isol

ODE

$$eq := \begin{cases} u'' - L\cdot\theta''\cdot\sin(\theta) - L\cdot\theta'^{2}\cdot\cos(\theta) + \omega^{2}\cdot u - g \\ L\cdot\theta'' - u''\cdot\sin(\theta) + g\cdot\sin(\theta) \end{cases}$$

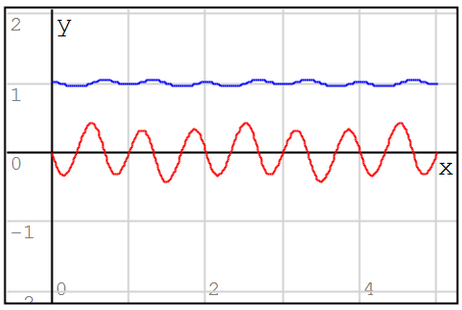
$$\theta''_{eq} := isol\left(eq_2, \theta''\right) \qquad u''_{eq} := isol\left(eq_1 \middle| \theta'' = \theta''_{eq}, u''\right)$$

$$D(t, \varphi) := \left| \begin{array}{l} [u\ u'\ \theta\ \theta'] := [\varphi_1\ \varphi_2\ \varphi_3\ \varphi_4] \\ u'' := u''_{eq} \\ [u'\ u''\ \theta'\ \theta''_{eq}]^{T} \end{array} \right. = \begin{bmatrix} \varphi_2 \\ -\dfrac{g\cdot\left(-1+\sin(\varphi_3)^2\right) - L\cdot\varphi_4^{\,2}\cdot\cos(\varphi_3) + \omega^2\cdot\varphi_1}{1-\sin(\varphi_3)^2} \\ \varphi_4 \\ -\dfrac{\sin(\varphi_3)\cdot\left(-L\cdot\varphi_4^{\,2}\cdot\cos(\varphi_3) + \omega^2\cdot\varphi_1\right)}{\left(1-\sin(\varphi_3)^2\right)\cdot L} \end{bmatrix}$$

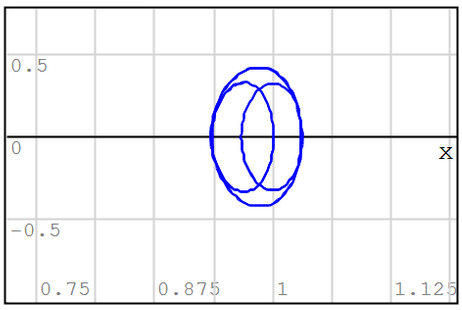
$L := 0.5$ $k := 800$ $m := 80$ $g := 9.81$ $\omega := \sqrt{\frac{k}{m}}$ $IC := \text{stack}(1, 0, 30\ \text{deg}, 0)$

$te := 5$ $N := 500$ $S := \text{Rkadapt}(IC, 0, te, N, D)$

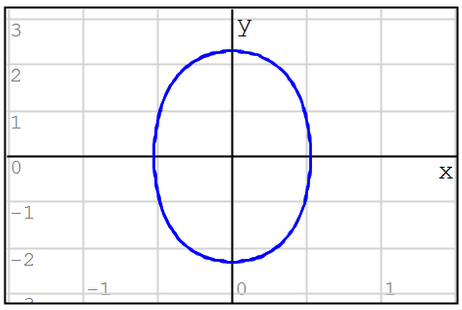
u(t) & θ(t)



u'' vs u'



θ'' vs θ'



$\begin{cases} \text{augment}(\text{col}(S, 1), \text{col}(S, 2)) \\ \text{augment}(\text{col}(S, 1), \text{col}(S, 3)) \end{cases}$ $\text{augment}(\text{col}(S, 2), \text{col}(S, 3))$ $\text{augment}(\text{col}(S, 4), \text{col}(S, 5))$

Utils

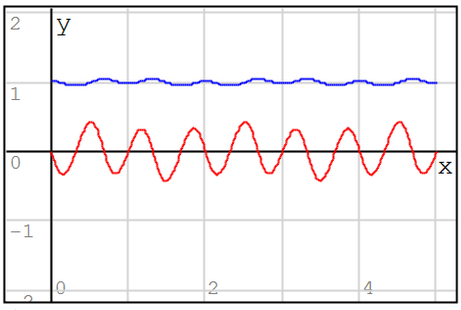
RKA

Vibrating Pendulum

$$DE := \begin{cases} u''(t) - L\cdot\theta''(t)\cdot\sin(\theta(t)) - L\cdot\theta'(t)^{2}\cdot\cos(\theta(t)) + \omega^{2}\cdot u(t) - g = 0 \\ L\cdot\theta''(t) - u''(t)\cdot\sin(\theta(t)) + g\cdot\sin(\theta(t)) = 0 \\ u(0) = 1 \\ \theta(0) = 30\ \text{deg} \\ u'(0) = 0 \\ \theta'(0) = 0 \end{cases}$$

