.9
Bal		-1.4	-2.3	-0.8			2.5	4	1.3	3			-1.2	-1.5	-0.7	
CO	-0.7		2		-0.4	1.3		-1.1		-0.7	0.7	-0.6		1.5		-0.3
Bal		-0.6	-1	-0.3			0.4	0.7	0.2	0.5			-0.6	-0.7	-0.3	
CO	-0.3		0.3		-0.2	0.2		-0.5		-0.3	0.1	-0.3		0.3		-0.1
Bal		-0.1	-0.2	-0.1			0.2	0.3	0.1	0.2			-0.1	-0.1	-0.1	
CO	-0.1		0.2		0	0.1		-0.1		-0.1	0.1	0		0.1		0
Bal		0	-0.1	0			0	0.1	0	0			0	-0.1	0	
CO	0		0		0	0		0		0	0	0		0		0
Bal		0	0	0			0	0	0	0			0	0	0	
Final BM	23.9	47.8	-73.1	25.3	12.6	20.9	41.8	286.1	22.1	-350	11	-64.1	-128.3	196	-67.7	-33.9