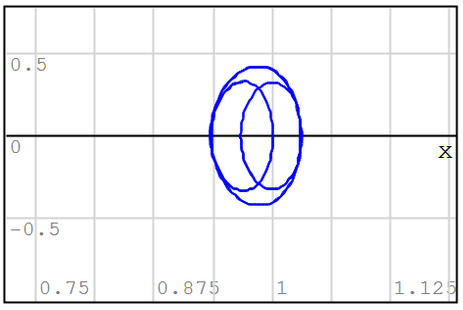
$$S := RKA\left(DE, \begin{pmatrix} u(t) \\ \theta(t) \end{pmatrix}, \begin{bmatrix} -1 \\ -10 \end{bmatrix}, te, N\right)$$

u(t) & θ(t)



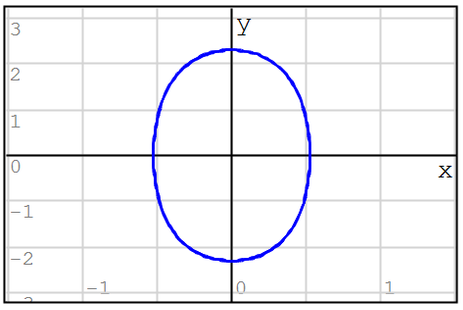
$$\begin{pmatrix} \text{augment}(\text{col}(S, 1), \text{col}(S, 2)) \\ \text{augment}(\text{col}(S, 1), \text{col}(S, 3)) \end{pmatrix}$$

u'' vs u'



$$\text{augment}(\text{col}(S, 2), \text{col}(S, 3))$$

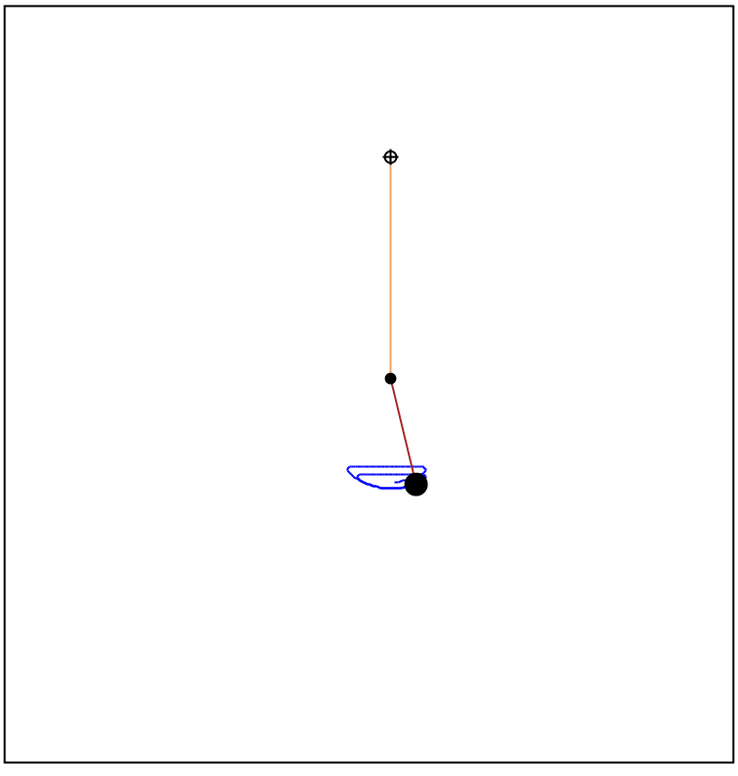
θ'' vs θ'



$$\text{augment}(\text{col}(S, 4), \text{col}(S, 5))$$

$$\tau := [1..N] \qquad u(\tau) := \text{col}(S, 2)_{\tau+1} \qquad \theta(\tau) := \text{col}(S, 3)_{\tau+1}$$

$$x(\tau) := L \cdot \sin(\theta(\tau)) \qquad y(\tau) := u(\tau) + L \cdot \cos(\theta(\tau))$$



Alvaro